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Electrical Engineering Testing

The Directly-Useful **D.U.** Technical Series

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Electrical Engineering Testing

A PRACTICAL WORK

ON CONTINUOUS AND ALTERNATING CURRENTS
FOR SECOND AND THIRD YEAR STUDENTS AND ENGINEERS

BY

G. D. ASPINALL PARR

M.Sc.; M.Inst.E.E., A.C.G.I.

Diplomée in Physics and Electrical Engineering
of the Central Technical College, City and Guilds of London,
Formerly
Head of the Electrical Engineering Department, The University, Leeds,
Chairman of the North Midland Section of the Inst.E.E.,
Examiner in Electrical Engineering in the Universities of Leeds and New Zealand,
Technical Adviser to the Mexican Government,
Associate Member of the Institute of Mechanical Engineers.

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EDITORIAL NOTE

The Directly-Useful Technical Series requires a few words by way of introduction. Technical books of the past have arranged themselves largely under two sections: the Theoretical and the Practical. Theoretical books have been written more for the training of college students than for the supply of information to men in practice, and have been greatly filled with descriptions of an academic character. Practical books have often sought the other extreme, omitting the scientific basis upon which all good practice is built, whether discernible or not. The present series is intended to occupy a midway position. The information, the problems, and the exercises are to be of a directly-useful character, but must at the same time be wedded to that proper amount of scientific explanation which alone will satisfy the inquiring mind. We shall thus appeal to all technical people throughout the land, either students or those in actual practice.

PREFACE

This work is intended to form a systematic course of instruction in the very extensive field of testing connected with pure Electrical Engineering. While much has been written, from time to time, about the more elementary branch of testing in Electrical Physics, relating in a measure to Electrical Engineering, I believe that, so far, no extensive attempt has been made to treat the more advanced and practical portions of the subject in that systematic manner which it requires.

I therefore venture to think that the present work, which not only embodies much of, if not all, the experimental work that it is usually possible to do at most colleges, but also many tests on heavier electrical machinery, together with a highly descriptive course on jointing Electric Light cables, should be eminently suitable for constituting the electrical laboratory practice in the second and third years of a complete course of instruction in Electrical Engineering, and in addition should be of considerable service to the electrical engineer in electrical works and central stations. As far as possible the tests have been arranged in the order in which they may be worked, when used as a course for students, but there are exceptions to this rule, owing to the advisability of keeping tests of a similar nature together. I have endeavoured to make the tests as complete and descriptive as possible, and applicable in the case of any college and testing room. Each test comprises—an *Introduction* giving the chief features, advantages, and disadvantages of the test, condensed as much as possible; the *Apparatus* necessary; the *Observations* to be carried out, in other words, a complete and carefully arranged digest of the method of carrying out the actual test, with a *Diagram of Apparatus* and connections represented symbolically; a *Tabular Form* indicating the most convenient and proper way of recording the observations; and finally, *Inferences* which can be drawn from the results of the test. These latter if conscien-

tiously worked out are calculated to cause the experimenter to think and reason for himself.

Following the series of tests is an Appendix, containing the Algebraical solutions of the various formulæ met with in the tests, and these the student is strongly urged not to refer to until he has tried, by all the means in his power, to solve the inference for himself.

The Appendix also contains complete descriptions and illustrations of a large amount of the apparatus which may be employed in carrying out the tests, a considerable proportion of it being such as will be found in almost every college and testing-room.

Useful constants and tables which are frequently needed in electrical engineering tests are added at the end of the book. A good deal of the apparatus illustrated has been constructed by the mechanical assistants, Messrs. John Watkinson and Herbert Addy, of the Electrical Engineering Department of the University, Leeds.

In conclusion, I wish to express my sincere thanks to my valued friend, Mr. Charles Mercer, M.A., for the very considerable amount of trouble he has taken in producing the photographs from which many of the illustrations are obtained; to Dr. John Henderson, for permission to use the tables of squares and reciprocals of numbers; to Messrs. Kelvin & James White, for permission to use the tables of doubled square roots; to His Majesty's Stationery Office, for allowing me to use the tables of logarithms and anti-logarithms; to Messrs. Longmans, Green & Co., for their kindness in lending eight illustrations and a little printed matter from *Practical Electrical Testing;* to Mr. Herbert Addy, for the trouble he has taken in making the drawings from which the illustrations of joints, made in electric light cables, are taken, and also for reading through the proofs simultaneously with myself; and further to Messrs. Nalder Bros. & Co., Kelvin & James White, Siemens Bros. & Co., Crompton & Co., and Evershed & Vignoles, for their kindness in lending me the blocks of some of the illustrations of the very excellent apparatus and appliances made by them.

G. D. A. P.

The University, Leeds.

PREFACE TO THE FOURTH EDITION

While opportunity has been taken, in previous editions, to both enlarge and improve the book, the scope of it has undergone very considerable extension and rearrangement in the present edition. Of some 132 extra pages of new matter, no less than 116 represent entirely new tests, including a little additional theoretical explanatory matter to the previously existing tests, while the remainder comprise descriptive matter and tables of useful figures.

Some of the new tests are of a direct current nature and bring the direct current portion of the book more thoroughly up-to-date, but the remainder, forming by far the greater proportion of new tests, relate to alternating currents. This branch of practical work—always more difficult to understand than that of direct currents—has therefore been greatly strengthened by additional matter dealing with modern theory, laboratory and commercial tests supplemented by vector diagrams which enable the phase relations between current and pressure to be more easily understood.

Further, the greater portion of the work has been completely rearranged so as to have all tests of a like nature together, though not necessarily numbered in the order in which they should be taken. The author therefore hopes that this edition will be found to offer many advantages over previous ones, and he will welcome notification of any errors which may have escaped observation before going to press.

I would like to thank my friends who have so kindly helped me to read through the proofs of this edition simultaneously with myself; also Messrs. Evershed & Vignoles, Nalder Bros. & Co., and Elliott Bros. Ltd., for their kindness in lending me additional blocks of illustrations of apparatus; and Messrs. The London Electric Wire Co., Smiths Ltd., for permission to reprint their Tables of Resistance Wires.

G. D. A. P.

January 1922.

CONTENTS

ELECTRICAL ENGINEERING TESTING

Curve Plotting.

Introduction.—The practice of recording the results of any measurements or tests graphically, as well as in tabular form, in cases where this is possible, cannot be too strongly urged, and is a most important, as well as, in many cases, an indispensable operation. More especially is this the case with a large number of physical measurements, and particularly so with a majority of tests in Electrical Engineering. The practice of curve plotting, as these graphical representations may otherwise be termed, presents the following important features—

(*a*) It enables the nature of the variation of one quantity with another to be seen at a glance much more clearly than is possible by aid of a table of results.

(*b*) It enables errors in experimental observations, of which there are sure to be some, to be corrected comparatively easily, which in a majority of cases would be impossible from the table of results.

(*c*) In the case of the calibration of instruments it enables the law of that under test to be readily observed.

(*d*) It has the enormous advantage of enabling any intermediate value between those actually observed and tabulated, to be at once obtained accurately. This, it will readily be conceded, is the most important and valuable feature of all, and the ease, as well as the rapidity with which the operation of obtaining intermediate values can be accomplished, will be dependent on the scales chosen in originally plotting the curve in question.

It may therefore be profitable to indicate the mode of procedure in plotting curves, and with a view to exemplifying it, the results of a particular test are given in Table I., and the corresponding curve or graphical representation in Fig. 1. They relate to the determination of the Brake Horse Power (B.H.P.) of an electromotor and the corresponding value of its efficiency at each load.

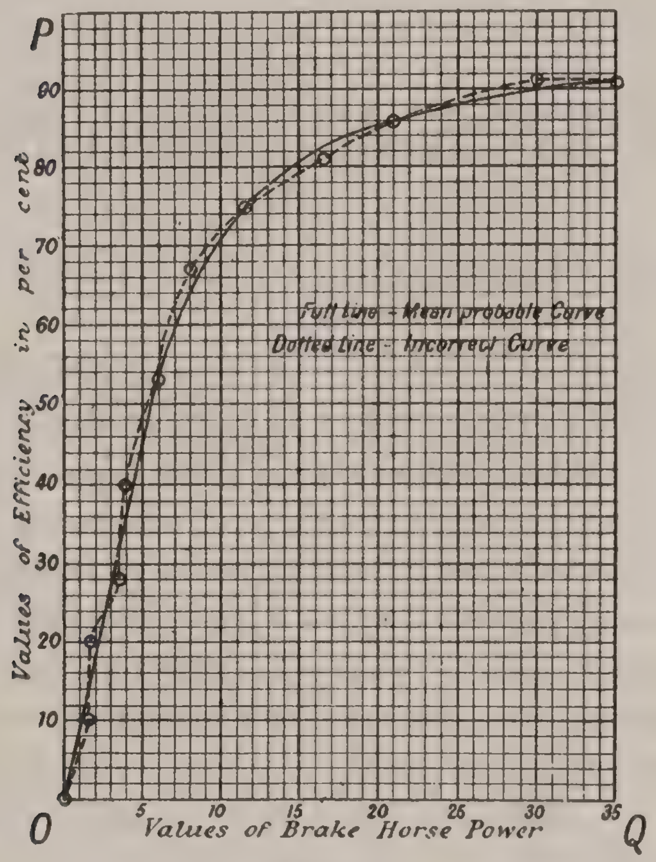


Fig. 1.

TABLE I.

B.H.P.	Efficiency.
1·5	10
1·75	20
3·5	28
4·0	40
6·0	53
8·0	67
11·5	75
16·5	81
21·0	86
30·0	91
35·0	91

In testing work generally at least six or eight different determinations throughout the range should be made, where possible, in order that the curve may be drawn more accurately. In most cases a curve constructed on three or four points only would be practically useless and could not be depended on.

Directions for Plotting.—(1) Assuming that all the results have been worked out numerically and entered up in tabular form, the first thing to note is what two sets of quantities have to be plotted together, and secondly, the largest value of each set, for beyond this the scale need not extend.

(2) The left-hand vertical and bottom horizontal sides OP and OQ respectively of the squared sheet of curve paper are termed the *axes* and are rectangular. They intersect in a point O called

the *origin*. Distances measured vertically are termed *ordinates*, and those horizontally, *abscissæ*.

(3) Carefully note which set of readings have to be plotted on the ordinates and which on the abscissæ, and then choose the scales of the axes OP and OQ such that they are as long as possible and include the maximum values to be plotted. Also, if possible, arrange such that one of the smallest divisions represents a simple *whole number* of one digit. For example, if 33 was the largest number to be plotted and the side of the squared paper contained 100 divisions, let 1 division represent 0·5 only, whence 66 will give the 33; this is far more convenient a scale for future reference in obtaining intermediate values than 1 division representing 0·33 (*i. e.* 99 to give the 33 approx.). While it is a great advantage for the numerical length of the axes to be as large as possible, so as to enable the curve to be drawn larger and more accurately, the length should be decided by considerations of future reference to it for intermediate values as just mentioned.

(4) The axes must be numbered every 10th division, and under no circumstances with the numbers obtained from experiment.

(5) Write along each axis the nature of the quantity plotted on it.

(6) Each point must be plotted by finding the point of intersection of the axes representing the two corresponding quantities under consideration at the moment and a distinctive mark there made.

(7) When all the points are plotted, a *mean curve*, as shown by the *full line*, Fig. 1, must be drawn through as many points as will allow of a *uniform line* being drawn.

Some points are always sure to lie on either side of this mean line and denote experimental errors. The object of the curve is to correct for these.

(8) In some tests, as for example in "characteristic" determinations with direct current generators, it often happens that curves cross one another and lie close together. In such cases they must be drawn *thin* and a different notation for the respective sets of points used, such as that represented in Fig. 2.

All confusion will thus be avoided.

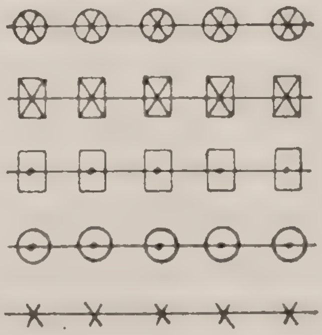


Fig. 2.

Calibration and Standardization of Electrical Measuring Instruments.

General Remarks.—This subject is perhaps one of the most important in connection with electrical testing, and we shall therefore devote some considerable attention to it. It will at once be obvious to any one, without any consideration, that a measuring instrument which is reading incorrectly, or one that has been calibrated so long ago that its present readings may not be true, is a useless instrument, or even worse than this, as one is unconsciously liable to take its reading as correct. The importance of correct reading and accurately calibrated measuring instruments cannot be over-estimated, for on their being so hangs the whole crux of further testing, the *results* of which would otherwise be *quite worthless.* This the author would emphasize most strongly, for it is unfortunately his experience, and no doubt that of many more like him, that the average experimentalist is only too ready to take the scale reading of any instrument as correct without in the least troubling himself as to whether it actually is so or not. This no doubt arises from the little extra trouble required to calibrate such instruments prior to starting some particular test.

Measuring instruments may change their constants and develop errors in their scale readings either from continued *use*, *abuse*, or in *transit* from one place to another, some of course being much more susceptible to alteration than others. Hence in all cases where it is desired to obtain accurate results and do good work, the instruments should be re-calibrated and re-standardized frequently, and a calibration curve drawn whenever possible with the date of the test inserted. At the very least six determinations should be made, wherever possible, but preferably ten or twelve, as it is not possible to draw a reliable calibration curve on less than six points. In all cases it is of the utmost importance to see that the connecting wires or cables do not magnetically affect the instruments, for it must be carefully remembered that a wire carrying a current, no matter whether it is straight or otherwise, acts as a magnet.

Such inductive effects will be minimized by running or twisting the "lead" and "return" together, when the two equal and opposite magnetic effects neutralize. An ordinary flexible twin-lead is non-magnetic externally, but it possesses a very small electrostatic capacity.

Instruments are usually calibrated by comparing their readings with those of very accurately calibrated standard instruments. *Simultaneous* readings must be taken on both to avoid errors due to variation in between. Ammeters are always connected *in series* and voltmeters are always connected *in parallel* with their standards.

In the calibration of voltmeters, the employment of keys in any of the branched or parallel circuits containing voltmeters to be calibrated is usually a source of inconvenience and should be avoided, for a key which places, say, a voltmeter of 1500 Ohms resistance in parallel with a similar instrument already reading, will cause this reading to decrease owing to the alteration of the P.D. at the terminals due to inserting such a low resistance meter, and the consequent reduction in the terminal combined resistance.

(1) Calibration of an Ammeter by comparison with a Standard D'Arsonval Ammeter.

Introduction.—When a standard current measurer, such as a Kelvin *standard balance* or a *potentiometer set,* is not available for comparing the ammeter to be tested with, the following method of calibration may conveniently be employed. It consists in using a good reflecting D'Arsonval galvanometer in conjunction with a low resistance composed of platinoid or other suitable material having a small temperature co-efficient of resistance, which should preferably be known. The resistance of the D'Arsonval galvanometer may conveniently be something like 2000 to 4000 times that of the low resistance to which it is shunted. The instrument, its scale, and the resistance should be permanently fixed and standardized carefully by means of a copper or silver voltameter. Then if the current which produces a full scale deflection, with a certain known resistance in series with the galvanometer, is accurately known, the current producing any

other deflection with the same resistances will be very approximately in direct proportion and therefore at once known. Some slight corrections might be necessary for great accuracy when subsequently using these particular *constants*, due to alteration of resistance through change of temperature and to the deviation of the D'Arsonval readings from the direct proportional law, for which correction see Appendix, p. 490.

Apparatus.—Secondary battery B capable of giving the maximum current required; reflecting D'Arsonval galvanometer G (p. 569); switch S; key K; carbon rheostat R (p. 597); resistance box (r); ammeter A to be calibrated; low resistance PP either of the form shown (p. 605), or simply a sheet of the metal.

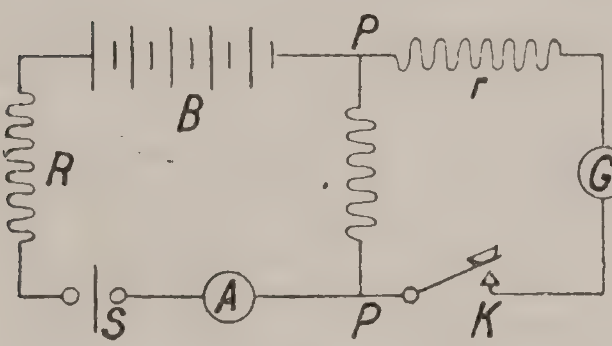


Fig. 3.

N.B.—It is assumed that the galvanometer and low resistance in combination has been carefully standardized previously and now constitutes the standard D'Arsonval ammeter.

Observations.—(1) Connect up as in Fig. 3, and adjust the pointer of A to zero and the spot of light from G to the left-hand end of the scale used as a temporary or false zero in this test.

(2) Insert the proper resistance in (r) as given from the *constants* of standardization for the maximum current to be measured and corrected for the temperature of the room at the time of the test.

(3) With R large, close S and adjust the current through the ammeter to be calibrated to about $\frac{1}{10}$th of the maximum scale reading by means of R. Then note simultaneously its reading A and the deflection d on G when K is pressed.

(4) Repeat 3 for about ten different readings on A *rising* by about equal increments to the maximum with no decreasings of current.

(5) Repeat 3 and 4 for a similar *descending* set of the same readings on G, noting the corresponding ones on A, avoiding all increasings of current, and tabulate your results as follows—

NAME . . . DATE OF TEST . . .
Ammeter tested : No. . . . Type . . . Resistance (r)= . . . Ohms.
Temperature of Room= . . . °C. Resistance of (G)= . . . Ohms at . . . °C.

Reading on Ammeter tested.		Deflection on D'Arsonval. d.	Corrected Readings of D'Arsonval. D.	True Current *i. e.* (D) reduced (a) Amps.	% Error of Ammeter tested.
Ascending A.	Descending A.				

(6) Plot curves having values of true current (a) as abscissæ and A as ordinates.

Inferences.—Enumerate any sources of error in ammeters generally. What can you infer from your experimental results? Why should the current be so carefully *increased only* in 4 above and *decreased only* in 5 above?

(2) Calibration of an Ammeter by comparison with a Kelvin Composite Balance used as a Centi-ampere Meter.

Introduction.—The following is a convenient and ready means of calibrating any ammeter reading up to 1 ampere, employing a Kelvin composite balance used in the manner mentioned, as a standard for comparison. A complete description of the construction and manipulation of the instrument will be found on p. 554, to which a reference should be made and the constants obtained therefrom.

Apparatus.—Kelvin composite balance $K.B.$ (p. 554); ammeter A to be tested; switch S; adjustable resistance R (p. 600, *et seq.*); source of current C at a P.D. of from 40 to 60 volts.

Observations.—(1) Connect up as in Fig. 4, adjusting both instruments carefully to zero. Make quite certain that the connections are as indicated.

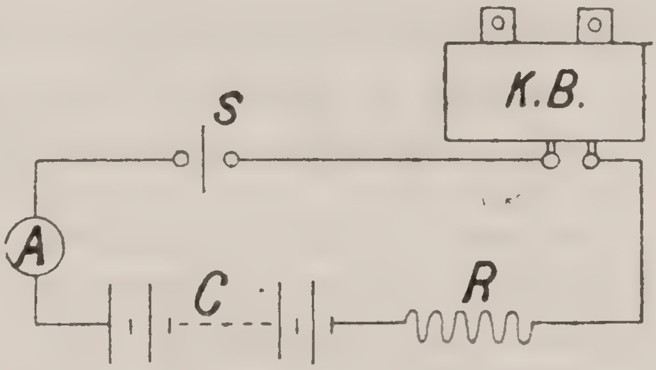


FIG. 4.

(2) Turn the switch in front of the balance to "volts" so as to place the fixed and movable fine wire coils in series with each

other and with the circuit. Now adjust the balance and its sensibility by employing the proper weights as given in the table of constants (p. 556), so that the maximum current to be measured on A would give a reading on $K.B.$ as nearly right across the scale as possible.

(3) With R as large as possible close S and obtain about $\frac{1}{10}$th of the maximum scale reading on A by varying R. Note this and simultaneously the corresponding position (d) of the slider of $K.B.$

(4) Repeat 3 for about ten different values of current on A (by altering R) *rising* by about equal increments to the maximum.

(5) Repeat obs. 4 for a similar set of *descending* values of current, and tabulate your results as follows—

NAME . . . DATE . . .
Composite Balance used: No. . . . Constants=. . . Ammeter tested. . .

Slider Reading (d).	True Amps. $K.d.$	Reading on A.		% Error of A.	Mean % Error.
		Ascending.	Descending.		

(6) Plot a calibration curve for the ammeter tested having readings on A as ordinates and "True Currents" as abscissæ.

Inferences.—What sources of error are ammeters in general liable to? Can anything in particular be inferred from your experimental results?

(3) Calibration of an Ammeter by comparison with a Kelvin Composite Balance used as a Hekto-ampere Meter.

Introduction.—The Kelvin composite balance can be used as a standard ammeter for the measurement of direct currents up to about 600 amperes, and hence any other ammeter can be readily calibrated by comparison with it. A description of the construction of the balance will be found on p. 554, together with the method of using it to measure heavy currents. In this connection it will be seen that the current to be measured passes through the thick fixed wire coils only, which act on the movable

coils of fine wire carrying a small auxiliary current from preferably an independent source of E.M.F. The accuracy therefore of the calibration will depend on the accuracy with which the current through the moving coils is measured, and this is a disadvantage in the use of the composite balance for current measurement.

Apparatus.—Kelvin composite balance *K.B.*; ammeter *A* to be tested; low reading accurately calibrated ammeter (*a*); rheostats *R* (p. 597) and *r* (p. 600); switches S_1 and S_2; battery *C* capable of giving the current corresponding to the highest scale reading on *A*.

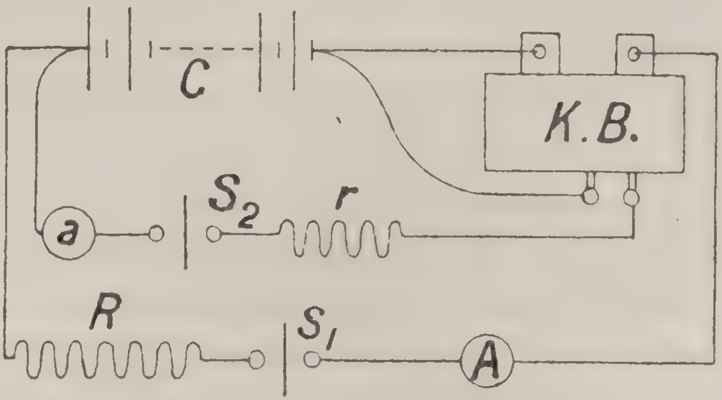


Fig. 5.

Note.—If a second battery, such as six small cells, is available the auxiliary current circuit may preferably be fed from it, for then this current will remain unaffected when the main current through *A* is altered.

Observations.—(1) Connect up as in Fig. 5 if only one battery is used, and adjust the pointers of *A* and *a* to zero, and also that of *K.B.* in the manner mentioned on p. 556. Make quite certain that the connections are as indicated in the diagram.

(2) Turn the switch on the balance (in front) to "*Watts*" so as to put the fine wire movable coils only in connection with the small terminals, and therefore in series with the auxiliary circuit.

(3) Adjust the balance and its sensibility by employing the proper weights as given in the table of constants (p. 557) so that the maximum current to be measured on *A* would produce as nearly as possible a full scale reading on *K.B.*

(4) With *r* at its maximum close S_2 and adjust the current through the fine wire movable coil to its proper value, as given with the *constant*, by varying *r*. *Always make quite sure that it has this value before taking each reading on K.B.*

(5) With *R* large, close S_1 and obtain about $\frac{1}{10}$th of the maximum scale reading on *A* by varying *R*. Note simultaneously the reading on *A* and the position (*d*) of the slider of *K.B.*

(6) Repeat 5 for some ten different values of current on A (by altering R) *rising* by about equal increments to the maximum.

(7) Repeat obs. 6 for a similar set of *decreasing* currents and tabulate as follows—

NAME . . . DATE . . .

Composite Balance: No. . . . Constants used . . . Current in Moving Coils = . . . Amps.
Ammeter tested . . . Type . . . No. . . . Range . . .

Slider Reading (d).	True Amps. $K.d.$	Reading on A.		% Error of A.	Mean % Error.
		Ascending.	Descending.		

(8) Plot "calibration" curves for the ammeter tested having readings on A as ordinates and true currents as abscissæ.

Inferences.—What sources of error are ammeters in general liable to? Can you infer anything in particular from your experimental results?

(4) Calibration of an Ammeter by comparison with a Kelvin Centi-ampere Balance.

Introduction.—Any ammeter reading up to 1 ampere can be readily calibrated by comparison with a Kelvin standard centi-ampere balance. A full description of this instrument, together with the table of constants, will be found on p. 546, and the method of using it is precisely the same as that of the composite balance used as a centi-ampere meter, except that there is no switch at all on the balance in question. The operator should refer to the Appendix (p. 546) for details in connection with the balance.

Apparatus.—This, with the exception of the balance, is precisely the same as is required for the corresponding calibration by the composite balance.

Observations.—These, together with the diagram of connections and the inferences, are precisely the same as for the test on p. 7, and will not therefore be repeated here. The operator must refer to the similar test using the composite balance.

(5) Calibration of a Direct Current Ammeter. (Crompton Potentiometer Method.)

Introduction.—This method is a very convenient and accurate one for calibrating ammeters, and in it the measurements are referred to and obtained in terms of a standard Clark cell and standard resistance. The principle of the method is a direct application of Ohm's Law, and consists in measuring the fall of potential down an accurately known standard low resistance connected up in series with the circuit through which the current to be measured is passing. This fall of potential is measured in terms of the E.M.F. of the Clark's cell through the medium of the potentiometer, employing the principle of the Clark-Poggendorff method for comparing two or more E.M.F.'s. A detailed description of this will be found in a separate work by the author on *Practical Electrical Testing* for 1st and 2nd year students and others. The Crompton potentiometer is a specially arranged form of comparing instrument by means of which the calibration can be easily and quickly carried out. Before proceeding further the operator should refer to a detailed description of this piece of apparatus which will be found in the Appendix (p. 510) together with the method of using it. The present method possesses the all-important advantages that the measurements are all in terms of the Official Board of Trade standard—the Clark cell—that their accuracy is great, and without any very special means this can be obtained to at least 1 in 1000, and that the range is almost illimitable from 0 continuously up to maximums commonly met with in practice. The accuracy of the results in the present method is more particularly dependent on that with which the standard low resistance is known. The value of this must be such that the fall of potential down it due to the maximum current to be measured is not greater than 1·5 volts, while at the same time the carrying capacity must be such as to allow it to pass this current without sensible heating, which would thereby alter its resistance.

Apparatus.—Crompton potentiometer P (p. 510); secondary battery B capable of easily giving the maximum current required for a full scale reading on the ammeter A to be calibrated;

switch S; one secondary cell (b) for the "working cell" of the potentiometer; accurately known standard low resistance R (p. 605); sensitive D'Arsonval or moving coil galvanometer (g) (p. 569); standard Clark cell E; carbon rheostat (rh) (p. 597).

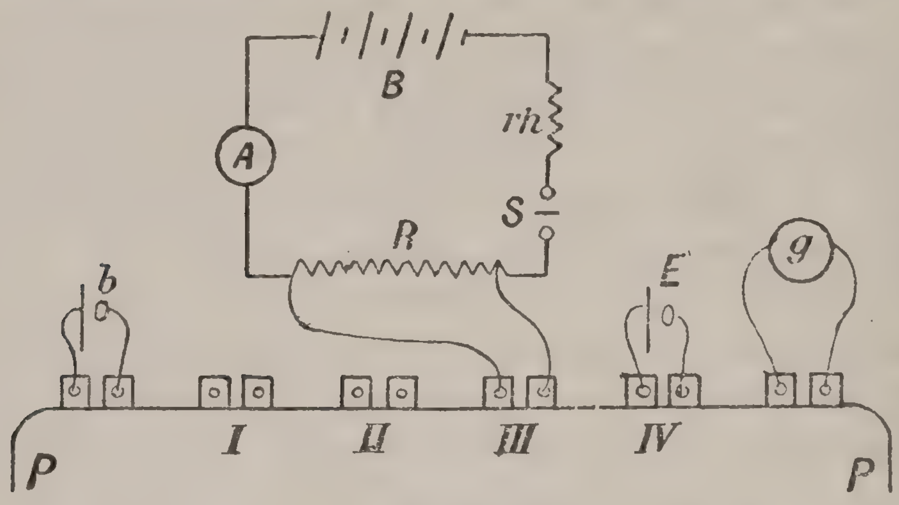


Fig. 6.

Observations.—(1) First place the levers of G and E (Fig. 208) on studs 14, and that of H on studs 1 or 2 for precaution. Now connect up as in Fig. 6, in which only the row of terminals on the potentiometer P is shown symbolically for brevity.

(2) Adjust the galvanomter g approximately, and A carefully, to zero, levelling them if necessary. See that a suitable low resistance standard R is employed, and one that will give a fall of potential at its ends not exceeding 1·5 volts ($= A \times R$) for the maximum current to be used (*vide* p. 13).

(3) "Set the potentiometer" by the standard cell in the way described on p. 514, the contact lever H above referred to being on studs *IV*, thus inserting E (Fig. 6) in the circuit of g, which must be done so that its E.M.F. opposes that of b. Now close S and adjust rh so as to obtain about $\frac{1}{10}$th of the full scale reading on A.

(4) With the positions of the resistances G and G_1 (Figs. 207 and 208) as found in 3, *unaltered*, turn the lever of H to studs *III* so as to throw into circuit with g the fall of potential down R which as seen is across terminals *III*. Now adjust the lever of the resistances E (Fig. 207) and the sliding key C so that no deflection occurs on g on pressing this latter.

N.B.—If it is impossible to get a balance owing to the deflection of g being always to one side, the P.D. across *III* is assisting, instead of opposing (as it should be) the fall due to b in the

stretched wire; the wires from R to III must then be interchanged.

If the lever at E is on stud 1 (literally 1000) and the slider C at 315 on the scale, the P.D. across R = ·1315 volts and the true current A_1 through A if $R = 0·1$ Ohm is $\frac{·1315}{0·1} = 1·315$ amps. Note simultaneously the readings on P and A when balance is obtained. Turn H to IV again and see whether the balance in obs. 3 still holds. If it does not, re-set P.

(5) Take about ten different scale deflections on A *rising* by about equal increments to the maximum by varying (rh) and repeat 4, noting the new values of P and A.

(6) Repeat 4 and 5 for a similar *descending* set of readings on A and tabulate your results as follows—

NAME . . . DATE . . .
Clark Cell: No. . . . Temperature =. . . ° C.: E.M.F. assumed = . . . volts.
Potentiometer setting: E on . . . C at . . . Standard Low Resistance R = . . . Ohms.

Ammeter Reading A.	Potentiometer Reading.		True P.D. across (R) V.	True Current $A_1 = \frac{V}{R}$ amps.	Error of Ammeter ($A_1 - A$).	% Error.
	Stud of E.	Position of Slider (C).				

It should be carefully noted whether a "Clark" cell or "Carhart-Clark" cell is being used before setting the potentiometer in 3, and that the assumed E.M.F. for this purpose at the particular temperature is correct, the temperature coefficient of E.M.F. being very different in these two cells (*vide* p. 643).

Standard Weston (cadmium) cells have an international value of E.M.F. of 1·0183 volts at 20° C., with a temperature coefficient of − 0·0000398 volt per rise of 1° C., between 0° and 40° C. In 1908, for a range of 0° C. − 40° C., Wolff obtained the relation giving the E.M.F. at °t C., namely—

$$E_t = E_{20} - 0·0000406(t - 20) - (9·5 \times 10^{-7})(t - 20)^2$$
$$= 1·0183\{1 - 0·0000398(t - 20)\} \text{ volts approx. (see table 643)}$$

(7) Plot calibration curves for the ammeter tested having values of A as ordinates and true amps A_1 as abscissæ.

Inferences.—What can you infer from your experimental results, and can you suggest any sources of error which might vitiate the results?

(6) Determination of the "Constant" of a Galvanometer or Ammeter by Means of a Copper Voltameter.

Introduction.—From the results of a large number of tests it has been found that, using the necessary precautions, the constant of an electric current instrument can be obtained with certainty to within $\frac{1}{10}\%$ of absolute accuracy by the electrolysis of copper. The voltameter (V) should consist of three or more pure copper plates dipping into a saturated solution of copper sulphate contained in a suitable glass or earthenware vessel, there being one more "anode" than "cathode," and the two sets arranged alternately with an anode at each end. The plates should be as square as possible, and placed from $\frac{1}{2}''$ to $\frac{3}{4}''$ apart; if too close, polarization will take place when strong currents are used, and the current density (reckoned in amps. per sq. cm.) is too great. There should not be less than 30 sq. cms. per amp.; if there is, the plate surface will be too small and the deposit on the cathode irregular, some of it falling to the bottom of the vessel. The resistance of V will also become high and variable, due to the formation of copper oxide, and will give trouble in keeping the current constant. The solution should be a saturated one (sp. gr. 1·211) of pure copper sulphate crystals and distilled water, with 1% by vol. of strong sulphuric acid added, which is necessary to insure success. The vol. of solution should be about 1100 cc. per amp. The anodes may be made of about No. 18 S.W.G., and the cathodes or gain plates of No. 30 S.W.G. pure copper, all edges and corners being smooth and rounded. The electrochemical-equivalent (Z) of any substance, in this case copper = No. grms, deposited by 1 coulomb.

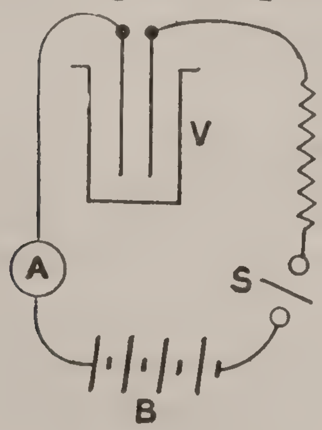


FIG. 7.

TABLE II.

Cathode area in sq. cm. per amp.	Values of (Z) for Copper.		
	2° C.	12° C.	23° C.
30	0·0003289	0·0003290	0·0003289
50	88	87	86
100	88	84	83
150	87	81	80
200	85	79	77
250	83	78	75
300	82	78	72

Apparatus.—Voltameter (V); rheostat R (p. 597); switch S; ammeter A to be standardized; secondary battery; drying cupboard D, not shown; acid bath.

Observations.—(1) Connect up as indicated in Fig. 7, and adjust A to zero. Light the gas-jet under the steam boiler of D, after seeing that the latter contains enough water.

(2) Determine the necessary area of cathode, and hence the number of gain plates required for the current to be used, reckoning both sides in contact with liquid as the effective area of cathode.

(3) Carefully clean the cathodes all over with fine emery cloth until quite bright, then dust with a *dry clean cloth*, and *do not touch the part to be immersed with the fingers.* Clean the anodes if they look dirty.

(4) Carefully weigh the gain plates on a chemical balance to 1 m.g., and note their weights (W_1) grams.

(5) Insert the same area of trial plate to act as cathode, so as to adjust the current to the value required, then remove them, making sure that the + of battery is joined to anode.

(6) Insert the weighed gain plates, and at a convenient and noted instant of time switch on, quickly adjusting the current to its proper value.

(7) Keep it flowing for at least thirty minutes, and maintain it constant all the time by altering R, if necessary. (Note.—1·177 grams of copper (cupric) are deposited per amp.-hour approx.)

(8) Note the exact instant of switching off. Very carefully remove the gain plates so as not to scratch them, rinse in acidulated water to prevent the nascent copper oxidizing, then in clean water, and place in D to dry.

(9) When dry and cool re-weigh the gain plates and note the weights W_2 grams.

(10) Repeat 2–9 for one or two other current strengths and tabulate as follows—

NAME . . . DATE . . .

Cathode Area = . . . sq. cms. per amp. Temperature of Bath = . . . ° C. Z = . . .

Weight of plates in grams.		Deposit W $=(\Sigma W_2 - \Sigma W_1)$	Time in Secs. (t)	Reading of A.	True Amps. $\frac{W}{Zt}$	% Error of A.
Before ΣW_1	After ΣW_2					

(7) Calibration of Direct Current Voltmeters. (Poggendorff's Method.)

Introduction.—The following method is a convenient one for enabling low-reading voltmeters up to about 3 or 4 volts to be easily and rapidly calibrated by comparison with one or two Clark's standard cells. Some convenient form of metre bridge, either circular or ordinary, is required. The principle of the method will be seen to be practically the same as the "Clark-Poggendorff" method of comparing two E.M.F.'s.

Apparatus.—Metre bridge of some convenient type, either the ordinary straight or circular form; secondary battery *B* giving an E.M.F. a little in excess of the maximum voltage to be recorded on the voltmeter *V* to be calibrated; key *k*; carbon rheostat *R* (p. 597); sensitive galvanometer *G* (p. 572); high resistance (r) of about 10,000 ohms; standard cell (S) of known E.M.F. It will be observed that the metre bridge, of whatever form is used, is represented symbolically by *PQ* in Fig. 8, and if the general scheme of the connections is understood, there will be no difficulty with them when using any form of bridge.

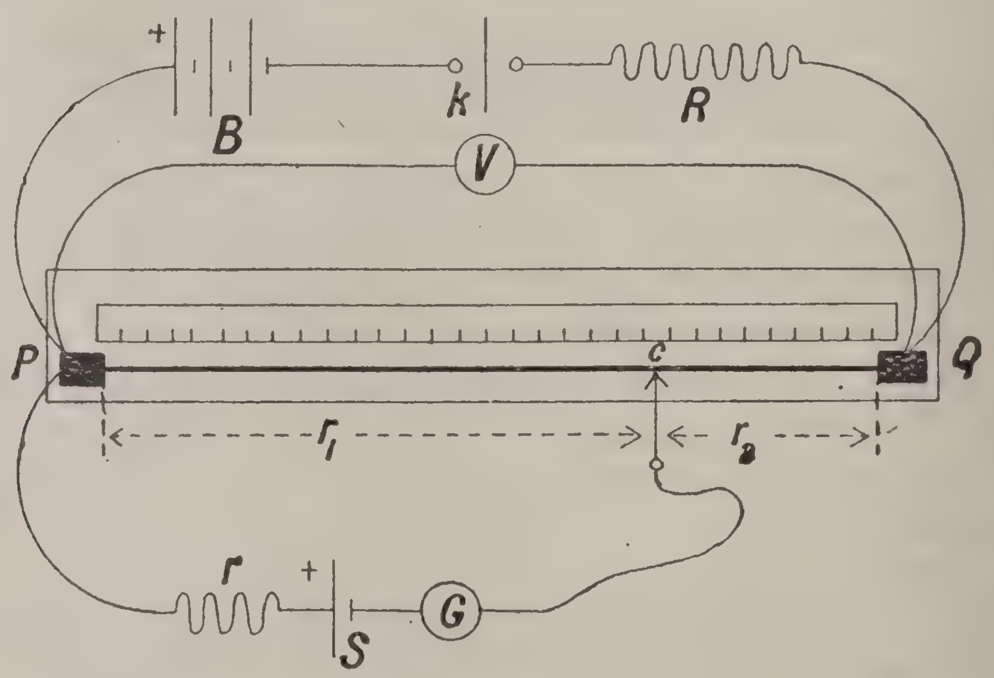


Fig. 8.

Observations.—(1) Connect up as in Fig. 8, adjusting *V* carefully to zero and *G* to about zero. See that like poles of *B* and *S*

are connected to the same end P, when the respective E.M.F.'s will oppose one another.

(2) Adjust R to a high value and likewise (r), which might preferably be about 10,000 ohms, and is merely for the purpose of preventing the standard cell S from sending but a very minute current when its circuit is closed at the tapping key or rolling contact C, which at first might be far from the correct position of balance. When C is moved to such a position that the deflection on G is small, (r) may temporarily be cut out of circuit so as to make the sensitiveness of the test a maximum.

(3) Now close K and alter R so as to obtain about $\frac{1}{10}$th of the maximum scale reading on V.

(4) Find a position for C such that on making contact with the bridge wire by means of it, no deflection occurs on G, r being manipulated as in obs. 2. Note the reading on the voltmeter V calibrated and the position of C where $PC = r_1$ and $QC = r_2$.

(5) Re-insert r and repeat obs. 3 and 4 for about ten readings on V rising by about equal increments to the maximum.

(6) Calculate the true volts V_T from the relation—

$$V_T = \frac{r_1 + r_2}{r_1} \times \text{E.M.F. of standard cell,}$$

and tabulate your results as follows—

NAME . . . DATE . . .

E.M.F. of Standard Cell = . . . Volts at . . . ° C. r = . . . Ohms, G = . . . Ohms } for reference only.

r_1	$r_1 + r_2$	Reading on V.	True Volts V_T	% Error of Voltmeter tested.

Note.—The E.M.F. of a Clark's standard cell = 1·4340 legal volts at 15° C., and its E.M.F. at other temperatures may be found from the relation—

$$\text{E.M.F.} = 1{\cdot}4340\{1 - 0{\cdot}0007\,(t - 15°)\}\ \text{legal volts,}$$

where 0·0007 is the *temperature coefficient* of the cell and t = its temperature in degrees Centigrade. For a Carhart-Clark cell the coefficient is 0·00038. For table of E.M.F.'s see p. 643.

(7) Plot a calibration curve of the voltmeter under test having values of V as ordinates and V_T as abscissæ.

Inferences.—Does the accuracy of the above test depend on

anything in particular? Show how the relation given in 6 can be obtained and state any assumptions made in deducing it.

(8) Calibration of a Voltmeter by Comparison with a Standard D'Arsonval Voltmeter.

Introduction.—The method may conveniently be employed for calibrating a voltmeter when neither a Kelvin *standard balance* nor a "potentiometer set" is available for use as a standard with which to compare the instrument under test. In the present case a fairly sensitive and good form of D'Arsonval galvanometer combined with a high resistance placed in series constitutes the standard voltmeter which together with its scale is permanently fixed up. The arrangement is very carefully standardized and its *constants* found with the aid of a Clark's standard cell and Clark's potentiometer method (p. 16), and thus a reliable standard voltmeter is obtained.

If the voltage which produces a full scale deflection with a certain resistance in series with the instrument at a given temperature is known, then that causing any other deflection under the same conditions will be in direct proportion and therefore at once known. For considerable accuracy some small corrections would be necessary in using these constants at some other time owing to the difference in temperature altering the resistances of the galvanometer coil and those in series with it, and to the D'Arsonval not exactly fulfilling the direct proportional Law, for which correction see p. 490.

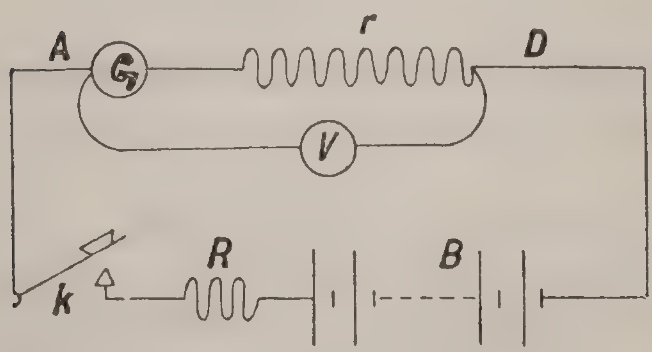


Fig. 9.

The D'Arsonval and its resistance can be arranged in one of two ways—(*a*) as represented in Fig. 9, assuming that a sufficiently high adjustable known resistance for placing in series with it is available. If then the "figure of merit" of G, *i.e.* the amperes (a) per scale division, is accurately known, the extra high resistance (r) necessary to be placed in series with it so that the

required maximum voltage applied to AD (Fig. 9) may produce a full scale deflection on G, can be found by Ohm's Law as follows—

If (d) = the maximum scale reading to be obtained by a maximum voltage V, then if (g) = the resistance of the galvanometer G, we have $(r + g) = \frac{V}{c} = \frac{V}{ad}$

$$\therefore\ r = \left(\frac{V}{ad} - g\right) \text{ ohms.}$$

(b) If G is very sensitive and sufficient resistance is not available the arrangement in Fig. 10 may be used, r now taking the form of two resistance boxes r_1 and r_2.

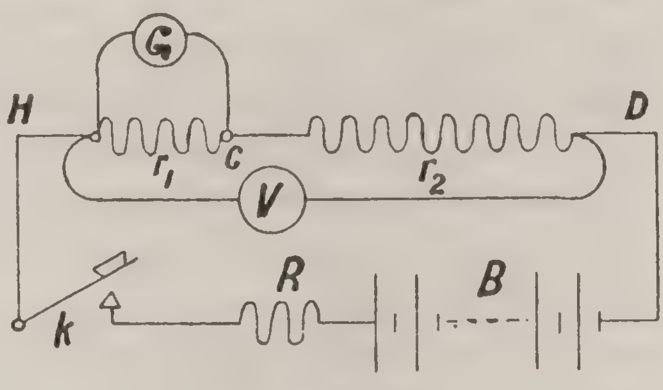


Fig. 10.

In this case r_1 will be small compared with the resistance of G and with $(r_1 + r_2)$, and this sum large compared with the resistance of V, the voltmeter to be calibrated.

If V and v are the voltages across HD and r_1 respectively and (g) the resistance of G, whose "figure of merit" is (a),

$$\text{then } V : v = r_2 + \frac{r_1 g}{r_1 + g} : \frac{r_1 g}{r_1 + g}$$

and if (d) has the same meaning as before $v = adg$,

$$\text{whence } V \frac{r_1 g}{r_1 + g} = adg \left(r_2 + \frac{r_1 g}{r_1 + g}\right)$$

assuming r_1 to be negligibly small compared with g and r_2 we have $\frac{r_1}{r_2} = \frac{adg}{V}$ approximately, which is obvious from a consideration of Ohm's Law.

Referring to Fig. 8 it will at once be seen that discarding the cell S, the standard D'Arsonval G with its extra resistance r may be employed to actually measure the P.D. between P and C, whence knowing the ratio of PQ to PC the true volts corresponding to any reading on V (Fig. 8) can at once be obtained. Thus the arrangement just referred to practically brings us to that shown in Fig. 10.

Apparatus.—Secondary battery B of a sufficient number of cells

to give the highest reading on the voltmeter (V) to be calibrated; a fairly high resistance D'Arsonval galvanometer G (p. 569); spring tapping key k; one or two high resistance boxes r_1 and r_2; variable unknown high resistance rheostat R; this latter, however, will be of no use if the resistance of the rest of circuit is very high, and in this case the voltage due to B will have to be varied by altering the number of cells in the battery.[1]

Observations.—(1) Connect up as shown in Fig. 10, which latter arrangement will be found to be the one most common in practice. Adjust the spot of light of G to the left-hand end of the scale as a temporary or false zero so as to obtain a full scale deflection up to the other end for the maximum voltage to be measured.

(2) Insert the proper resistances in r_1 and r_2 as given from the *constants* of standardization for the maximum voltage to be measured and corrected for the temperature of the room at the time of the test.

(3) Close k and adjust R, or alter the number of cells in B so as to obtain about $\frac{1}{10}$th of the maximum scale reading on V and note simultaneously the readings on V and G.

(4) Repeat 3 for about ten different readings on V *rising* by about equal increments to the maximum.

(5) Repeat 3 and 4 for a similar *descending* set of the same readings on G, noting the corresponding ones on V, and tabulate your results as follows—

NAME . . . DATE . . .
Voltmeter: No. . . . Type . . . Resistance r_1 = . . . Ohms.
" r_2 = . . . "
Temperature of room = . . . °C. Resistance of G = . . . Ohms at . . . °C.

Reading in Volts on Voltmeter tested.		Deflection on D'Arsonval D.	True Voltage, *i.e.* D reduced (v) Volts.	% Error of Voltmeter tested.
Ascending V.	Descending V.			

[1] Instead of the variable high Resistance rheostat (R) shown in tests 8–11 which may not be available, the following arrangement for varying the voltage across the parallel combination of voltmeter tested and standard, will be found convenient, namely—connect two variable lamp resistances R_1 R_2 in series across the supply and shunt one of them (R_1) with the voltmeter and standard in parallel, the lamps of each rheostat being operated in parallel and being for a voltage = to that of the supply. Then by keeping 1 lamp in R_1 and varying R_2, the volts across R_1 can be varied from $\frac{1}{2}$ that of the supply to its full value, while by keeping 1 lamp in R_2 and varying R_1 the volts across R_1 can be varied from $\frac{1}{2}$ that of the supply to 0; thus covering the full range of supply volts on R_1.

(6) Plot "calibration curves" having values of true voltage (v) as ordinates and V as abscissæ.

Inferences.—Enumerate any sources of error in voltmeters generally. State clearly what you can infer from the results of your tests.

(9) Calibration of a Voltmeter by Comparison with a Kelvin Composite Balance used as a Voltmeter.

Introduction.—The composite balance when used in conjunction with separate anti-inductive resistances may be conveniently employed as a standard direct or alternating current voltmeter capable of measuring pressures up to about 600 volts. The following method assumes the use of such an instrument, and the reader should refer to p. 554, *et seq.*, for the construction and mode of using this form of balance and for the table of constants and sensibilities.

Apparatus.—Adjustable, fairly high resistance R (p. 603); switch S; voltmeter V to be calibrated; Kelvin Composite Balance $K.B.$, with its non-inductive resistance r (p. 553); battery of secondary cells capable of giving the maximum voltage to be used, which we therefore assume to be direct.

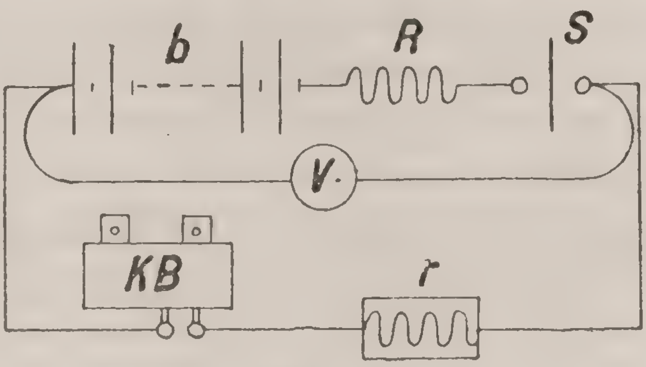


FIG. 11.

Observations.—(1) Connect up as in Fig. 11, adjusting both instruments carefully to zero, and make quite certain that the connections are as indicated.

(2) Turn the switch in front of the balance to "Volts" so as to place the fixed and movable fine wire coils in series with one another across the terminals. Observe that the anti-inductive resistance r is numbered the same as, and therefore belongs to the balance in use, and make quite sure of having the correct resistances in it in use (p. 556).

(3) Adjust the balance and its sensibility by employing the proper weights as given in the table of constants (p. 556), so that

the maximum voltage to be measured on V would produce as nearly as possible a full scale reading on $K.B.$

(4) With R as large as it can be, close S, and obtain about $\frac{1}{10}$th of the maximum scale reading on V by altering R. Note simultaneously the reading on V and position (d) of the slider of $K.B.$

(5) Repeat 4 for some ten different values of voltage on V (by altering R or the number of cells in the battery) rising by about equal increments to the maximum.

(6) Repeat obs. 5 for a similar set of decreasing voltages, and tabulate your results as follows—

NAME . . . DATE . . .

Composite Balance: No. . . . Constants . . . Voltmeter tested. . . . Temp. = . . . °C.

Slider reading d.	Volts $K.d.$	Corrected Volts V_T.	Reading on V.		% Error of V.
			Ascending.	Descending.	

N.B.—The values V_T are the readings on the standard corrected for temperature.

(7) Plot a calibration curve for the voltmeter tested having readings on V as ordinates and true volts V_T as abscissæ.

Inferences.—What sources of error are voltmeters in general liable to? Can anything in particular be inferred from the above results?

(10) Calibration of a Voltmeter by Comparison with a Kelvin Centi-ampere balance used as a Voltmeter.

Introduction.—The Kelvin Standard Centi-ampere balance when used in conjunction with extra anti-inductive resistances constitutes a most convenient standard voltmeter with which to compare any other voltmeter to be calibrated, up to about 800 volts.

For larger voltages, up to 2500 volts, special non-inductive high resistances are provided for inserting in series with the coils of the instrument. The combination then constitutes a standard voltmeter by means of which any other voltmeter, either for direct or alternating currents, reading up to 2500 volts, can be calibrated, by comparison, in the ordinary way.

A complete description of this balance, together with the

method of using it, will be found on p. 546 *et seq.*, where the table of constants is given.

In the present test the apparatus, observations and inferences are precisely similar to that of the corresponding calibration by means of the composite balance used as a voltmeter, the centi-ampere balance being substituted for this latter. They will consequently not be repeated here, but may be seen on p. 21.

(11) Calibration of a Direct Current Voltmeter. (Crompton Potentiometer Method.)

Introduction.—The method is a very convenient and accurate one for the purpose, and consists in calibrating the voltmeter to be tested in terms of the E.M.F. of a Clark's standard cell, a known fraction only of the voltage applied to the instrument being actually compared with the standard E.M.F. The principle of the method is identical with that of the "Clark-Poggendorff" method for comparing two or more E.M.F.'s, a full description of which will be found in a separate work, by the author, on *Practical Electrical Testing*, for 1st and 2nd year students and others. The Crompton Potentiometer is a specially arranged form of comparing instrument by means of which the calibration can be easily and quickly carried out. A detailed description of it will be found on p. 510, to which the reader should in the first instance refer. There are three extremely important features in connection with the present method, using the potentiometer; firstly, the enormous range of applicability, for the instrument can be used equally well in the measurement of current and resistance as well as voltages from 0 to almost any practical amount; secondly, the measurements are in terms of the official Board of Trade Standard—the Clark cell—though any other standard can be used; thirdly, the accuracy is great and under ordinary conditions the measurements are accurate to at least $\frac{1}{10}\%$, and with care, using a very accurately adjusted instrument, accuracy to something like $\frac{1}{50}\%$ can be obtained. In this form of potentiometer the highest voltage which can be compared directly is 1·5 volts, and hence the fractions employed of all higher pressures to be measured must fall within this limit.

Apparatus.—Crompton potentiometer P (Fig. 208); secondary battery B capable of giving the maximum voltage required for a

full scale reading on the voltmeter V to be calibrated; key or switch S; one secondary cell (b) for the "working cell" of the potentiometer; "volt box," *i.e.* divided resistance acd for obtaining a fraction (less than 1·5 volts) of the total P.D. to be measured (p. 521); sensitive D'Arsonval or moving coil galvanometer (g) (p. 569); standard Clark cell E; fairly high resistance rheostat R (p. 603). See footnote, p. 20.

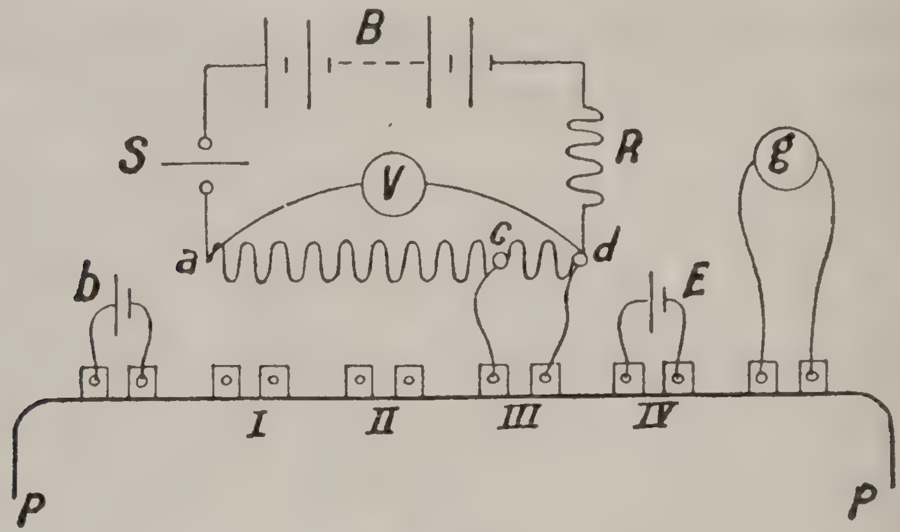


FIG. 12.

Observations.—(1) After placing the levers of G and E on studs 14 and that of H on studs 1 or 2 for precaution, connect up as in Fig. 12, in which only the row of terminals on the potentiometer P is shown, symbolically, for the sake of brevity.

(2) Adjust the spot of light of the galvanometer to somewhere about zero on the scale, and the resistance ad and cd in the "volt box" so that $cd = \frac{1}{100}$ of ad, supposing, of course, that not more than 150 volts is across ad. Carefully level and adjust V to zero if it requires it.

(3) "Set the potentiometer" by the standard cell in the way described on p. 514, the contact lever H, Fig. 208, p. 512, being on studs IV, thus inserting E in the circuit of g in such a way as to oppose that of b, close S, and adjust R so as to obtain the full scale deflection on V.

Note.—This last operation will only be possible when R is comparable with the parallel resistance of ad and V. If both these are very high then altering R will have very little effect on the reading of V unless R is high also in comparison.

(4) With the positions of the resistances G_1 and G (Fig. 208, p. 512), as found in 3, *unaltered*, turn the lever of H to studs

III so as to throw into circuit with g the $\frac{1}{100}$th part of the voltmeter P.D., which was across terminals *III*. Now adjust the lever of the resistances E, Fig. 208, and the sliding key C, Fig. 208, so that no deflection occurs on pressing this latter.

N.B.—If no balance can be obtained owing to there being no reversal of deflection on g, the fractional P.D. across *III* is assisting instead of opposing (as it should be) the fall of potential due to (b) in the stretched wire. The wires from cd to *III* must then be interchanged. If the lever at E is on stud 12 (literally 12,000) and the slider C at 625 on the scale, the voltage across ad, i. e. at the terminals of $V = 126{\cdot}25$ volts. Note these positions on PP, and simultaneously the reading V on the instrument to be calibrated.

(5) Reduce V by about $\frac{1}{10}$ either by cutting out cells in B or by altering R or both, and repeat 4, noting the new values of V and PP. Turn H to IV for a moment and see whether the balance in obs. 3 still holds, if not re-set as in obs. 3 above.

(6) Repeat 4 and 5 for some ten or twelve different readings on V *decreasing* by about equal amounts to the lowest.

(7) Repeat 4 to 6 for a similar *ascending* set of observations, and tabulate your results as follows—

NAME . . . DATE . . .

Clark Cell: No. . . . Temperature = . . . ° C: E.M.F. Assumed = . . . Volts.

Volt Box: Fraction of total used $\frac{cd}{ad}$ = . . . Potentiometer setting E on . . . C at . . .

Voltmeter Reading V.	Potentiometer Reading.		True Volts Across.		Error of Voltmeter $V_1 - V$.	% Error.
	Stud of E.	Position of Slider C.	Fraction cd v.	Voltmeter $V_1 = \frac{ad}{cd}\ v$.		

As it may sometimes be the case that a Carhart-Clark and not a Clark's standard cell has to be used, care should be taken that the E.M.F. assumed at the particular temperature is correct, the temperature coefficient of E.M.F. being very different in the two cases (*vide* pp. 17 and 643).

(8) Plot a calibration curve for the voltmeter tested having values of V as ordinates and true volts V_1 as abscissæ.

Inferences.—What can you infer from the results of your test? Are there any sources of error which might vitiate the results?

(12) Calibration of a High Tension Alternating Current Voltmeter.

Introduction.—In the ordinary high tension systems of distribution of electrical energy by alternating currents, the average working pressures are about 2000 or 2500 volts. The "electromagnetic" and "hot wire" types of voltmeters are unsuitable for measuring such high pressures, which can best be dealt with by means of a third class of instrument known as the electrostatic voltmeter, a description of two forms of which will be found in the Appendix (p. 562).

These instruments are almost universally employed for measuring alternating current pressure, and they have the all-important advantage of being unaffected by variation of frequency. Owing usually to the difficulty experienced in obtaining direct current pressures of the above magnitude for testing purposes, alternating currents have nearly always to be employed for calibrating high tension voltmeters. Thus it will be seen that none of the preceding methods are available in the present case, but the calibration can be effected by what may be termed the "fractional potential difference" method, using an accurately calibrated low tension electrostatic voltmeter for comparison in the manner to be described later.

This low tension voltmeter, which may conveniently be one of Lord Kelvin's multi-cellular electrostatic instruments reading to, say, 150 volts, should be very carefully calibrated by one of the preceding methods—preferably the *potentiometer*, one using a Clark's cell as a standard of E.M.F. and direct current pressures of course. For accurate work, however, the following remarks should be observed. In these voltmeters the movable needle system is usually aluminium and the fixed system brass, whence owing to aluminium being electro-positive to brass, the instrument will read from 0·2 volt to 0·3 volt too low when the $+^{ve}$ pole of the direct current source is connected to needle, and the same amount too high if the $-^{ve}$ is joined to the needle system instead. In calibrating this low reading voltmeter by the potentiometer, it should be connected up through a reversing key to the rest of the apparatus and the *mean* of the readings, before

and after reversing the polarity on its terminals, taken as the correct one for alternating currents, since when used with such the above-named error does not occur. Again with direct currents an electrostatic voltmeter passes no current, but owing to it possessing a perfectly definite though very small capacity, it will behave like a condenser with alternating currents, *i.e.* a pulsating or "charge and discharge" current will be set up in its circuit. Thus it will be seen that if, as in the present method, such an instrument is shunted across part of a circuit carrying an alternating current, the current in the voltmeter branch may be quite comparable with that in the main circuit, in other words the P.D. between the points to which it is shunted would be lowered somewhat by the voltmeter, and ∴ would not bear to the whole P.D. the ratio of the resistance of the two portions of the circuit. To avoid such an error the resistance of the main circuit should be such that the maximum pressure to be used sends a sensible current such as from $\frac{1}{4}$ to $\frac{1}{2}$ an ampere through it, which will consequently be very large compared with the current in the voltmeter branch. Thus the presence of this latter will not affect the value of the P.D. between the two points to which it is applied, and consequently the ratio of the whole P.D. to the fraction thus tapped will equal the ratio of the whole resistance to the fraction across which the electrostatic voltmeter is placed.

Apparatus.—Alternator A, capable of supplying a low pressure, and of being driven at any required speed by a direct current, electromotor (preferably), or other prime mover, its exciting circuit E consisting of the field coils of A (shown), together with switch, rheostat, ammeter and source of excitation (not shown), but which can be varied so as to vary the E.M.F. of A; step-up transformer T capable of increasing the pressure from that of A to the maximum required for a full scale reading on the high tension voltmeter V to be calibrated;

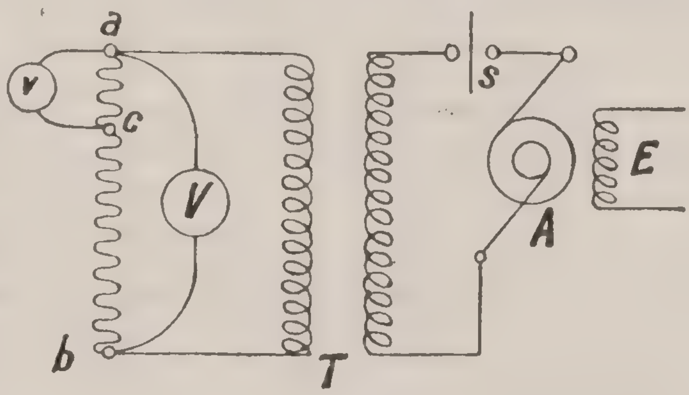


FIG. 13.

low tension electrostatic voltmeter (v) (p. 562), reading to say 150 volts, and which has been previously carefully calibrated by reference to a standard Clark cell on the potentiometer; switch s. A divided non-inductive high resistance (acb) capable of standing the highest pressure to be used on V, and of carrying an appreciable current, say of the order of $\frac{1}{4}$ to $\frac{1}{2}$ an ampere continuously without excessive heating.

Caution.—Under no circumstances whatever is any part of the high tension circuit to be touched while "alive," and the india-rubber gloves are to be worn throughout the test by the operator reading the electrostatic voltmeters.

Observations.—(1) Connect up as in Fig. 13, and carefully level and adjust the pointers of V and v to zero. For the high tension side use well-insulated wires for the connections, and keep them in mid-air as much as possible.

(2) If the voltmeter V to be calibrated reads up to, say, 2500 volts, and v to only 150 volts, place this latter across a convenient fraction ($a\,c$), say $\frac{1}{20}$th of the whole non-inductive resistance ($a\,b$), which in this case may conveniently be something like 5000 Ohms.

(3) Start the alternator A, close s, and then adjust the speed and excitation so as to obtain the lowest scale reading on V. Note simultaneously that on (v) also.

(4) Repeat 3 for ten or twelve different voltages on V, rising by about equal increments to the maximum, and tabulate as follows—

NAME . . . DATE . . .

H.T. Voltmeter tested: No. . . . Type . . Range . . . Made by . . .

Whole resistance R_{ab} = . . . Ohms. Fraction used R_{ac} = . . . Ohms. Ratio $\frac{R_{ab}}{R_{ac}}$ = . . .

V.	Reading on v.		True voltage $V_1 = \frac{R_{ab}}{R_{ac}} v_1$	% Error of meter tested 100 $(V - V_1)$.	Mean Error.
	v.	Corrected v_1			

(5) Plot a calibration curve for the high tension voltmeter having values of V as ordinates and true volts V_1 as abscissæ.

Inferences.—Enumerate what you consider to be the advantages and disadvantages of electrostatic voltmeters.

(13) Complete Test of both Direct and Alternating Current Ammeters and Voltmeters for the various sources of Errors.

Introduction.—The principle involved in the action of any type of ammeter or voltmeter will come under one of the following heads—

(1) *Heating* effect of the current or P.D. to be measured.

(2) *Electrostatic* effect of attraction or repulsion between fixed and movable conducting surfaces, close to, but insulated from one another, when electrified to opposite potentials.

(3) *Electro-magnetic* effect of the current in a coil of wire on iron or *vice versâ*.

(4) *Electro-dynamic* action of the current in one part of circuit on the same current in another part of that circuit, causing electro-dynamic attraction or repulsion between the two.

There are briefly eight principal sources of error to which ammeters and voltmeters in general are liable, namely—

(*a*) Error in the calibration owing to the standards employed in the two cases being different.

(*b*) Error through a partial demagnetization of the permanent field, causing an alteration in the sensibility, in the case of permanent steel magnet instruments.

(*c*) Error caused by the sensibility of the instrument being temporarily altered by external magnetic influence.

(*d*) Error due to a current producing a different deflection depending on the magnitude of the current previously measured compared with the one in use at the time.

(*e*) Error due to the instrument giving a different scale reading for different directions of the *same* current.

(*f*) Error in the case of voltmeters due to alteration of the resistance of the instrument caused by change of temperature, whether from the room or passage of the current.

(*g*) Error in alternating current instrument due to alteration of frequency.

(*h*) Error due to friction at the pivots in all classes.

Error (*a*) applies to all four classes of instruments, and cannot very well be remedied except by re-calibration.

It is clear that classes 1 and 2, being entirely non-magnetic,

may be dismissed as not being liable to magnetic errors. Class 3, however, and 4, which latter type may or may not contain iron, are liable to large errors arising in the case of direct currents from the retentivity of the iron used (error *d*), or magnetic hysteresis, and from the proximity of, and the external magnetic effect of currents in neighbouring wires and of magnets (error *c*). In the case of alternating currents from hysteresis, eddy currents in the metal work about the instrument re-acting on the coil, and change of frequency (error *g*) in the alternating current.

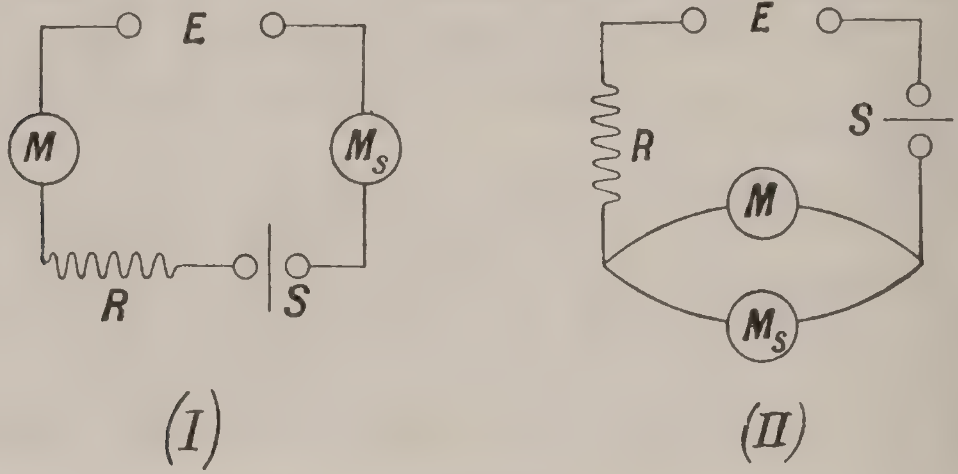


Fig. 14.

In this latter class of work the instruments should contain no iron at all, and very few metal fittings. In good direct current instruments where iron is a necessity, it should be *very soft*, and well laminated, and there should not be much of it.

Apparatus.—Ammeter or voltmeter M to be tested; suitable variable rheostat R, which in the case of the ammeter test must be capable of carrying the maximum current required by M, and in the case of the voltmeter test must have a large resistance so as to be quite comparable with that of M; switch, or key S; source of electrical supply E, whether direct, or alternating current; standard accurately calibrated ammeter or voltmeter M_s which must not contain any iron, and preferably no metal fixings.

N.B.—The standard M_s may be either a Kelvin standard balance (p. 546); Siemens electro-dynamometer (p. 577); Cardew, or electrostatic voltmeter (p. 562), preferably the latter, which not only is non-magnetic, but also has no temperature error. See footnote, p. 20.

Observations.—(1) If an ammeter is being tested, connect up as in Fig. 14 (I), but if it is a voltmeter, then as in Fig. 14 (II). Care-

fully adjust the pointers of M and M_S to zero if they require it, levelling them also when necessary so that all moving parts can move quite freely. Place M and M_S at some distance apart so that there can be no possibility of one affecting the other. Also run the connecting leads close together so that their magnetic effect due to the current in them may not affect the instruments.

(2) Error due to External Magnetic Effect. (A.)

With R at its maximum, close S, and obtain about $\frac{1}{4}$ full scale reading on M. Note the corresponding reading of M_S, which must be kept quite constant and steady when no magnet is near. Now move a powerful permanent magnet in a *plane perpendicular to the axis* about which the moving system of M oscillates and passing through its centre, the axis of the magnet pointing to M always, and its pole nearest to M being moved so as to be always at about 12″ say from M. Note the alteration (if any) of the reading of M.

(3) Repeat 2 for a full scale deflection on M.

(4) Repeat the last part of 2, pertaining to the motion of the magnet, with *no current flowing*, i. e. *S open*, and tabulate your results as shown in the table below.

N.B.—The magnet *must not* be allowed to *affect* M_S in any way, and the latter must be far enough away to ensure that this is the case.

If M is an alternating current instrument, and an alternating supply is being used, the *frequency* must be maintained *constant* in 2 and 3 above, as well as the reading of M_S in the particular test.

(5) Error due to Retentivity or Residual Magnetism. (B.)

With R a maximum, close S, and *carefully* take a gradually *increasing* set of about ten simultaneous readings on M and M_S from the lowest to full scale, by gradually diminishing R, gently tapping the instruments to eliminate any pivot friction.

(6) Repeat for a similarly obtained decreasing set, the *same* scale reading of the standard M_S being obtained when descending as was obtained on ascending. Tabulate your results as indicated in the following table.

N.B.—This test of course only applies to direct current instruments, and the error in question may amount to over 20%.

Standard instrument used : Type . . . No. . . . Maker . . . Constant= . . .
Instrument tested : Type . . . No. . . . Maker . . .

A. External Magnetism.			B. Retentivity or Residual Magnetism.				
Position of influencing magnet.	Reading on		Reading on			% Error.	Frequency (constant) if alternating current used.
	Ms.	*M.*	*Ms* ascending and descending.	*M* ascending.	*M* descending.		

(7) Error due to variation of "frequency" with Alternating Current instruments. (C.)

Close S, and adjust R to give some convenient scale deflection on M and M_s, which latter must be kept constant by means of R. Now vary the frequency, by altering the speed of the alternator if this is under control, from the smallest to the greatest possible so as to obtain about ten different values, and note the simultaneous readings on M and M_s at each.

(8) Error due to Eddy Currents in metal fixings with Alternating Currents. (D.)

If either M or M_s possesses a moving coil, the terminals of which can be got at to send a current through this coil only of the instrument, as in either a Kelvin balance, Siemens dynamometer, or Parr direct reading dynamometer instruments. Proceed as follows—Adjust the pointer of this instrument carefully to zero, and send the maximum alternating current through this moving coil alone, noting whether it deflects. If it does, eddy currents are being set up in the metal fixings and re-act on the moving coil causing it to deflect.

(9) Repeat 8 for the same current at different frequencies, and tabulate your results as in the following table—

Standard instrument used : Type . . . No. . . . Maker . . . Constant= . . .
Instrument tested : Type . . . No. . . . Maker . . .

C. Effect of Frequency.			D. Eddy Currents.		
Frequency ∽ per sec.	Reading on		Frequency ∽ per sec.	Reading on	
	Ms.	*M.*		*Ms.*	*M.*

(10) Error due to Reversal of Current through the instrument. (E.)

Connect up as shown in Fig. 14, but instead of connecting M directly in series as shown, join it up now to the circuit through a reversing switch or key, so that the current through it may be reversed in direction though that in the rest of the circuit and therefore through M_s is still unidirectional.

With R large, close S, and obtain say $\frac{1}{5}$ full scale deflection on M, noting that on M_s which must be constant; now reverse current in M and again note its value for the same one as before on M_s. Next re-reverse and note it again.

(11) Repeat this operation for about 5 scale readings on M up to the maximum at roughly equal intervals.

N.B.—This test of course only applies to direct current instruments. Tabulate as follows—

Standard instrument used : Type . . . No. . . . Maker . . . Constant = . .
Instrument tested : Type . . . No. . . . Maker . . .

E. Effect of Current Reversal.				F. Heating Effect.		
Reading of				Time of Running Hours.	Reading on	
M_s (constant).	M direction of current.				M_s.	M.
	←	Reversed. →	Re-reversed. ←			

(12) Error in Voltmeters (only) due to Heating of Coils by passage of Current. (F.)

Close S, and adjust R to obtain about $\frac{1}{2}$ scale reading on M, note the corresponding reading on M_s which must be an electrostatic voltmeter. Maintain M_s constant for, say, quarter of an hour and again read M.

(13) Repeat 12 for a full scale reading on M, and tabulate as before.

N.B.—The error in voltmeters due to change in the temperature of the room is readily calculable when the latter is obtained by a thermometer.

(14) Plot the following curves for tests—

B. Having readings on M as ordinates, and M_s as abscissæ for both ascending and descending curves.

C. Having readings on M as ordinates and frequency in ∽ per sec. as abscissæ.

D. Having readings on M as ordinates and frequency in ∽ per sec. as abscissæ.

Inferences.—State very clearly and concisely what you can infer from the results of your observations.

(14) Calibration of a Wattmeter by Comparison with a Kelvin Composite Balance used as a Wattmeter.

Introduction.—The following is a convenient method of calibrating a Wattmeter by means of direct currents, using a Kelvin composite balance as the standard Wattmeter with which to compare the one to be tested. The construction of the balance is detailed on p. 554, where the mode of using it as a Wattmeter is also given, and it will merely suffice to say here that it is used very similarly to the Hekto-ampere meter, the only difference being that as a Wattmeter, the fine wire movable coils (only) are placed in series with an extra anti-inductive resistance across the mains supplying the power measured by both Wattmeters. It may here be noted that it is not necessary for the current through the thick winding and the pressure across the thin coils to be developed by one and the same source. For since the Wattmeter deflection is ∝ to the products of the currents flowing through the two coils, clearly these may come from two totally different sources. In fact it is distinctly preferable to have them separate when possible, for then the variations of the main current will not affect the constancy of the pressure on the fine coils.

This same test serves to determine the "*constant*" (K_W say) of the Wattmeter, or in other words the *number* by which the scale reading must be multiplied so as to obtain the power in Watts.

The following reasoning will no doubt render this clearer.

Assuming the general principle and construction of, suppose, a Siemens Wattmeter to be understood. Let C and c = the currents flowing through the fixed thick- and movable thin-wire coils respectively when a deflection of the torsion head and its pointer on the scale is D° or divisions. Then the force acting between the coils is $\propto C \times c$, but $(c) \propto$ to the pressure V at the terminals. Hence the deflecting couple acting between the coils $\propto C \times V \propto$ Watts. Now when the index is brought back to 0 again by turning the torsion head, thereby twisting up the spring and introducing the control, we have—torsion of spring $\propto$ Watts $\propto CV$; but the force of torsion is $\propto$ to angle of torsion of such a spring,

$$\therefore D \propto CV$$

or $KD = CV =$ Watts measured and causing a deflection D, where K is the "*constant*" of the Wattmeter tested. It may be found that K is not perfectly constant throughout the whole scale. In this case the Watts should be obtained from a calibration curve rather than by the product KD.

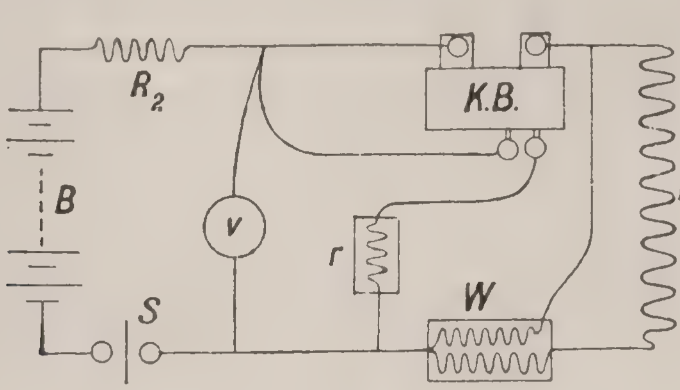


Fig. 15.

Apparatus.—Kelvin composite balance ($K.B.$) (p. 554), with its anti-inductive resistance r (p. 553); switch S; variable power-absorbing resistance R (p. 606); accurate voltmeter V (preferably electrostatic); main current battery B; Wattmeter (W) to be calibrated; [pressure battery b and adjustable resistance R_1 if available (Fig. 16)]; adjusting rheostat R_2 (p. 597).

Observations.—(1) Connect up either as in Fig. 15 or 16, and in the present test assume the latter for actual experiment, and make quite certain that the connections are as indicated in Fig. 16.

(2) Carefully level the instruments that require it, adjusting their pointers to zero, and if W has a suspended coil see that this is quite free to move.

Note.—Care should be taken to run the "leading in" and "out" wires carrying the main current to W, and in the rest of the circuit close together or twisted in order that the currents flowing in them shall exert no magnetic influence on the instruments.

(3) Turn the switch on the balance to "*Watts*" so as to place the movable fine wire coils across the small terminals. Observe whether (r) is numbered the same as, and therefore belongs to the balance in use, and make quite certain that the correct resistance is being used in (r) (p. 553).

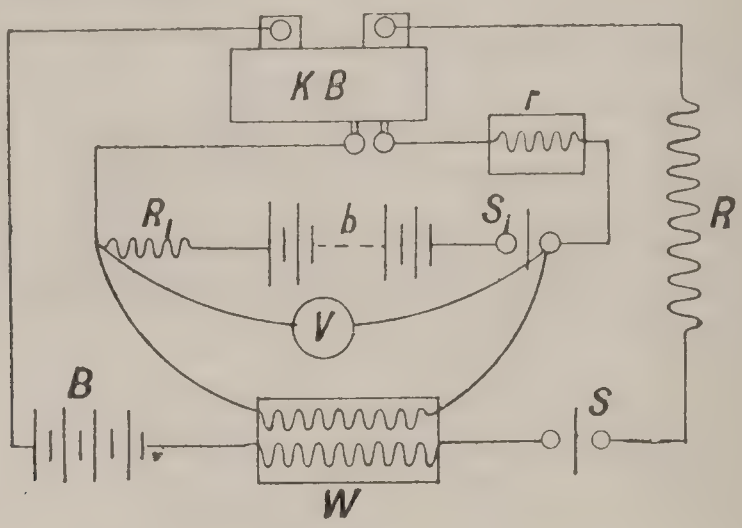


FIG. 16.

(4) Adjust the balance and its sensibility by using the proper weights as given in the table of constants (p. 558), so that the maximum Watts to be measured on W would give, as nearly as possible, a full scale reading on *K.B.*

(5) With R_1 fairly large, close S_1 and adjust the voltage as read off on V to the desired amount by altering R_1, and then maintain this voltage constant, observing that it is so before taking every reading.

(6) R being fairly large, close S, and alter R so as to obtain about $\frac{1}{10}$th of the maximum scale reading on W. Note simultaneously the reading on W and position (d) of the slider of *K.B.*

(7) Repeat 6 for some ten different deflections on W (by varying R) *rising* by about equal increments to the maximum, the pressure remaining constant all the time.

(8) Repeat obs. 7 for a similar set of *decreasing* readings on W, and tabulate your results as follows—

NAME . . . DATE . . .
Composite Balance: No. . . . Constants used K_B . . . Temperature . . . °C
Wattmeter tested: No. . . . Maker . . . Range . . . Maker's Constant . . .

Slider Reading d.	True Watts $K_B \cdot d$.	Reading on W.		Constant $K_W = \frac{K_B \cdot d}{\text{mean } D}$	% Error of Wattmeter	Mean % Error.
		Ascending D.	Descending D.			

(9) Plot a "calibration" curve for the Wattmeter tested having values of D as ordinates and true Watts as abscissæ.

Inferences.—What can be inferred from the results of your test? Are Wattmeters subject to any sources of error, and if so, how can they be minimized or got rid of?

(15) Calibration of a Wattmeter by Comparison with a Standard Ammeter and Voltmeter.

Introduction.—The following method of calibration by direct currents entails the use of an accurately calibrated standard ammeter and standard voltmeter. These may be either Kelvin balances or ordinary instruments which have recently been carefully compared with accurate standards, and a record of the calibration curves of which are obtainable.

It should be remembered that the constant of a Wattmeter obtained with direct currents will only be true for alternating currents providing the self-induction of the fine wire moving coil or its circuit is practically zero or very nearly so. In other words, the instrument must contain no iron and also be very nearly "non-inductive." This is a matter of great importance, for Wattmeters are in most cases only required to measure power in alternating current circuits.

Apparatus.—Standard ammeter (A) and voltmeter (V); Wattmeter (W) to be calibrated with its anti-inductive resistance (r) if there is one; battery of secondary cells B; switch S; suitable resistance R for absorbing power (p. 606), which must be non-inductive if alternating currents are employed; carbon rheostat (Rh) (p. 597).

Observations.—(1) Connect up as shown. Carefully level all the instruments, adjusting their pointers to zero, and see that

the swing coil of the Wattmeter is quite free to move. Care should be taken to run the "leading in" and "out" wires carrying the main current to the Wattmeter, close together or

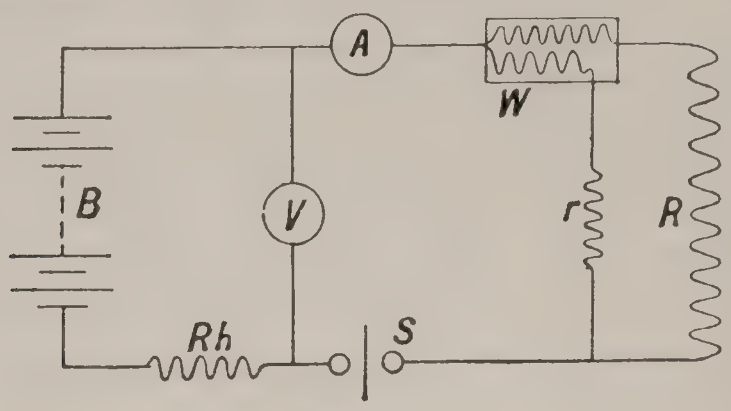


FIG. 17.

twisted. Also the main wires of the rest of the circuit close together in order that the current flowing in them shall exert magnetic influence on any of the instruments.

(2) R being at its maximum value, close S, and adjust R so as to obtain about $\frac{1}{10}$th of the full load current through W, the pressure being maintained at standard voltage by varying the carbon rheostat (Rh). Note the readings of all the instruments.

(3) Repeat 2 for about ten different readings on W rising by about equal increments to the maximum current allowable, and calculate for each the percentage error of the Wattmeter and the mean. Tabulate as follows—

NAME . DATE . . .

Non-inductive Wattmeter: No. . . . Maker . . . Temperature = . . . °C.

True Volts. V.	True Amps. A.	True Watts. $W=AV$.	Wattmeter Reading. D.	Wattmeter Constant. $K=\frac{W}{D}$	Percentage Error of Wattmeter.	Mean Error.

(4) Plot a curve having values of (W) as abscissæ and the corresponding Wattmeter readings (D) as ordinates.

(16) Calibration of a Wattmeter with Alternating Currents. (Three-Voltmeter Method.)

Introduction.—Wattmeters form a class of measuring instrument the chief application of which consists in measuring

accurately the power taken up in alternating current circuits. The great value of a Wattmeter in such measurements practically disappears with direct currents as the individual factors of power, namely "volts" and "amperes," are usually here required, and in addition the product of the two can easily be obtained and at once gives the "true power." With alternating currents, however, this last remark is not true, and herein lies the great value of the properly constructed Wattmeter, in that it measures the true power in such a circuit. For it to be capable of doing this, however, it must be carefully constructed, and there must be *no iron* and preferably no other metal work near the coils. Wattmeters when used on alternating current circuits are liable to the following sources of error: (*a*) owing to the fine wire coil possessing some self-induction and consequently impedence, the current in it is not able to rise to the same maximum strength which it would do for a direct P.D. of similar magnitude; (*b*) this impedence causes a lag in phase of the current in the fine wire coil behind the P.D. across which it is placed.

(*c*) A third source of error common also to all voltmeters, and occurring both with direct and alternating currents, is that due to the alteration of the resistance of the fine wire coil due to change of temperature, and which can be minimized in the manner described later on. From the preceding remarks it will therefore be evident that when a so-called "Non-inductive Wattmeter" is calibrated with direct currents (which is usually the case) its "*constant*" so obtained will *not* be *correct* for alternating currents. The instrument will also read differently for variation of the "frequency" of the current even though the actual power being measured remains the same. Thus a Wattmeter may with advantage be calibrated with alternating currents on a circuit having the same "constants," namely voltage, frequency and "wave form," etc., as that in which it is eventually desired to measure the power. The calibration can be performed by what is commonly known as the 3-voltmeter method of measuring power in alternating current inductive circuits, and by it the "*true power*" may be obtained with almost any degree of accuracy desired by using an accurately calibrated voltmeter and by repeating the observation two or three times, noting the *mean*. It has the advantage that only

one alternating current voltmeter is required, though three similar ones may be used if available.

Apparatus.—Alternator D and its exciting circuit (not shown) under rheostatic control or other convenient source of alternating current supply; inductive portion PQ of the circuit in series with a strictly non-inductive portion QR; two 2-way keys k_1, k_2 (p. 587); Cardew or low-reading electrostatic voltmeter V accurately calibrated; main switch S; Wattmeter W to be calibrated; alternating current ammeter to indicate the current merely for reference only.

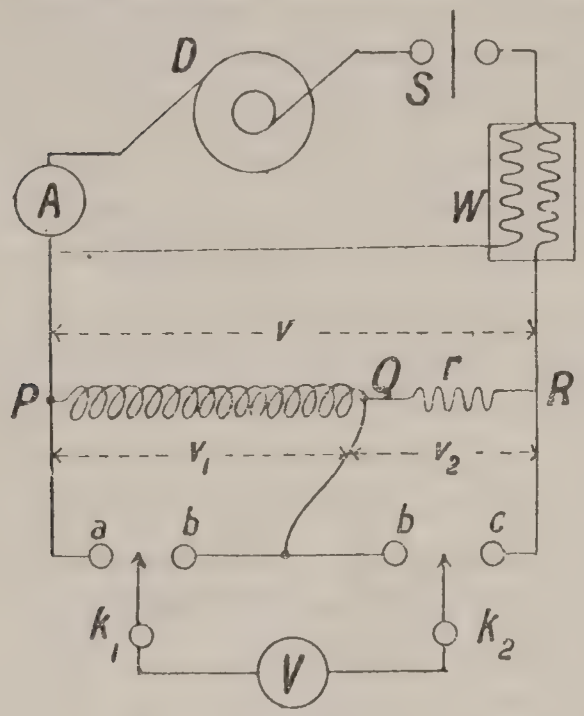


Fig. 18.

Note.—The resistances of PQ and QR should both be fairly small compared with that of the voltmeter V.

Observations.—(1) Connect up as in Fig. 18, and adjust the pointers of all the instruments to zero, levelling such as need it. See that all lubricators in use feed properly, and then start D.

(2) Adjust the speed of D so as to obtain the desired "frequency," say 100 $\sim$ per sec., at the same time varying the excitation to get the proper voltage, suppose 100 volts across PR. Adjust the resistance of PR so as to pass about $\frac{1}{10}$th of the full load current (necessary to give a full scale reading on W) through W. Then the speed and voltage being constant, note the reading on A, W, and in quick succession the voltages V_1, V_2 and V across PR, PQ, and QR respectively by moving k_1 and k_2 simultaneously.

(3) Repeat 2 for about ten different currents rising by about equal increments to the maximum allowable.

(4) Calculate the power absorbed in PR from the relation

$$W_1 = \frac{1}{2r}(V^2 - V_1^2 + V_2^2) \text{ Watts,}$$

where r is the ohmic resistance of QR.

If r is unknown or liable to be altered by the heating effect

of the current, its value $r = \frac{V_2}{A}$ may be substituted in the above relation. If the current and voltage are sine functions, $\cos. \theta = \frac{V^2 - V_1^2 - V_2^2}{2V_1V_2}$.

(5) Repeat 2—4 for a different frequency, say 60 $\sim$ per sec., to see whether the Wattmeter "constant" (K) alters, and tabulate your results as follows—

NAME . . . DATE . . .

Wattmeter tested: No. . . . Maker . . . Range . . . Temperature . . .

Speed r.p.m.	Frequency $\sim$ per sec.	Power Factor W_1/AV	Angle of Lag $\theta°$	r Ohms or V_2/A	Current in Amps. A.	Volts.			Power.		Wattmeter.		Error.	
						V	V_1	V_2	Apparent AV.	True W_1	Reading d.	Constant $K = W_1/d$	%	Mean.

Note.—Errors made in measuring the voltages V, V_1 and V_2 or in the graduation of the voltmeter scale will have least effect on the results when $V_1 = V_2$. If the formula in 4 is used with the substituted value of (r), this latter may consist of glow lamps, as the resistance *may* vary with the different mean current strengths.

(6) Plot a calibration curve for the Wattmeter tested, having values of deflection d as ordinates and true power W_1 as abscissæ.

Inference.—Prove the formula given in 4 and state any assumption made in obtaining it. What inferences can you draw from the results of your test? and explain why the resistance of the voltmeter V should be large compared with either PQ or QR.

(17) Calibration of a High Tension Wattmeter. (By Ohm's Law, using an auxiliary transformer.)

Introduction.—It is not always possible in actual practice and testing work to avoid the use of a Wattmeter on a high tension circuit, as for instance would be the case in measuring the efficiency of a high tension transformer run off the terminals of a high tension alternator. The Wattmeter in such a case should be a specially arranged one for the following reasons—

(1) Owing to the high pressure in the fine wire moving coil circuit, an extremely high *non-inductive* resistance, capable of standing the full pressure across its terminals, would otherwise have to be put in series with the fine wire swing coil if an ordinary Wattmeter was employed.

(2) Owing to the difficulty in obtaining the above resistance.

(3) The risk entailed in handling such an instrument, and of the breakdown of the insulation of the whole arrangement under the high pressure.

The best arrangement of a high tension Wattmeter, and which gets over these difficulties, is that shown symbolically in Fig. 19, together with the connections for its calibration.

The Wattmeter *W* consists of an ordinary Siemens electro-dynamometer; the mercury cups, forming the terminals of the swing coil, are connected to a separate pair of terminals by the side of the other pair forming those of the fixed coil. There is thus no electrical connection between the fixed and moving coils. These latter are connected to the low pressure coil of a small auxiliary transformer *T*. The high tension side of *T* is placed across the high pressure mains (*m.m.*), hence the moving coil of *W* passes a current which depends on the E.M.F. of *m.m.*, and at the same time there is no fear from a breakdown of insulation since both coils of *W* are passing ordinary currents. The actual current which *T* sends through the swing coil of *W* may be as small as convenient.

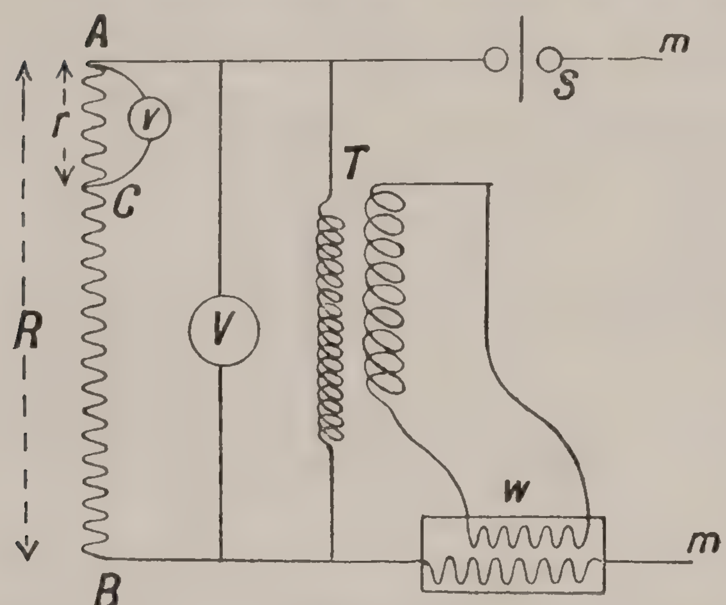


FIG. 19.

Apparatus.—High tension Wattmeter *W* to be calibrated arranged as mentioned above, with its fixed and movable coils separate. Small auxiliary (H.T.) transformer *T*; high and low tension electrostatic voltmeters *V* and *v* respectively; strictly non-inductive resistance *ACB* capable of being placed

across the (H.T.) mains *m.m.*, and of carrying enough current at that pressure to enable a considerable scale deflection to be obtained on W. A part AC of the whole resistance AB should have such a value (r) and be of such a carrying capacity as not to be heated and changed by the current through ACB and as will have a P.D. across its terminals capable of being read on v.

Note.—As a precaution, india-rubber gloves must be worn, and an india-rubber mat provided to stand on.

Observation.—(1) Connect up as in Fig. 19, and adjust the instruments carefully to zero.

(2) Close switch (S) and adjust V to read the desired amount which W has to deal with on future occasions. Note the readings of V, v, and W, and tabulate as follows—

NAME . . . DATE . . .

Wattmeter tested: No. . . . Maker . . . Range . . . Temperature . . .

Non-Inductive Resistances.		Voltages.		Current through ACB $A=\frac{V}{r}$	True mean Watts in ACB $W_1=VA$	Reading on Wattmeter d_w	Constant of Wattmeter $W_1/d_w=K$.
R.	r.	V.	v.				

Inferences.—Is the method liable to any sources of error? and if so, state them.

(18) Calibration of an Electricity Meter (on Constant Supply).

Introduction.—An electricity meter, which performs the same kind of office to a consumer of electrical energy that a gas-meter does to one using ordinary gas, is an electrical instrument that requires carefully calibrating or standardizing at some time or another. There are a great number of different forms of electricity meters, but they all come under one or other of four main classes, namely—ELECTROLYTIC, THERMAL, MOTOR, CLOCKS AFFECTED. It is not, however, our intention to dilate on these further as their theory and description comes under the scope of the ordinary text-book, but there are some points in general which may be remarked. Practically all meters measure one or other of two things, namely,

(*a*) Ampere-hours, when they are called *quantity-* or *Coulomb-meters*, (*b*) Watt-hours, when they are termed *Energy-* or *Joule-meters*. In most cases, though by no means all, meters are graduated and read directly in the official "Board of Trade Unit" (1000 Watt-hours). It must not, however, be supposed that because a meter *reads* Board of Trade units on its dials it is a *true* energy meter in the real sense of the word, *i. e.* a meter containing a current and pressure coil acting on one another in a suitable manner, for if the pressure is pre-assumed and taken as being constant it is an easy matter to graduate and calibrate the dials of a Coulomb-meter to read directly in B.O.T. units. This it may be remarked is generally done now.

Apparatus.—Accurately calibrated ammeter A, and voltmeter V; secondary battery B, or steady source of supply; switch S; power absorbing resistance L, or bank of lamps (p. 598); rheostat R (p. 597), which might be required for adjusting the pressure on the mains; meter M to be tested. When possible, it is best and most economical in power used, to employ two distinct circuits, one giving the necessary voltage for the fine wire coil of M, if there is one, the other the necessary current through the current circuit of M. These two sources must be secondary batteries if possible so as to be quite constant.

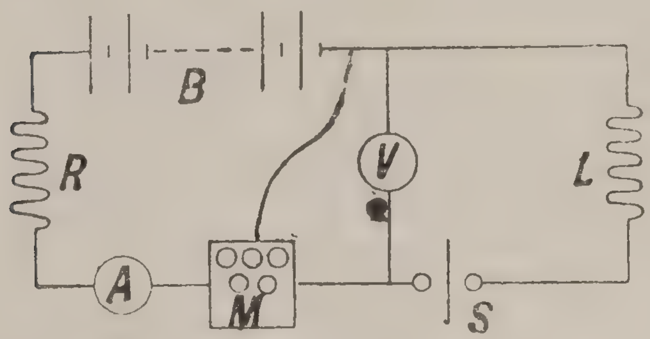


FIG. 20.

Note.—If the meter to be tested is an alternating current one, then A and V should be alternating current instruments, such as a Siemens electro-dynamometer and electrostatic voltmeter respectively, or the author's instruments. R also should be non-inductive, and L may preferably consist of a bank of lamps (p. 598) or other non-inductive resistance. If an accurately calibrated non-inductive Wattmeter is available, then the true power given to L can at once be obtained irrespective of the nature of R and L and also without using A. In such cases the *meter* should be *tested* at *different frequencies* and on *inductive loads*.

Observations.—(1) Fix up the meter in a position as nearly vertical as possible and connect it in circuit so as to register the

quantity (amp.-hours) or energy (Watt-hours) as the case may be, given to L.

(2) If the meter is intended for use on 100 or 200 volt circuits, close S and vary L so as to absorb the full load current of the meter, and adjust R so that V reads the required voltage.

(3) Open S and take the dial readings of the meter.

(4) At a known tabulated instant, switch on, and keep the current (A) and pressure (V) constant for about $\frac{1}{2}$-hour by altering R and L if necessary. Then switch off at a noted instant and take the dial readings again.

(5) Repeat 2, 3, and 4 for $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{10}$ full load currents through the meter, and tabulate as follows—

NAME . . . DATE . . .
Meter tested: Type . . Maker . . . No. . . .
Amps. (full load)= . . . Voltage (if any) . . . Dial reading in . . .

Meter reading		Amount Registered	Amps.	Volts.	Time.	True amp. or Watt-hours.	% error of Meter.
Start Q_1.	End Q_2.	Q_2-Q_1.	A.	V.	T (Hours).		

Inferences.—State the chief conditions which a meter of the above type should fulfil.

(19) Complete Test of an Electricity Meter.

Introduction.—In order to completely test an electricity meter in the way that would be advisable with any new type of instrument, three or four additional tests other than the preceding one should be carried out and are as follows:—

(*a*) *Starting power of the Meter.*—Obtained by carefully measuring the *least* current or Watts that will just cause the meter to start. It is obvious that such should not exceed the amount used up in the smallest lamp employed.

(*b*) *Effect of external Magnetism.*—Which can be investigated by sending a steady current or number of Watts through the meter according to what it measures, and then bringing a strong magnet into a number of different positions about the outside of the meter. The record of the instrument taken over a sufficient

period for each position of the external magnet should remain unaltered.

(*c*) *Power absorbed in the Shunt-coil.*—Some meters possess a fine shunt-coil of considerable resistance for the purpose of providing the instrument with a sufficient magnetic field to enable it to start with a very small current. The amount of power absorbed in this coil, if there is one, should be carefully measured and the cost of it per annum calculated on the basis of say 3*d.* to 4*d.* per B.T.U. if the supplier pays for it as he should. It should also be observed whether this coil is across the lamp or supply side of the meter in order to see if the consumer or supplier pays for the power so wasted, for in the aggregate the cost of this may amount to a considerable sum in the course of the year.

(*d*) *Gradual deterioration of working parts.*—This is most important, but can only be determined by a "*time test*" extending over a considerable period amounting to months.

Thus to furnish a true and accurate report on an electricity meter, investigations (*a*—*d*) should be undertaken, and in addition the test immediately preceding them.

(20) Measurement of a Resistance heated by an Electric Current.

Introduction.—When a resistance is heated by the passage of a current its value so heated may or may not be very different from that when it is cold. Thus, for example, a resistance composed of the alloys Manganin or Eureka, etc., would alter its resistance very little for considerable changes of temperature; whereas if made of carbon, the specific resistance of which diminishes rapidly as the temperature rises, the resistance would be very different when hot to what it would be while cold. In practice, however, one of two conditions may occur in connection with heated resistances, namely, (1) The type and form may be such that the temperature does not fall very quickly immediately the current is cut off, thus enabling time measurements of resistance to be taken with, say, a Wheatstone Bridge or other suitable means, and the resistance *hot*, at the moment of breaking the current, to be obtained graphically by plotting the results; owing,

however, to the difficulty of taking rapid time measurements of resistance and the introduction of other errors we shall not consider this method further. (2) The type of resistance may be such that the temperature falls very rapidly and far too quickly to enable any measurements, as in 1 above, to be taken. Such is the case with the filament of an electric incandescent lamp, and in order to obtain its resistance (warm), accurately, *during the passage of a current,* and which is absolutely necessary, another method has to be employed other than that of the Wheatstone Bridge, etc.

Though the results of the present test will disclose the fact, it may be mentioned here that the filament resistance of an electric glow-lamp when burning normally is very different from that when cold. Thus this latter, which can best be obtained by the Wheatstone Bridge, would not in any way represent the true resistance while the filament is under working conditions.

We will assume that the resistance in question is of the nature of an electric glow-lamp and therefore cools far too rapidly to allow of the employment of the first method mentioned. For a concrete case suppose that the resistance of the filament of an electric incandescent lamp is required at different luminosities, *i. e.* when different currents are passing through it. This resistance will diminish as the current increases, or as the temperature increases, owing to the specific resistance of carbon diminishing as the temperature rises. This property of carbon, in having a *negative* coefficient of variation of resistance with temperature, should be remembered as compared to the same property of all the metals and nearly all the alloys, of which "*Manganin*" may be cited as an exception in having a negative coefficient. The present method is a direct application of Ohm's Law, and consists in measuring the current through the lamp and the voltage across its terminals. There are, however, some precautions which have to be adopted in order to obtain the *true voltage and current,* and these we may now point out in connection with the two arrangements possible with this method. In each of the cases I. and II. (V) represents the voltmeter, (A) the ammeter, and R the glow-lamp or other resistance to be measured while hot. The rest of the main circuit is omitted for simplicity, but may comprise a suitable secondary battery and a rheostat for varying the current

through R. In *Case I.* the voltmeter V is connected directly to the terminals of R, hence the ammeter A will measure the *sum* of

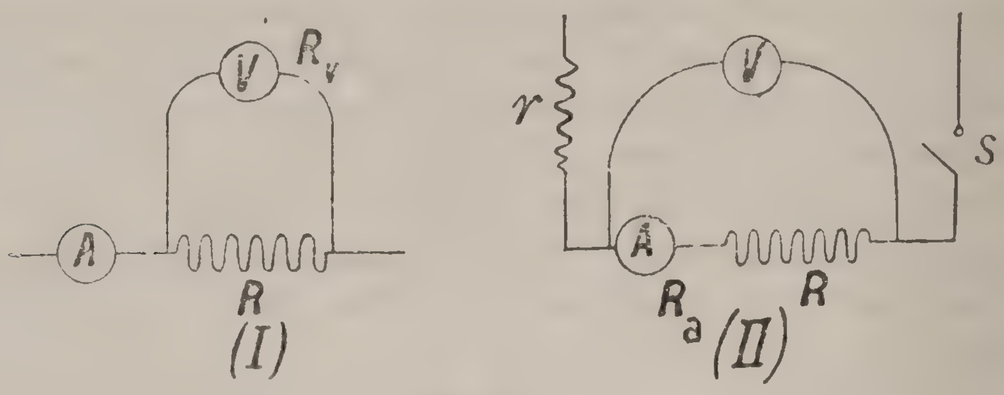


FIG. 21.

the currents through V and R together. But the method requires the actual current through R only, which is found as follows—

Let R_V = the true resistance of the voltmeter in ohms.

Then $\frac{V}{R_V}$ = the true current flowing through it in amperes, whence the *actual current through* $R = A - \frac{V}{R_V}$ amperes and the resistance of R (hot) $= \dfrac{V}{A - \frac{V}{R_V}}$ ohms.

To see the magnitude of the error caused by neglecting the correction for the voltmeter current, suppose $V = 100$ volts, $R_V =$ 10,000 ohms and A reads 0·61 ampere. Then without correction: R (hot) $= \frac{V}{A} = \frac{100}{0{\cdot}61} = 163{\cdot}93$ ohms, and with correction R (hot)

$$= \frac{V}{A - \frac{V}{R_V}} = \frac{100}{0{\cdot}61 - \frac{100}{10000}} = \frac{100}{0{\cdot}60} = 166{\cdot}66 \text{ ohms.}$$

In other words the error made in the resistance, neglecting the correction, = 1·64%. It should, however, be remembered that the average commercial voltmeter has a resistance much less than 10,000 ohms, and hence the above error would be much greater when using such. If R_V = Infinity then no correction is necessary; for, the resistance of R (hot) then $= \dfrac{V}{A - \frac{V}{R_V}} = \dfrac{V}{A - \frac{V}{\infty}} = \dfrac{V}{A}$ ohms or the voltmeter passes *no current.*

Such is the case with any electrostatic instrument such as the Kelvin multicellular voltmeter, the resistance of which is practically infinite. It is therefore a good one to use for the purpose.

In *Case II.* the voltmeter is connected across the ammeter and resistance combined, and it will therefore measure the *sum* of the voltages across R and A. But the actual voltage across R only, is required by the method, and this is obtained as follows—

Let R_A = the true resistances of the ammeter in ohms.

Then AR_A = the true voltage across the ammeter terminals, whence, the *actual voltage across* $R = V - AR_A$ volts and the resistance of R (hot) $= \dfrac{V - AR_A}{A}$ ohms.

To see the error caused by neglecting the voltage lost in the ammeter. Let V read 100 volts and A read 0·6 ampere, assume $R_A = 0{\cdot}1$ ohm, which is about the value for an instrument reading such small currents. Then we have

$$\text{without correction } R \text{ (hot)} = \frac{V}{A} = \frac{100}{0{\cdot}6} = 166{\cdot}66 \text{ ohms.}$$

$$\text{and with correction } R \text{ (hot)} = \frac{V - AR_A}{A} = \frac{100 - 0{\cdot}06}{\cdot 6} = \frac{99{\cdot}94}{\cdot 6}$$

$$= 166{\cdot}57^{\omega}.$$

Or the error made in not allowing for the loss of voltage in A is 0·054%. Thus, although the resistance of V can easily be measured on a Wheatstone Bridge and that of A either by the bridge or by the "fall of Potential" method (*vide* p. 84), when these cannot be readily obtained, we see that Case II. will give the best results and the freest of the two from error by neglecting any corrections.

Apparatus.—Accurate ammeter A and voltmeter V; the resistance R or glow-lamp to be measured while hot. The rest of the main circuit, if this is not already set up, comprising secondary battery, rheostat r, switch s, etc. Arrangements should be at hand for measuring the resistance of V and A, viz.—P. O. Bridge, galvanometer, Leclanché cell and standard known 0·1 ohm resistance, etc.

Observations.—(1) Connect up as in Case II. Fig. 21, and adjust the pointers of V and A to zero.

(2) Measure the resistance of the ammeter, voltmeter, and lamp (cold) by suitable means.

(3) Take a series of simultaneous readings on A and V for different voltages, rising by intervals of 10% from 0 to within about 5% of the normal, and then by steps of 1% to 5% above normal voltage

(4) Repeat (3) above, using a metal filament lamp instead of the carbon one, and tabulate thus—

NAME . . . DATE . . .

Nature of Resistance tested . . . Resistance Cold = . . . Ohms.

Ammeter ,, R_A = . . . ,,

Voltmeter ,, R_V = . . . ,,

Reading on Voltmeter.	True Volts (V) across Lamp.	Current through Lamp (A) Amps.	Resistance (hot) $\frac{V - AR_A}{A}$ Ohms.	% increase or decrease of resist. when hot.

(5) Plot curves for each lamp having values of lamp resistance as ordinates and corresponding currents through it and voltages across it, both as abscissæ.

(21) Measurement of the Efficiency and Candle Power of Electric Glow Lamps.

Introduction.—At the present day, when new forms of electric incandescent lamps are frequently making their appearance before the general public, or otherwise, it becomes of scientific interest and often of practical importance to thoroughly test the advantages claimed for these particular forms, and to discover their disadvantages. Amongst others, the chief investigations should be—

(α) The efficiency *at* and *about* the normal or rated voltage, stamped on the lamp by the makers. This is reckoned in "Watts per candle" for commercial purposes, though it should more correctly be termed its "in-efficiency." The number of "candles per Watt" more properly denotes the efficiency of a glow lamp as a light-emitting source.

(β) The candle power (C.P.) at the rated voltage and in a given direction.

(γ) At what efficiency, the total cost of operating this particular form of lamp, is a minimum.

These three investigations are of considerable moment to the user of such lamps, and the results practically decide whether his annual expense of electric lighting, using such a form of lamp, would be greater or less than with the present lamps in use. It

may probably be the case that it is impossible to do anything with the investigation marked (γ), owing to there being insufficient data to hand, the data required being (1) the cost of energy supplied, (2) the cost of lamp, (3) rate of variation of the life with Watts per candle, which is the most difficult item of the

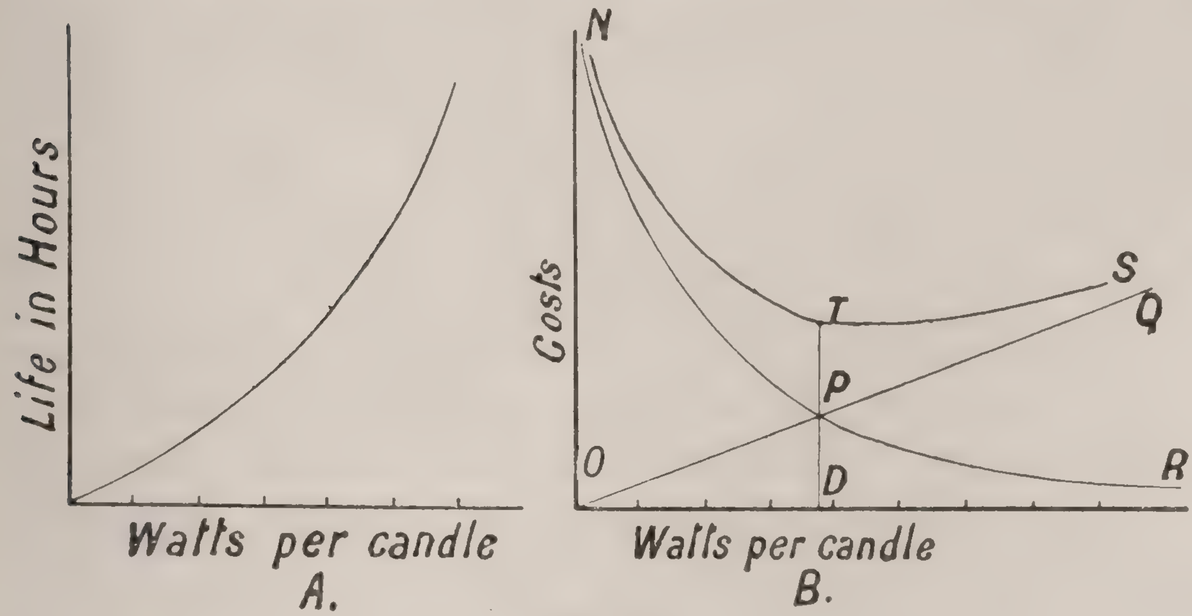


Fig. 22.

three to obtain. For purposes of discussion, however, as this particular question is of considerable interest, we will assume that very carefully-made tests give the relation between the life in hours of the lamp and the efficiency, or Watts per candle used, as shown in curve *A*, Fig. 22. Then on plotting curves *B* of the same fig., the cost of lamp renewals per hour will give us the curve *NPR*, and the cost of energy per candle-hour will give us the straight line *OPQ*. Summing the ordinates of the two curves, we get the third curve *NTS* convex to the abscissæ and representing the total cost of the only two sources of expenditure, namely, cost of lamps and cost of energy. If now an ordinate through the lowest point *T* of this third curve cuts the abscissæ in the point *D*, then *OD* (in this case 3·8) gives the efficiency, or Watts per candle, at which the total cost of operating this form of lamp is a minimum for the particular electrical supply taken.

The two first-named investigations (α and β) are contained in the following relations, which must be determined, viz.—the variation of C.P. with (1) amperes, (2) volts, (3) Watts, (4) efficiency (Watts per candle), (5) resistance of filament, and (6) the cost per candle-hour for energy with efficiency.

The candle power may be obtained by employing some convenient form of photometer, such as any one of those described on p. 589. We shall, however, here assume the use of a Bunsen grease-spot photometer, the carriage of which slides along an ordinary straight graduated bank or bench containing the standard of light at one end and the glow lamp to be tested at the other. If now C = the C.P. of the standard of light, and (d) its distance from the grease spot when "balance," in the manner to be described later, is obtained, also if D = distance between standard and lamp to be tested, then the C.P. of this lamp is

$$K = C\,\frac{(D-d)^2}{d^2}.$$

Now to facilitate working out the results of the tests, a calibration curve for the photometer bench may be drawn from calculations in which values of (d) are abscissæ, and the corresponding ones for $\frac{(D-d)^2}{d^2}$ as ordinates. Thus the ordinates of this curve × by the C.P. of the standard will give directly the C.P. of the lamp to be tested corresponding to the particular distance (d). The values of $\left(\frac{D-d}{d}\right)^2$ for various values of d when D=2, 3, 4, 5 and 6 metres, are given in the Table, p. 651, and they will be found to save a great deal of time in working out the results of photometric tests in general. Intermediate values not given in the table can best be obtained from the curve plotted between the numbers pertaining to the value of D used in the test and the values of $\left(\frac{D-d}{d}\right)^2$.

Referring to the above formula for calculating the unknown C.P., we see that any error made in reading the true position of the carriage carrying the "grease spot" or other balancing device will have minimum effect when $K = C$ or $d=(D-d)$, *i. e.* when this carriage is at the *midway* position between the two sources of light. The same kind of thing occurs in the case of measurements of resistance by the "Metre Bridge." To fulfil, however, the relation just mentioned, it will in most cases be necessary to employ a subsidiary or intermediate standard of light, such as a good electric glow lamp, which has itself been very carefully standardized by reference to an ordinary smaller standard.

Considerable difficulty may sometimes be experienced in balancing on the photometer with the naked eye when different sources of light are being compared owing to the difference in colour of the lights. In such cases it is of advantage to observe the "sight-box" containing the Bunsen grease spot or other arrangement through coloured glass when balancing; that known commercially as "signal red" and "green" being the best for the purpose, and two pieces should be chosen, so that on looking through the two together in bright daylight next to no light passes through them. Thus on balancing the sight-box by observing through each separately, the mean of the readings will afford a correction to a certain extent for any difference in colour of the two sources of light.

The principal object gained in using coloured glasses is that the eye then observes a less bright surface, and is consequently better able to gauge its illumination relatively to the surrounding surface. It is a fact that when the eye looks at a very bright surface, the pupil of the eye partially contracts, thus causing the effect of temporary partial blindness, hence the use of coloured glass to prevent this.

It should be carefully remembered that if the resistance of the voltmeter, which measures the pressure, is not very high compared with that of the lamp (say exceeding 100 times), and the ammeter resistance not very low compared with that of the lamp (seldom the case), then corrections must be applied to one or other of these instrument readings, in order to obtain either the true voltage on the lamp or true amps. through it. For such see test on p. 48.

Apparatus.—Low reading ammeter A with long open scale (p. 559); high resistance voltmeter V; adjustable fairly high resistance R (Fig. 272); secondary battery and photometer bench DD complete, containing "Methven screen" 2 C.P. standard (C) of light (p. 595), or Fleming standard glow lamp; glow lamp G to be tested; and "sight-box" B containing a Bunsen grease spot (p. 592), or Flicker photometer head; switch S.

N.B.—For particulars on the adjustment and use of the Methven standard, see Appendix, p. 595. The lamp G to be tested may be run as low as will give measurable luminosity, but not higher than 5% above its normal voltage

Observations.—(1) *Fix*, in a manner most convenient for calculation, *the distance D* between standard and lamp to be tested (500 cms. say), and adjust the standard to the certified standard value of C.P.

If a standard glow lamp is used instead of *C* connect it to a constant voltage supply through an exactly similar circuit to that shown for *G*, but without an ammeter (*A*).

(2) Connect up the glow lamp as indicated in Fig. 23, and adjust the pointers of *V* and *A* to zero, levelling the instruments carefully if necessary.

(3) Measure the resistance of the lamp filament while cold by means of the Wheatstone Bridge, and note its value.

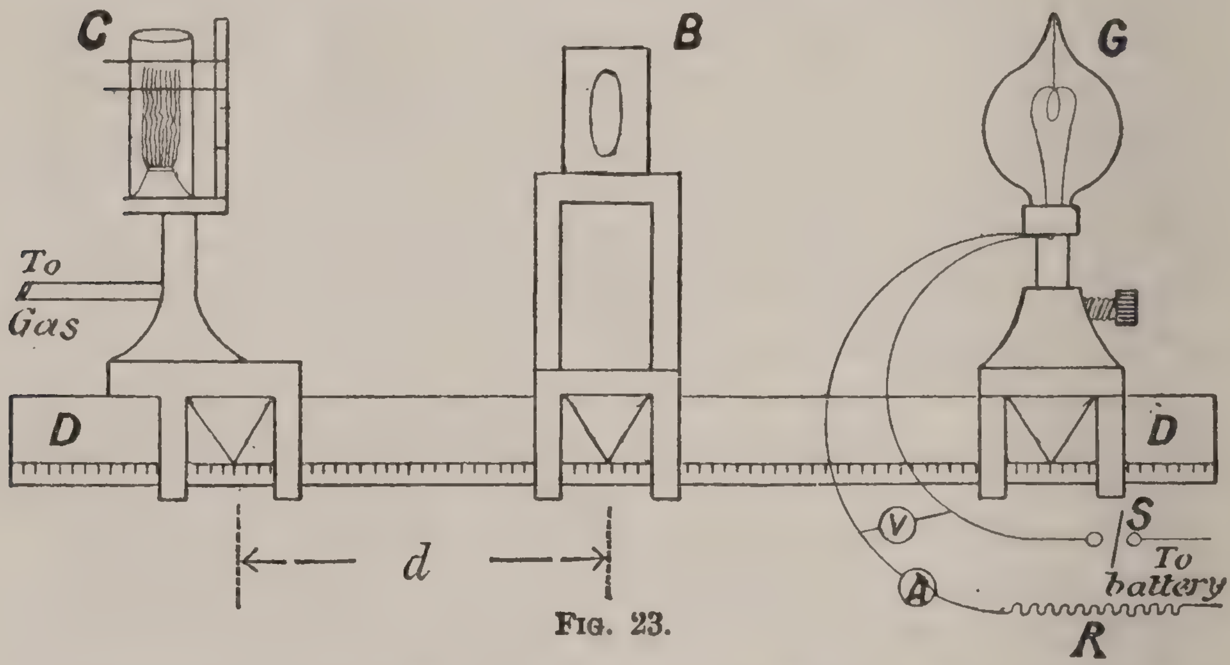


FIG. 23.

(4) Close *S* and adjust *R*, so as to obtain just measureable luminosity on *G*, then move the carriage carrying the "grease spot" until the whole surface of the latter appears equally illuminated. *Great care being taken to keep the standard of light properly adjusted* all through the tests at its certified value.

N.B.—The *true* scale position will be found more accurately by taking the *mean* of the two positions of the carriage when the spot is *just perceptibly darker* and *lighter* respectively than the surrounding paper, or with a Flicker head when the Flicker disappears.

Note in each case the distance (*d*) from standard to grease spot when the *plane of the lamp filament* is (*a*) parallel, (*b*) perpendicular to the axis of the bench, coloured glass being used in each case if necessary.

(5) Repeat 4 in 10% intervals of voltage across the lamp terminals up to within about 5% of the normal voltage for the lamp tested, and afterwards in 1% intervals to the maximum allowable.

(6) Calculate the C.P. (K) of the lamp at each voltage from the relation

$$K = C\,\frac{(D-d)^2}{d^2}$$

where C = C.P. of the standard and (d) = *mean of all* the bench readings of the grease spot as found in 4 above. Tabulate all your results as follows—

NAME . . . DATE . . .

Lamp tested: Make . . . Class . . . Normal Volts . . . Normal C.P. . . Resistance Cold . . .
Standard of Light: Type . . . C.P. = . . . D = . . .

Current (A) Amps.	P.D. (V) Volts.	Power (W) Watts.	Resistance (hot) $r=\frac{V}{A}$ (Ohms).	Plane of Filament and axis of Bench.				Total mean position (d)	C.P. K	Efficiency in Watts per candle.	Cost per candle-hour for power P
				Parallel.		Perpendicular.					
				Spot just perceptibly							
				dark	light	dark	light				

(7) Plot the following curves, to the same pair of axes and same scale for C.P. (K) on the ordinates, between K and (1) amperes, (2) volts, (3) Watts, (4) resistance, (5) distance (d), (6) efficiency, (7) the cost P.

(8) Calculate the ratio of the resistance "hot" to that "cold" for the lamp filament.

Note.—In calculating the cost P at the various C.P.s, assume that electrical energy costs 6*d.* per Board of Trade unit (1000 Watt-hours).

Inferences.—State at some length all the inferences which can be drawn from the above experimental results and curves.

(22) Variation of Candle Power with direction around an Electric Incandescent Lamp.

Introduction.—With the introduction of new designs of electric glow lamps at the present day it is of considerable interest and often of importance to see the way in which the magnitude of the C.P. along a fixed or given direction changes as the lamp is turned

through various angles in both horizontal and vertical azimuths. The glow lamp to be tested should be capable of being turned in any direction about a point which is the centre of the principal part of the filament, and, further, this point must be in a line with the standard of light and centre of the Bunsen grease spot or other "sight-box."

If the lamp thus adjusted is supplied at constant voltage and the C.P. measured at different angles in the horizontal plane as the lamp is turned completely through the circle, then the *mean* of all these C.P.s gives what is termed the "*mean horizontal C.P.*" If in addition the C.P. is now measured all round a vertical circle, the plane of which successively makes different angles with the axis of the photometer bench around a horizontal plane, then the mean of all the results will give what is termed the "*mean spherical C.P.*," and it will be found that the ratio

$$\frac{\text{mean spherical C.P.}}{\text{mean horizontal C.P.}} = K,$$

a constant which may be determined in the manner to be described presently.

The variation of C.P. around the lamp, as found in the present test, can best be seen by plotting "polar curves" for the different planes in which the filament is turned, using "polar co-ordinates." We will now consider briefly the approximate general form and method of plotting such curves.

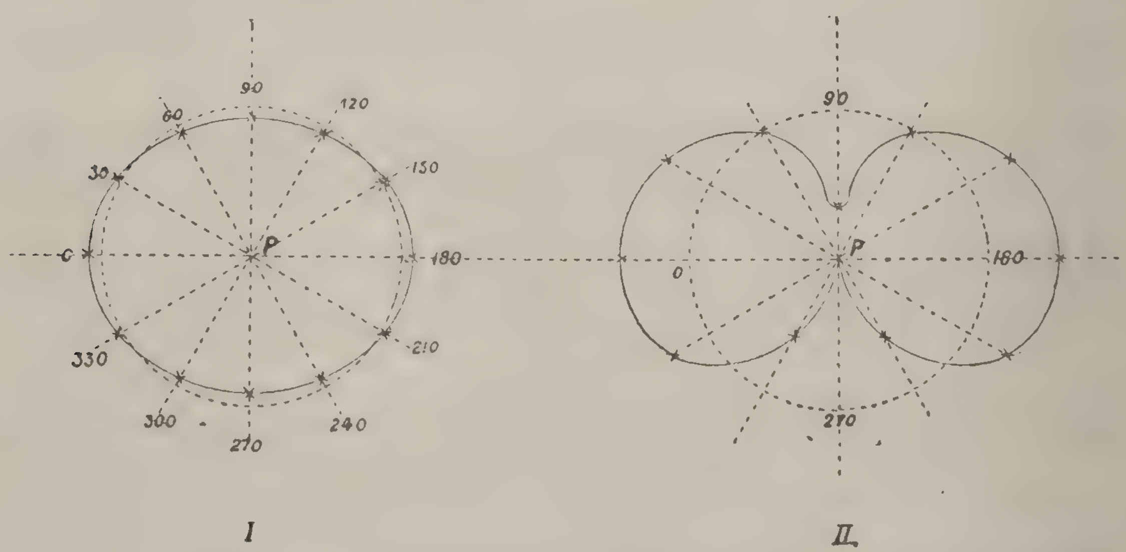


Fig. 24.

Fig. 24 I. and II. represent polar curves showing the distribution of luminosity in a horizontal and vertical plane respectively.

To obtain I. take any point or pole (P), and with it as centre describe a circle, the radius of which represents to a suitable scale either the normal rated C.P. of the lamp, stamped on it by the makers, or else, the *mean horizontal C.P.* as determined from the experimental results. Divide the circle (shown dotted) into 12 equal divisions of 30° each all round from 0° to 360, and draw a radial line from P through each, then setting off the C.P.s measured at the respective angles on these radial lines, the continuous curve is obtained on suitably joining them. In Fig. 24 I. the starting-point 0° would be that position of the lamp when the plane of the filament is parallel to the axis of the photometer bench. In II. the datum circle is drawn in the same way as before (and is shown dotted), and on setting off the C.P. measured at the respective angles as the lamp is now turned in a *vertical plane*, the continuous curve, somewhat of the shape shown, will be obtained. This polar curve II. will only represent the distribution of C.P. in that particular vertical plane which makes some noted angle with the above-mentioned zero on the horizontal plane or circle.

Apparatus.—Precisely the same as that mentioned in the preceding test, with the single exception that the glow lamp is now held in a special form of holder capable of turning through known angles in horizontal and vertical planes.

Observations.—(1) Connect up as indicated in Fig. 23, and adjust the pointers of the instruments V and A to zero, levelling the meters if necessary.

(2) *Fix*, in a manner most convenient for calculation, *the distance D* between standard and lamp to be tested (500 cms. say) and adjust the standard of light used to the proper C.P., either using the gas carburettor or otherwise.

(3) Close S and adjust R so as to obtain exactly the normal rated voltage across the lamp terminals, which must be kept *perfectly constant* throughout the whole set of tests.

(4) Adjust the lamp and its holder so that the principal part of the filament can rotate in a horizontal or vertical plane about some fairly definite point, the line joining which to the standard passes through the grease spot and is parallel to the photometer bench.

(5) With the voltage at exactly the normal value and the index

at zero on the horizontal scale for the plane of the filament parallel to the bench, measure the C.P. every 30° on the horizontal scale as the lamp is turned round (*vide* obs. 4 and 6 of the last test).

(6) With the index at 0° on the last-named scale measure the C.P. every 30° on the vertical scale as the lamp is turned round.

(7) Repeat 6 for the index at 45° and 90° on the horizontal scale, and tabulate all your results as follows—

NAME . . . DATE . . .
Lamp tested :— Make . . . Class . . . Normal Volts . . . C.P. . . . Resistance Cold . . .
Standard of Light. Type . . . C.P. . . . $D=$ . . .

Horizontal scale readings.	Vertical scale readings for different horizontal angles.			Distance d	C.P. K	Mean K	Ratio $\frac{M.S.K.}{M.H.K.}$
	0°	45°	90°				

(8) Plot the polar curves for horizontal and vertical distributions in the manner set forth above. The latter for each of the angles 0°, 45° and 90°.

(23) Measurement of the Percentage Absorption of Light by different kinds of Shades and Lamp Globes.

Introduction.—In electric lighting particularly, and also in other methods of illumination, it is almost invariably the case that the lamp is partly or wholly enclosed in a globe or shade which is partly for use in softening the light, as we may express it, and partly for ornament. In arc lighting the opalescent globe is very generally used, while in the case of electric incandescent lighting either the bulbs of the lamps themselves are often made of opalescent, ground, frosted, coloured or other translucent forms of glass, or separate shades of such material enclose the ordinary clear glass bulb of the lamp. In all cases the result is a more evenly diffused light and a more uniform illumination, and one that is softer, so to speak, and less trying for the eyes. The introduction of such shades, however, usually diminishes considerably the outside illumination of the source owing to the absorption of light by the shade, but it should be borne in mind

that some of the light which is apparently absorbed is actually lost by reflection. As, in some cases, so much as 70% of the light produced is thus absorbed, it becomes of importance to determine the amount in particular cases, for it will be seen to materially affect the number of lamps really needed to illuminate satisfactorily a room of given area. The present test is arranged with the object of doing this, but no account will be taken of loss of light through reflection, as the measurements of this and absorption separately requires more elaborate methods.

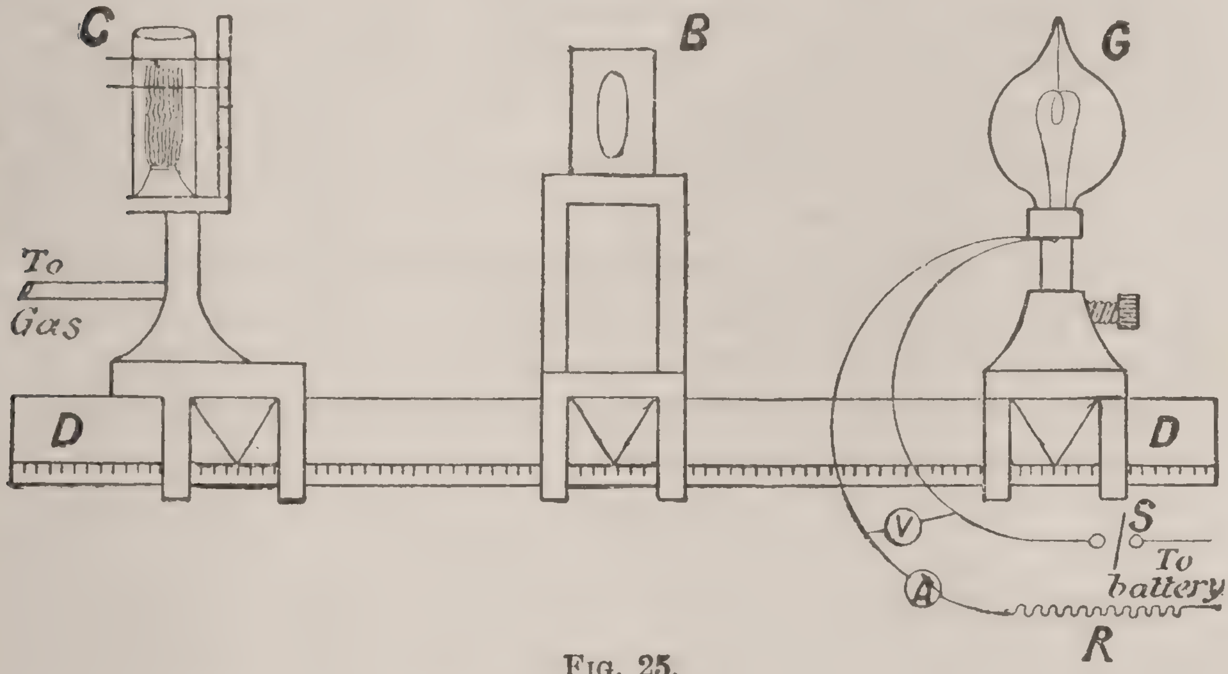


FIG. 25.

Apparatus.—Low reading ammeter A with long open scale (p. 559); high resistance voltmeter V; adjustable fairly high resistance R (p. 603); secondary battery and photometer bench DD complete, containing "Methven screen" or other standard (C) of light; "sight-box" B containing a Bunsen grease spot (p. 592); switch S; and an electric glow lamp G (say of 8 C.P.); shades to be tested.

N.B.—For particulars on the adjustment and use of the Methven screen standard of light, see Appendix, p. 595.

Observations.—(1) *Fix*, in a manner most convenient for calculation, *the distance* D between standard and glow lamp (500 cms. say), and adjust the standard to the proper C.P., either using the gas carburettor or otherwise.

(2) Connect up the glow lamp G as indicated in Fig. 25, and

adjust the pointers of V and A to zero if necessary and levelling them if required.

(3) Close S and alter R so as to obtain the *normal voltage* across the lamp terminals as read off on V. Then move the carriage carrying the "grease spot" until the whole surface of the latter appears equally illuminated, great care being taken to keep the standard of light properly adjusted throughout the tests.

N.B.—The *true* scale position will be found more accurately by taking the *mean* of the two positions of the carriage when the spot is *just perceptibly darker and lighter*, respectively, than the surrounding paper. In this way the mean distance (d) from standard to grease spot should be taken when the plane of the lamp filament is (a) parallel, (b) at 45°, (c) perpendicular to the axis of the bench, and the volts and current noted at each (which must be kept constant).

(4) Now place a given shade to be tested for absorption over this plain glass bulbed lamp and repeat 3 for this and all other shades in succession, keeping the lamp voltage quite constant.

(5) Calculate the C.P. (K) of the light with and without shades from the relation

$$K = C\,\frac{(D-d)^2}{d^2}\ \text{candles},$$

where C = the candle power of the standard and d = *mean* of all the bench readings of the grease spot as found in 3 above, and tabulate your results as follows—

NAME . . . DATE . . .

Glow Lamp: Normal Volts= . . . Nature of glass bulb= . . .

Standard of Light: Type . . . C.P. . . . Total distance D= . . .

Nature or kind of shade tested.	Volts V.	Amps. A.	Plane of Filament and axis of Bench						Total mean of all positions (d).	C.P. of light.		Absorption by shade $100\,\frac{K_0-K_1}{K_0}\%$
			Parallel.		At 45°.		Perpendicular.			Without shade K_0	With shade K_1	
			Spot just perceptibly									
			dark	light	dark	light	dark	light				

Inferences.—State carefully all you can infer from the results of your experiments and point out their bearing on the lighting of rooms in general.

The Photometry of Electric Arc Lamps.

General Remarks.—In photometric measurements of the present nature, many little difficulties exist which do not appear in the study of nearly all other sources of light. They arise from the fact that, in the first place, the intensity of the light is extremely fluctuating and very difficult to maintain constant for any appreciable length of time. In the next place, the intensities are usually so large that special standards of light have to be employed, and in addition, the general arrangements of the arc light source relatively to the photometer and standard have to be such that only a known fraction of the light to be measured is balanced against the standard. The chief difficulty, however, arises from the great difference between the colour of the arc light and that from all other common standards of light. So marked is this, both in the case of direct and alternating current arcs, that frequently the unpractised eye is unable to form anything like an accurate judgment between the amounts of illumination on a given surface due to each. Lastly, the arc is continually travelling round the carbons, thereby causing wide variations in the light emitted along the axis of the photometer bench. This effect on the correct readings of the photometer "*sight-box*" for given positions can be minimized by taking the mean of several readings of the sight-box on the bench for as nearly as possible the same values of voltage and current supplied to the lamp.

As is well known there are some characteristic differences between continuous (direct) and alternating current arcs. In the former, the positive and negative carbons burn away at rates approximately in the proportion of 12 : 7 respectively, which, however, may vary from 1·2 to 4, according to the quality and type of carbons and the voltages and current supplied to the arc, the first-named ratio only holding approximately true for ordinary lamps with carbons of equal diameter, and using from 10 to 12 amps. at 45 to 50 volts and run with continuous currents. Peculiar to this type of lamp is the formation of a recess or "*crater*," as it is commonly called, at the end of the $+^{ve}$ carbon, the $-^{ve}$ assuming a conical pointed shape at the end.

For lighting purposes the lamp is always connected up and suspended so that the $+^{ve}$ carbon is uppermost, and no second consideration is necessary to at once see that the distribution of light *all round* the arc (*i.e.* the spherical distribution) is far from being uniform. This determination, together with that of the spherical candle power (C.P.) for a given amount of power absorbed by the lamp and the regulation of the lamp mechanism, amongst other tests, is the object of the following investigation.

The distribution of intensity requires for its determination somewhat special arrangements for enabling the C.P. to be taken at different angles to the horizon line. This variation is measured in a vertical plane only, and there are several devices for carrying it out. One is to suspend the lamp from the ceiling by means of cords and pulleys or a pulley block, so that it can be raised or lowered to different heights above the central axis of the photometer bench; the light is then reflected by a suitably placed plane mirror (making an angle of 45° to the bench) along the axis of the bench to the sight-box; the centre of the mirror forming a right angle between the axis of the bench and the direction of the incident beam from the arc. The mirror is so arranged that the reflected rays always make an angle of 45° with the axis. The absorption of light or coefficient of reflection by the mirror at this angle is carefully measured and allowed for. Let this co-efficient or percentage of the total light striking the mirror, which is reflected, $= K$. Then C.P. of reflected beam $= \frac{K}{100} \times$ C.P. of beam from the arc, and this reflected beam is then measured against the standard. The distance of arc lamp from photometer sight-box then is reckoned as = distance between lamp and mirror + that between mirror and sight-box. This enables large C.P.s to be compared with comparatively small ones, which would otherwise necessitate a bench many yards long.

STANDARD OF LIGHT.—This should be as large as possible in magnitude, so as to keep the sight-box near the centre of the photometer bench, and thus minimize errors in its true position. In addition the colour of the light should approximate to that of the arc. The standard which fulfils these conditions in a fairly satisfactory manner is one consisting of an *over-run* glow lamp of,

say, 32 C.P. at normal voltage. If this is over-run some 5–8% in voltage, it will emit a much whiter light and one that is more nearly the colour of the arc. This lamp must be carefully standardized against a known standard of light at two or three definite and accurately noted voltages above normal. Probably 5% over normal will give a C.P. = about 40 or 45, and 7% over normal about 50—60 C.P. At this abnormal voltage the bulb will blacken inside fairly soon, and hence the lamp should not only be kept on for the shortest time, but should frequently be re-standardized.

Differences in colour between the two lights to be compared may to some extent be corrected by taking the readings of the slider carrying the sight-box when observing the balance of illumination of the latter through red and green glass in succession, the best kinds for the purpose being what are known commercially as "*signal-red*" and "*green*," and which should be so chosen that a bright sunlight viewed through the two together appears quite dark. The effect and object in using coloured glass is explained on p. 53.

There are really two kinds of efficiency determinations recessary in connection with arc lamps which are automatically self-regulating, namely—

(*a*) The "Commercial Efficiency" of the lamp as a whole reckoned in Watts per candle emitted, and which takes into account the total power in Watts absorbed by the lamp, *i.e.* in the arc and regulating mechanism.

(*b*) The "Nett Optical Efficiency," reckoned also as above, but taking into account only the power given to the arc itself and neglecting that absorbed in the regulating mechanisms.

The following tests are devised for the purpose of investigating these separately and comparing the results, and also of examining other very important points pertaining to arc lamps in general.

(24) Measurement of the Commercial Efficiency of an Arc Lamp.

Introduction.—In the present instance the photometer bench being of considerable length, presumably, the two sources of light are placed one at each end and in a line with the photometer

sight-box, which will in future be termed the *screen*, for brevity sake.

Apparatus.—Photometer bench *BB*; fitted with a Bunsen *grease-spot screen* (*G*) (p. 592), placed inside a sight-box to prevent stray light, due to reflection from the walls of the room, falling on the screen, the walls and ceiling being as *dull black* as possible; standard known source of light (L^s) consisting of an over-run 32 C.P. glow lamp, carefully standardized at a known voltage by a previous test.

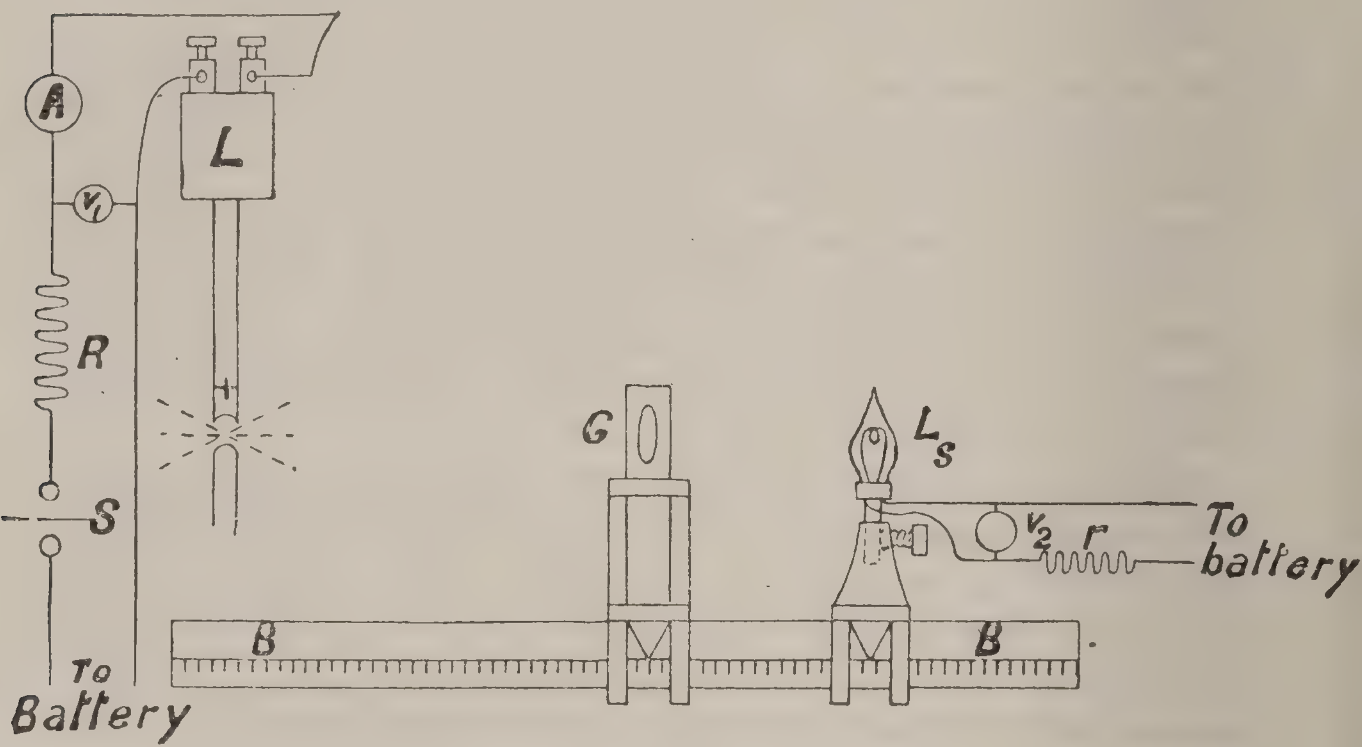


FIG. 26.

A voltmeter V_2, preferably the same used in the calibration of the glow lamp, together with a rheostat *r* (p. 603) for reproducing the voltage of calibration. Arc lamp (*L*) to be tested, supported on a suitable stand, and placed in circuit with an ammeter *A*; voltmeter V_1; rheostat *R* (p. 606), and switch *S*. A secondary battery should be available to feed both circuits in preference to a dynamo current, as the former gives a far more steady E.M.F. and better results than the latter.

Observations.—(1) Adjust the arc lamp in its cradle or stand, so that the point of contact of the carbons is at the same height above the bench as the centre of the screen and standard light. Centre the two carbons very carefully.

(2) Fix the distance *D* between arc and standard at some con-

venient amount for future calculations (say 600 cms.), and adjust the arc lamp so that the carbons are vertical.

(3) Connect up as shown in Fig. 26, and adjust the pointers of A, V_1 and V_2 to zero if necessary. Vary r so as to give L_s the voltage as read off on V_2, at which it was standardized last, when it will then give a definite known C.P.

(4) With R full in, close S, and vary R so that the arc just burns with the carbons in equilibrium. Now quickly adjust the position of the sight-box, so that the grease spot appears equally illuminated all over each side. Note its scale reading (d) from the standard, the volts V_1, V_2 and the amps. A.

Note.—V_2 must be kept rigorously constant during this and the following observations. The scale reading d can be obtained most accurately by reading it when the grease spot is just perceptibly darker and lighter respectively than the surrounding paper. This should be done, using red and green glass in addition to the naked eye, and the mean of all the readings recorded. The reading of V_1 and A must be constant during the taking of the above readings.

(5) Re-adjust R and repeat 4 for about ten different values of V_1, rising by about equal increments to 60 volts or so, V_2 being constant.

Each of the readings in 4 and 5 should consist of a group of 3 or 4 observations with V_1 and A constant, so that a mean may be taken which would allow for alteration in C.P. due to the arc travelling round the carbons.

Tabulate your results as follows—

NAME . . . DATE . . .

Arc lamp tested : No. . . . Maker . . . Type . . .
Type of carbons . . . Size of positive . . . Size of negative . . .
Standard light: Type . . . Candle power K_s = . . . D = . . . cms. Ammeter Resist. = . . . Ohms.

Volts on Standard V_2.	Amps. through arc lamp A.	Volts across lamp V_1.	Watts given to lamp AV_1.	Distance.		C.P. of "Arc" $K_a = \frac{(D-d)^2}{d^2} K_s$	Commercial Efficiency $\frac{AV_1}{K_a}$.	Costper candle hour for power.
				d.	$D-d$.			

(6) The voltmeter used for V_1, especially if a hot wire, must be shunted, not directly to the lamp, but to the lamp and ammeter A combined. The volts lost in A can be calculated and sub-

tracted from that shown on V_1 in order to get the *true volts* on the lamp.

The ammeter resistance is required for this correction and can be found approximately by the Wheatstone Bridge.

(7) Plot the following curves on the same sheet of curve-paper between C.P. as ordinates in each case, and (i) Volts V_1, (ii) Amps. A, (iii) Watts AV_1, (iv) Efficiency as abscissæ.

(8) Compare the above cost per candle hour for power at 6*d.* per unit at normal voltage across the arc lamp with that of a glow lamp of equal C.P., taking 3 Watts per candle at normal voltage. The price of energy being the same.

Inferences.—State clearly all you can infer from your experimental results.

(25) Determination of the Nett Optical Efficiency of an Arc Lamp.

Introduction.—This test is similar to the last, with the following exceptions—

Place the voltmeter V_1, which should be of a sensitive high resistance type, *across the arc* instead of the lamp terminals as in the preceding test. This can be done by connecting it to two spring clips, which make good contact with the carbons about two inches from their ends next to the arc. The ammeter (A) must now be so arranged in the circuit that it measures the current through the arc without taking into account that passed by any shunt-coil which the lamp may happen to have.

Apparatus.—The same as for the commercial efficiency test, and in addition the two necessary spring clips.

Observations.—Repeat those of the foregoing test exactly as there indicated.

Connect up as in Fig. 26, with the exceptions noted above as regards the position of the ammeter and voltmeter.

Compare the efficiencies obtained from the two tests, and also the costs per candle hour for power at the same price.

(26) Determination of the Distribution of Light from an Electric Arc.

Introduction.—The distribution in the case of an arc light is obtained by measuring the C.P. at various angles to the horizon, in one single vertical plane containing the central axis of the photometer bar.

To enable this to be done some arrangement, similar to the one briefly described under "General remarks" on arc light photometry, is necessary.

The author, however, has devised a simple form of "*cradle*" in which to fix the lamps, and which is illustrated and described in the Appendix, p. 587.

The difficulty arising in the use of such an arrangement is due to the fact that most arc lamps will not continue to self-regulate when placed in a slanting position. The author, however, finds that, with a suitable type of lamp, there is practically no difficulty from the above cause, until the cradle makes an angle of about 50° with the horizontal, and for just the one or two readings after this it is easy to help the mechanism by hand in order to maintain the "intake" of electrical power by the lamp constant.

Apparatus.—Precisely that mentioned for the test No. 24, p. 63, on "Commercial Efficiency," except that the cradle is now required where just an ordinary stand would have done in that test.

Observations.—(1) Repeat 1–3 of the above-cited test, seeing in addition that the carbons touch at a point which is in a line with the centre of the axle of the cradle, and that their axes coincide.

(2) Set the cradle with its pointer at zero, when the carbons will be vertically over one another. With R full in, close S, and vary R so that the lamp takes its normal voltage and current and burns quite steadily, then quickly balance (by moving the screen), in the manner set forth in observation 4 of the test cited; repeat this three or four times for the same values of A and V_1, and record the mean in the table.

(3) Repeat 2 above, for the *same values* of A and V_1, for every 10° through which the cradle is turned, up to 70° or 80°, when

the axis of the carbons will be nearly parallel with the photometer bench, and tabulate as in the table for test 24 cited, substituting the heading "Angle between carbons and horizon" for the cost, etc., in the last column of the table.

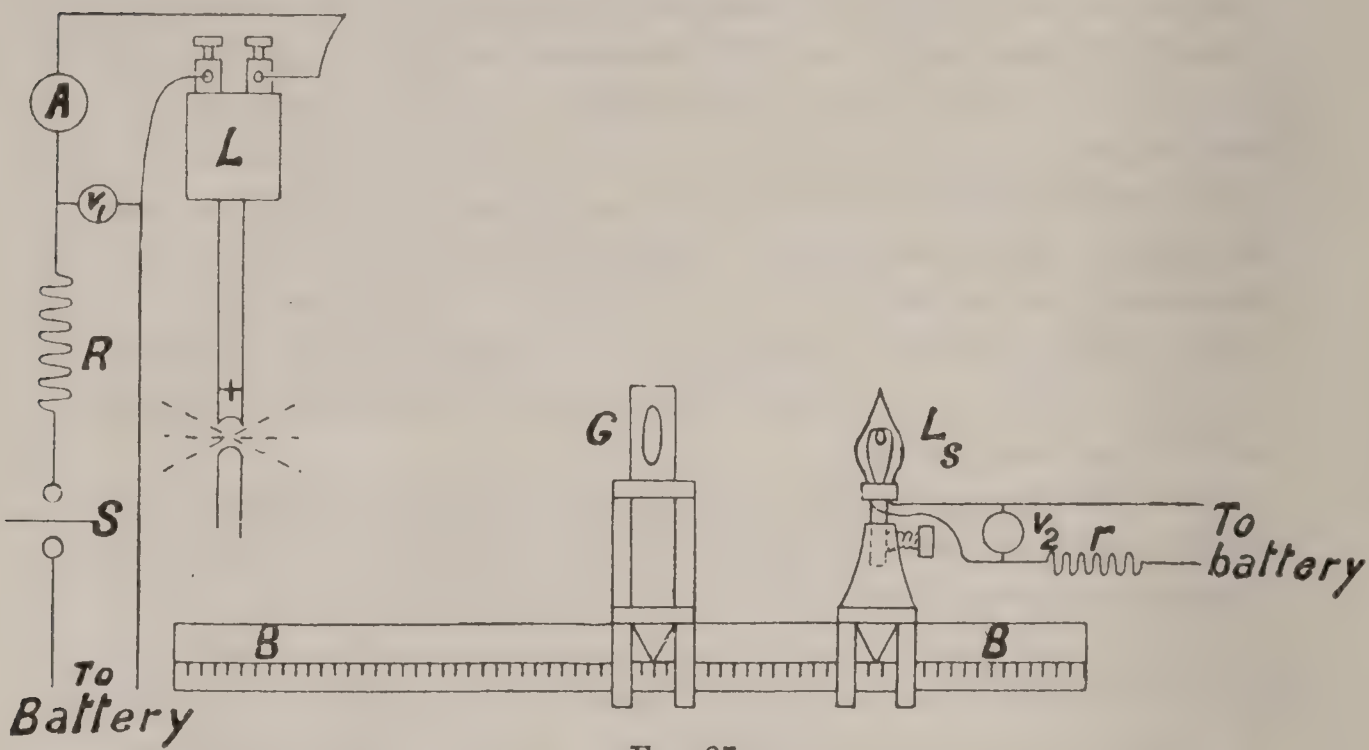


FIG. 27.

(4) Repeat 2–3 for a considerably lower current A than the normal, but one at which the arc burns properly.

(5) Plot the *Polar Diagram* or curves of distribution of light from the arc at various angles for each current used in the manner described below.

Inferences.—State clearly all you can infer from your experimental results, and point out their bearing on the lighting of streets and large areas by means of arc lamps.

Determine from your results the mean spherical efficiency which $= (\frac{1}{4}$ horizontal efficiency $+ \frac{3}{4}$ max. efficiency).

Note.—This may vary from 0·050 to 0·200, depending on the diameter and quality of the carbons, and is the ratio of the normal power in Watts, as given to the lamp in observation (2) above, to the mean spherical C.P. resulting.

PLOT OF POLAR DIAGRAM AND DISTRIBUTION OF LIGHT FROM ARC.

Let D be the junction of the + and − carbons, *i.e.* centre of arc.

Let DB represent the maximum C.P. obtained for some position of the arc, then with D as centre and DB as radius, draw the semi-circle ABC. Divide this into eighteen equal parts, each of which will therefore = 10° of arc, and draw radii to the points of intersection so formed. Now set off to the same scale as DB, the various C.P.s measured, along the respective radii from D, representing the angles in which they were measured. Then the curve $DGFH$ drawn through these points is the polar diagram of C.P.s from the arc.

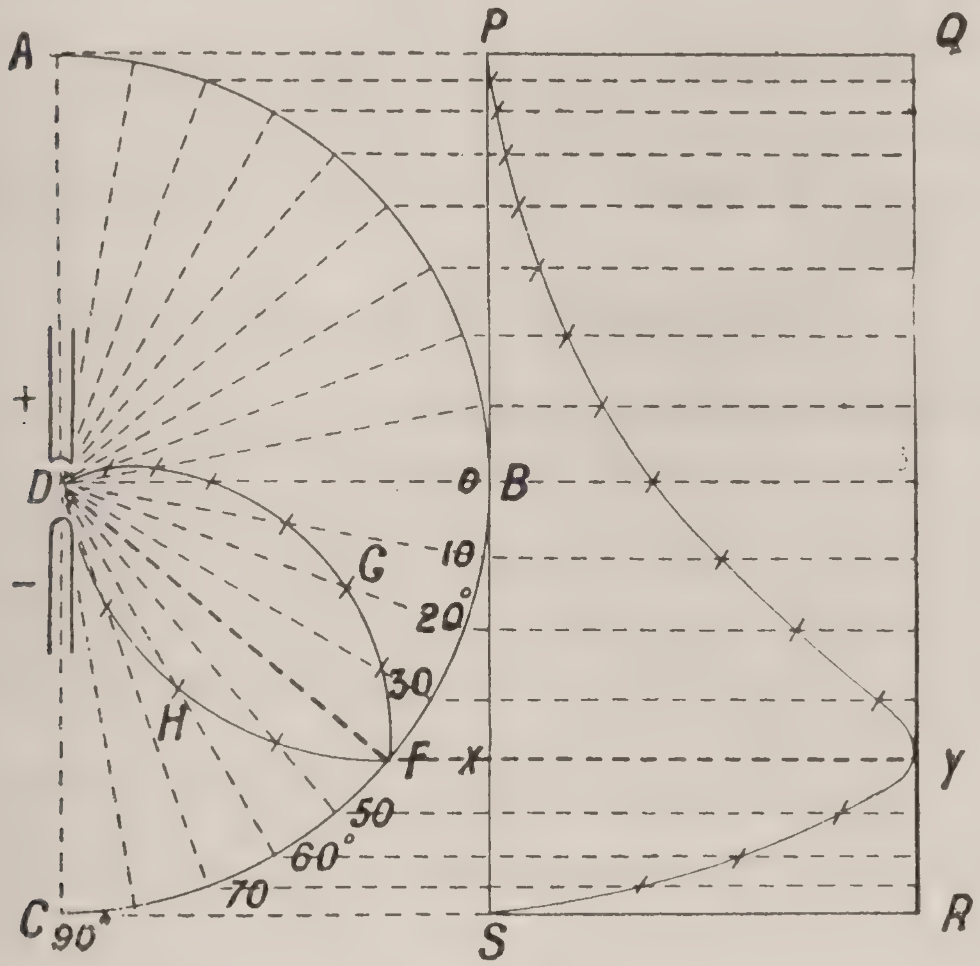


Fig. 28.

The distribution corresponding to this polar diagram is obtained as follows—

Draw PBS through B parallel and equal to ADC.

Through each of the points of division on the semi-circle ABC draw lines parallel to DB, and therefore perpendicular to PBS. From PS on these set off lengths proportional to the respective C.P.s at the corresponding angles. Thus, for instance, $XY = DF$ = maximum C.P., and so on for the rest. Now complete

the rectangle $PQYRS$ and draw the second curve PYS. Then we have for the arc lamp—

$$\text{Mean spherical C.P.} = \frac{\text{Area of curve } PYS}{\text{,, \quad ,, rectangle } PQRS} \times \text{max. C.P.}$$
$$= (\tfrac{1}{2}\,\text{horizontal C.P.} + \tfrac{1}{4}\,\text{max. C.P.})\ \text{approx.}$$

The curve PYS shows the manner in which the illumination of streets falls off with direct current arc lamps at different distances from the lamp for a given height above the ground.

(27) Other Tests on Arc Lamps.

Determination of the Effect of Carbons of Different Diameters and Quality on the Spherical C.P. and Spherical Efficiency.

Notes.—In this test care must be taken to vary *only one* thing at a time, as for instance—

(*a*) Vary the diameters only for exactly the same quality of carbon, all other conditions being constant.

(*b*) Vary the quality only for exactly the same diameter of carbon, all other conditions being constant, as for instance the amount of power supplied to the arc. The spherical C.P. and efficiency is then measured in each of the cases *a* and *b* in the manner just described.

The results should be tabulated in a convenient manner. If possible a curve should be drawn between each separate pair of variables, and, lastly, inferences deduced from the experimental results.

(28) Determination of the Relation between Voltage and Current respectively, and the loss in grammes per hour of Positive and Negative Carbons.

Notes.—In this test, as was also mentioned in the last, only one thing must be varied at one and the same time. Thus—

(*a*) For the same voltage, measure the loss in grammes of the

same or exactly similar $+^{ve}$ and $-^{ve}$ carbon occurring in the same interval of time with different currents.

(*b*) For the same current, measure the loss in grammes of the same or exactly similar $+^{ve}$ and $-^{ve}$ carbon occurring in the same time with different voltages.

The results should be tabulated in a convenient manner, and if possible the following curves drawn—

Two between volts and amps. respectively on the abscissæ and losses in grammes per hour of $+^{ve}$ carbon as ordinates.

Two between volts and amps. as before, with losses in grammes per hour of $-^{ve}$ carbon as ordinates.

All on the same sheet of curve paper.

Carefully deduce the inferences obtainable from the results of the tests.

(29) Relation between Voltage and Length of Arc (with Constant Current through it).

Introduction.—This test, like the next one, is important, as it indicates why certain types of lamps can be run at higher voltages than others, while in conjunction with the results of a corresponding test for obtaining the Polar Diagram of the lamp, the effect of the length of arc on the distribution of light over the lower hemisphere is clearly indicated. The reader should peruse the remarks under "introduction" in the next test which apply to the present one also.

Apparatus.—Precisely that for test No. 30.

Observations.—(1) Connect up as in Fig. 30, and set A and V to zero if necessary.

(2) With (R) *full in,* close S and "strike" the arc by bringing the carbons together for an instant, and then quickly separating them, R being reduced to keep the arc burning.

(3) By varying R, obtain a series of arc lengths between about $\frac{1}{8}''$ and the maximum possible by applying different voltages across it, the current being kept as constant as possible all the time at the most convenient value to be found by trial. Then after rapidly moving L to obtain the sharpest image I on G of each arc length, quickly measure I, and note x, y, A and V.

Note.—Time should be allowed for the carbons to burn to shape, and for the arc to become steady before readings are taken.

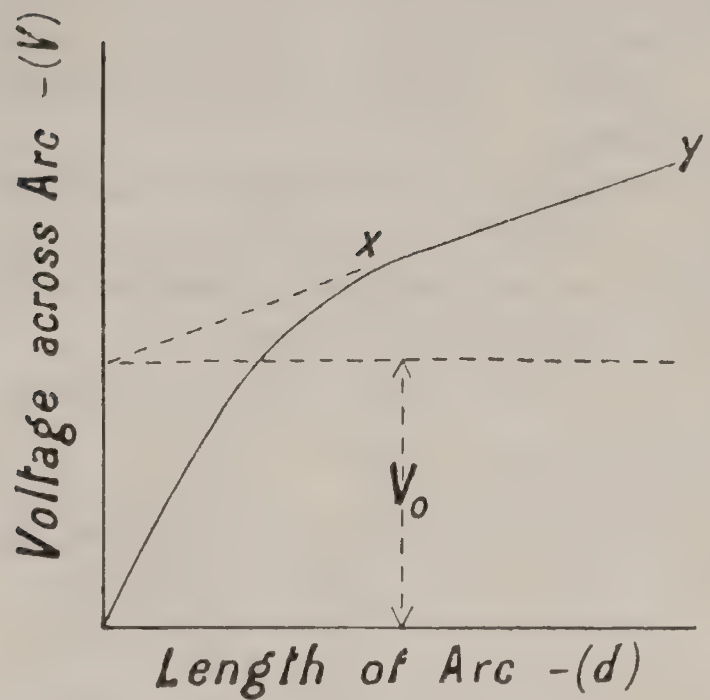


Fig. 29.

Tabulate your results exactly as in the last test, and plot curves having values of V and R as ordinates with length of arc (d) as abscissæ.

Find the constant (a) in the equation—

$$V = V_0 + a.d.$$

to the working part xy of the curve, Fig. 29.

Inferences. — What can you deduce from the results of your test?

(30) Relation between the Current through an Electric Arc and the Voltage across it (for a constant Length of Arc).

Introduction.—The present test has an important bearing on the supply of electrical energy to "open," "enclosed," and "flame" arc lamps in view of the length of arc normally employed in these three distinctive types being different in practice. The voltage across the arc can be obtained by a high resistance voltmeter connected to two spring clips placed on the carbons as close to their tips as is safe without risk of fusion.

The voltage thus measured will be that necessary for overcoming the apparent resistance of the arc made up of the back E.M.F. of the arc + the "ohmic drop" in the arc due to its ohmic resistance.

If the voltmeter is connected to the lamp terminals it will measure the above-named apparent resistances + the additional "ohmic drop" between carbon tips and terminals.

The length of arc may be found in one of two ways: (1) by throwing an image of the arc on to a screen at a known distance away, by means of a double convex lens, when from the length of image and the distances of the lens from arc and screen the length of arc itself is at once obtainable; (2) by placing a gauge of known length, about equal to that of the arc in front of the latter, and measuring the shadow of the gauge on the screen, when from the length of shadow, gauge and the distances, the length of arc is obtained.

Apparatus.—Hand-feed arc lamp B with terminals TT; double convex lens L, mounted on sliding base; ground or milk glass screen G; ammeter A; voltmeter V; variable rheostat R; switch S and source of supply E.

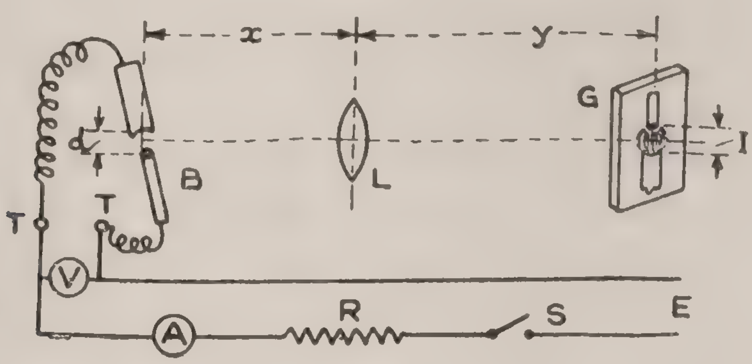


FIG. 30.

Observations.—(1) Connect up as in Fig. 30, and adjust the pointers of A and V to zero if necessary.

(2) With (R) *full in*, close S and "strike" the arc by bringing the carbons together for an instant, and then quickly separating them, R being reduced to keep the arc burning, which must be carefully watched.

(3) Adjust the arc to a convenient length, say $\frac{1}{8}''$ to $\frac{1}{4}''$, and move L until the clearest image I is obtained on G, then quickly measure the length of I on G, and note the readings of V and A and the distances x and y.

(4) With the length of I constant, vary R so as to obtain a series of values of V and A between 0 and, say, 25 amps.—x and y being constant, and the arc adjusted to keep I constant.

(5) Repeat 3 and 4 for constant lengths of arc of about $\frac{1}{2}''$ and $\frac{3}{4}''$, and tabulate your results as follows—

Distances.		Lengths of		Volts V.	Amps. A.	Apparent Resistances $R = \frac{V}{A}$ ohms.
x.	y.	Image (I).	Arc (d) $= \frac{x}{y} \times I$.			

(6) Plot to the same axes, curves having values A as abscissæ with both V and R as ordinates.

Inferences.—What can you deduce from the results of the test?

(31) Examination of Alternating Current Arcs.

General Remarks.—The alternating current arc possesses many characteristic and interesting features which are absent in the case of the continuous current arc. Thus for instance the two carbons consume away at approximately equal rates. The colour of the rays is quite different, being much more purple than in the direct current arc. Again, more energy is needed for the same volume of light emitted, and the arc gives out a rhythmic hum if it is burning properly.

In addition the true power W given to the lamp may be considerably less than the apparent power AV, *i. e.* the product of the alternate current ammeter and voltmeter readings. Thus the power factor which $= \frac{W}{AV}$ may be very low and even down to 0·50 in an alternating current arc lamp.

The following additional investigations should be carried out on this type of lamp, namely—

The effect on the C.P. of variations of (a) voltage, (b) current, (c) frequency, (d) quality of carbon.

The effect on the angle of phase difference or power factor of (e) quality of carbon, (f) cored and uncored carbons, (g) hissing of the arc.

The relative amounts of power absorbed by the arc itself and by the regulating mechanism should be investigated.

Many of the above tests can only be employed on hand-regulated lamps.

(32) Measurement of the Internal Resistance of Secondary Cells.

Introduction.—The following method is the best for measuring the working value of the internal resistance of a storage cell or battery of such. Owing to the very low resistance met with usually in this kind of cell the ordinary methods are practically inapplicable, and in the present case the cell is being tested more or less under working conditions.

If a battery is being tested the total internal resistance can be obtained at once, and if the cells are all of the same size, make, and type, the resistance of each cell can be deduced, probably with considerable accuracy, by dividing the total resistance so obtained by the number of cells and thus obtaining the average resistance per cell. It should be remembered that the internal resistance of any cell is not a fixed and invariable quantity but depends on several things, thus, for instance, on the density of the sulphuric acid solution which is continually changing according to the amount of discharge, or charge of the cell. It is interesting to note in this connection that the resistance of a solution of dilute sulphuric acid is least at a specific gravity of about 1220 and increases from this in either direction, *i.e.* for a rise or fall in density. Again, the internal resistance will depend on the condition the plates are in, and will be greater if they are "*sulphated*" than if in good condition.

Apparatus.—The cell or cells (B) to be tested; voltmeter V of sufficiently large resistance, and having a long open scale, enabling small differences to be read accurately; ammeter A capable of reading up to the maximum current to be taken from the cell; key K; switch S; carbon rheostat R (p. 597).

Observations.—(1) Connect up as in Fig. 31, and adjust the pointers of V and A to zero, levelling the instruments if necessary.

(2) With S *open*, close K and note the reading E on the voltmeter. This is therefore the E.M.F. of the cell in volts, since only an extremely small current is flowing.

(3) Close both K and S and adjust R so as to obtain about

$\frac{1}{10}$th of the maximum current output from *B*. Note simultaneously the readings on *A* and *V*, which latter now gives the *terminal* P.D. (*V*) in volts.

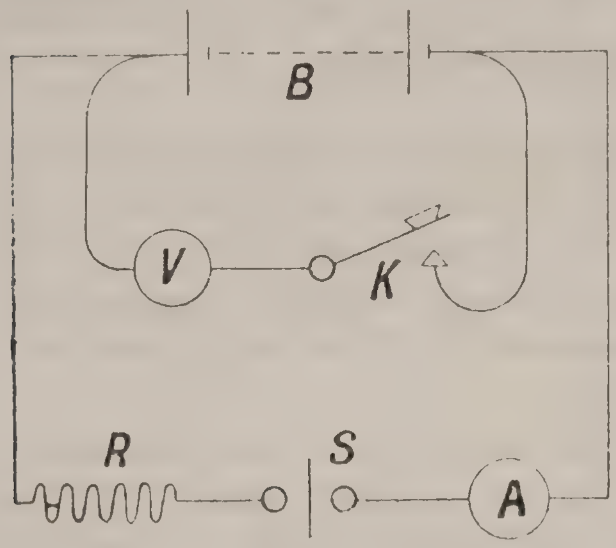


FIG. 31.

(4) Repeat 2 and 3 for about ten different currents rising by about equal increments to the maximum.

(5) Calculate the working value of the internal resistance *b* of the cell or battery from the relation

$$b = \frac{E - V}{A} \text{ ohms,}$$

and tabulate your results as follows—

NAME . . . DATE . . .

Cell tested : Make . . . Type . . No. of Plates = . . . Size of Plates = . .

Distance between Plates = . . . Approx. Sp. Gr. of Solution = . . .

E.M.F. E Volts.	P.D. V Volts.	Current A Amps.	Internal Resistance b Ohms.	Internal Resistance per Cell for a Battery.	Mean Internal Resistance.

(6) Plot 2 curves having values of *V* and (*b*) as ordinates and *A* as abscissæ. Show that the tangent of the angle of slope (from the horizontal) of the *V* and *A* curve = the internal resistance (*b*).

(33) Measurement of the Efficiency and Storage Capacity of Secondary Cells.

Introduction.—Secondary cells may be divided into two main divisions, namely—the "Fauré" or *pasted type*, and the "Planté" or *non-pasted type*. The chemical changes occurring in either class, during charge and discharge, are precisely alike, but the reader is referred to ordinary text-books of Electrical Engineering—for instance, *Electrical Engineering in Theory and Practice*, by the author—for such changes which hardly come under the scope of the present work.

The secondary or storage cell has taken up so prominent a

position at the present day in both electric lighting and electric traction that the method of measuring the efficiency and storage capacity of any type of cell, or perhaps more particularly the relative behaviour of different types under the same conditions, is a matter now of paramount importance to every electrical engineer. A good deal may be said with regard to the precise mode of testing such cells, and in this connection much depends on the duty which they have to perform in actual practice. Any laboratory test of such cells will be worthless almost, from a practical point of view, unless it is carried out under conditions as nearly as possible alike to those the cell will work under in its everyday use. Thus, for instance, take a battery employed for merely lighting purposes, say at a central electricity supply station. It is never resting idle and never merely giving either its full load discharge or any other constant output, for the load which it has to take varies with the hour, day, and season of the year, from often next to nothing, to full load and sometimes a considerable percentage overload for short periods. Thus it will be seen that in this instance any test to be of value must be carried out as nearly as possible under these conditions, and for months continuously, too, instead of, perhaps, only for two or three weeks always at full load and with, say, a night's rest in between each such discharge.

Again, in the case of electric traction work, the above remarks do not all apply, for instance, usually a battery used in this kind of work in sub-stations is relieved of discharge between midnight and about 7 a.m. in the morning, during which period it is charged. When used for portable work, as in autocars and tramcars, it is subject to both rapid and wide fluctuations of output and often to excessive jolting. Hence the test on a cell required for this kind of work should be a very stringent one, automatic jolting gear being provided to operate on the cell while being discharged, while this latter must often be abnormal. Practically the Fauré or pasted type of cell is the only one available for self-contained autocar traction, as weight forbids the use of the Planté type. As one instance of a traction type of pasted cell which will stand periods of excessive discharge and the *wash* of the solution against the plates and yet have a long *life*, the Headland secondary cell may be instanced, and tests extending over years amply justify this.

The efficiency of any secondary cell or battery can be reckoned in one of two ways, namely, the—

Quantity efficiency, or Ampere-hour efficiency

$$= \frac{\text{Ampere-hours given out}}{\text{Ampere-hours put in}}$$

Energy efficiency, or Watt-hour efficiency

$$= \frac{\text{Watt-hours given out}}{\text{Watt-hours put in}}$$

Each of these will depend to a certain extent on the relative periods of charge, rest, and discharge, and also on the current density or rate of discharge reckoned say in amperes per unit of area of positive plate. The greater this is the less will be the quantity efficiency, and also the energy efficiency, though the latter not to the same extent as the former.

It may here be remarked that the quantity efficiency may be as high as 94% when the current density is low and the cell used under favourable conditions, whereas the energy efficiency cannot exceed 80% from the fact that the average *normal voltage* of a cell *on discharge* is *2·0* volts approximate and the average *voltage needed to charge* being *2·5* about. These two efficiencies in practice may be taken more nearly as about 75% and 65% respectively.

The CAPACITY of any secondary cell may be expressed in one of two ways, namely, either as the ampere-hours or as the Watt-hours which it is capable of giving as a *useful discharge.* The term *commercial capacity* might be given to the number denoting the ampere-hours or the Watt-hours per lb. of plate (taking both $+^{ve}$ and $-^{ve}$ together) or per lb. of cell complete, including acid, etc.

At the present day, owing to there being so many forms and methods of building, the latter mode of reckoning the capacity is the only one available when comparing different types of cells.

A secondary cell may be charged either (1) at constant P.D. or (2) at constant current. In the first case a fairly heavy rush of current takes place at starting, and the method would be unsuitable for use on some types of pasted cells from the risk of the plates buckling. The second method is the one nearly always employed in practice and is the one which will here be considered.

The cell should not be discharged *normally* below 1·80 volts on closed circuit, since it will then become practically useless for lighting circuits, and there is also the danger of the plates "*sulphating*" rapidly below this limit. For the latter reason it should not be allowed to rest in this discharged condition.

Apparatus.—Cell B to be tested; sensitive voltmeter (V) with

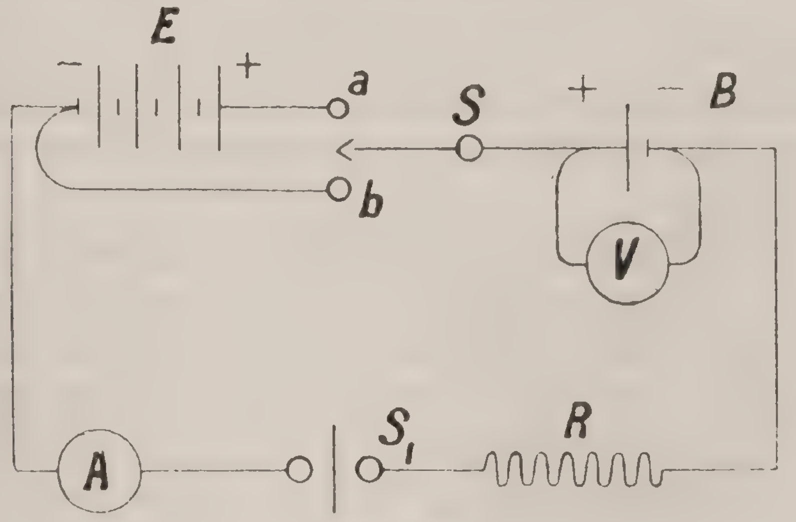


Fig. 32.

open scale; ammeter (A); switch S_1; carbon rheostat R (p. 597); two-way switch S; source of charging E.M.F. (E); hydrometer and weighing arrangements if the latter should be required.

Observations.—(1) Assuming that the cell to be tested is not already set up, but is still as received from the makers. First weigh each complete set of plates, "Positives" and also "Negatives," separately after dusting them. Also weigh the containing vessel, and the dilute sulphuric acid solution (of the specific gravity authorized for that particular cell), which is sufficient to cover the plates and be about one inch above their tops. Measure the size and thickness of the plates.

(2) Set up the cell properly, connecting up as indicated in Fig. 31, and adjust the pointers of V and A to zero if necessary.

(3) More as a matter of interest than otherwise, carefully note the sp. gr. of the acid solution before and immediately after putting it into the cell by means of the hydrometer, and then note the readings of this latter and the time frequently, while the sp. gr. is rapidly altering and until it becomes constant.

Note.—In all cases exercise great care in keeping the hydrometer away from the sides of the vessel and plates; if this is not done it will give totally erroneous readings due to adhesion.

(4) If the sp. gr. of the acid solution is constant note it, then with R at its maximum and S on contact a, note the time on closing S_1 and quickly adjust the current on A to the "normal" for this cell by means of R.

(5) Keep this current constant in strength until the acid becomes milky in appearance throughout—commonly known as *boiling* and due to bubbles of gas liberated from the plates. Note the readings of the hydrometer and voltmeter and the time frequently while they are varying rather rapidly, but less often as they vary more slowly, and, lastly, open S_1 when the cell is completely charged.

N.B.—Probably this first charge will last from at least 15 to something like 30 hours before the cell thoroughly comes up to the "boil," and in no case should it be stopped in the first 12 hours except for a minute or so. Beyond keeping the current constant from beginning to end the other readings during the middle stages of charge need probably be only taken every 1 or 2 hours about.

Tabulate your results as follows—

CHARGE.

NAME . . . DATE . . .

Name of Cell . . . Type . . . Normal rated capacity . . . Amp.-hours . .

Weight of plates: Positive= . . . lbs.: Negative= . . . lbs.: Vessel= . . . lbs.: Acid= . . . lbs.

Thickness of plates: Positive= . . . Negative= . . .

Total surface of Positive= . . . sq. ft.: Plate volume= . . . Acid volume= . . . Ratio= . . .

Number of Charge.	Amperes A.	Terminal P.D. (V) Volts.	Time in Hours.	Sp. gr. of Acid.	Input Amp.-hours.	Input Watt-hours.

(6) Take note of the period of *rest* (if any) which the cell has had since the last charge, note the open circuit P.D. at its terminals and the sp. gr. Then put S to (b) and close S_1 at a noted instant of time, quickly adjusting A to the *normal* discharge value for the cell, which must be kept constant by R. Note the P.D. on V and the sp. gr., and the time frequently while they are varying somewhat rapidly, but less often as they vary more slowly.

Open S_1 every half-hour, say for the shortest time necessary to just take the "open circuit" volts, noting the time at each.

(7) Continue the discharge until the terminal voltage falls to 1·80, then open S_1 and tabulate your results as follows—

DISCHARGE.

Number of Discharge.	Time in Hours.	Sp. gr. of Acid.	Amps. *A*.	Terminal Volts.		Internal Resistance *b*.	Output Capacity.		Efficiency.	
				Closed circuit *V*.	Open circuit *E*.		Amp.-hours.	Watt-hours.	Amp.-hour.	Watt-hour.

(8) Repeat tests 4–7 until the charge and discharge curves practically coincide, indicating that the cell has attained a good normal working state, and take note of the length of rests between charge and discharge.

(9) Take a discharge, as per 6 and 7, for 50% *under*- and also "*over*"-normal rate.

(10) At the conclusion of all the tests carefully observe whether any appreciable "buckling" or disintegration of the plates has occurred.

(11) Calculate the capacity of the cell in both amp.-hours and Watt-hours per lb. of total plates and per lb. of cell complete with acid. Also calculate the current density used per sq. ft. of + plate, reckoning both sides of each.

(12) Plot the following curves, all like ones being on one sheet—

(*a*) Internal resistance in ohms as ordinates and times in hours during discharge as abscissæ.

(*b*) Time in hours as abscissæ, with voltage and sp. gr. as ordinates in each case for both charge and discharge.

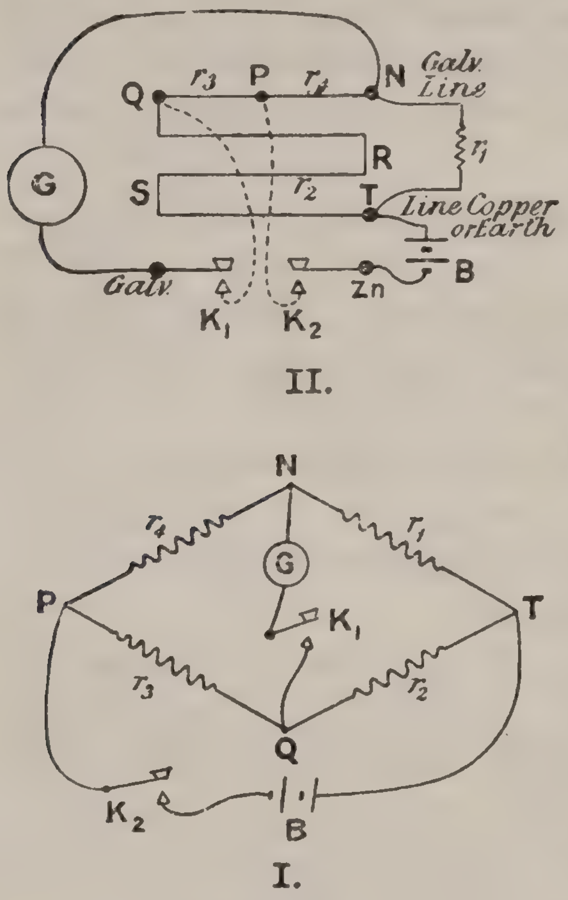
(*c*) Current density as abscissæ and amp.-hours output as ordinates.

(*d*) Current in amps. as abscissæ and the quantity and energy efficiency as ordinates.

(34) Measurement of Resistance by the "Post Office" Pattern of the Wheatstone Bridge.

Introduction.—It is assumed that the first principles of a Wheatstone Bridge (W.B.) have already been studied from an ordinary text-book. The Post Office (P.O.) pattern, Fig. 33 II., is merely a specially-arranged and compact form of W.B. placed in

a suitable box for portable purposes. If the principle and action of a W.B. is understood at all, and the stamping opposite the various terminals in the P.O. form observed, it ought to be impossible to couple up incorrectly. Each of the "proportional arms" r_3 and r_4 consists of three resistance coils of 10, 100, and 1000 ohms each respectively, hence the ratio $\frac{r_3}{r_4}$ or $\frac{r_4}{r_3}$ can be made a very simple number. *QRST* (Fig. 33 II.) is the "adjustable arm" r_2, and it consists of 16 different coils and one infinity plug either at *Q*, *R*, or *S*. The value of r_2 can be made anything from 1 to 11,110 ohms. Opposite two of the terminals (*N* and *T*) is marked (Galvanometer Line) and (Line Copper or Earth) respectively. This is because the P.O. form is primarily intended for measuring the metallic and insulation resistance of telegraph lines, and hence in the first case that line would be joined to *N* and *T*, and in the second case only one end to *N*, the other being free and insulated, *T* then being put to earth. As therefore we are measuring metallic resistance (r_1) it is put between *N* and *T*. The terminals to which the battery *B* must be connected are equally obvious. The white dotted lines on the top show where the under contacts of the keys K_1 and K_2 are joined to, inside the box. In any form of W.B. variation of the battery E.M.F. or its resistance or that of the galvanometer (*G*) has no effect on the accuracy of the measurement. The sensitiveness of the test, though principally depending on that of *G*, can be increased within limits by using a larger E.M.F. and making r_1, r_2, r_3 and r_4 as nearly equal as possible. *The battery key K_2 must always be pressed before the galvanometer key K_1* to allow the currents in the



II.

I.

FIG. 33.

arms to become steady before pressing K_1. The battery key should be pressed for no longer in order to prevent the coils being heated by the current and their resistance thereby altered. *It should also be broken last* to avoid the risk of damaging G by inductive kicks when measuring inductive resistances. In inserting plugs *press in lightly and give about $\frac{1}{8}$ of a turn* to insure good electrical contact. Reverse this operation when removing them. The ends of all connecting wires should be scraped clean.

Apparatus.—P.O. Bridge; sensitive galvanometer G (p. 571); 2 or 3 Leclanché cells B.

Observations.—(1) Connect up as indicated in Fig. 33 II., and adjust the galvanometer needle to zero.

(2) Note once for all the direction in which G deflects when (r_2) Fig. 33 II. is too large to give balance (done by taking out "Inf" in r_2 with, say, $r_3 = r_4 = 10$).

Note.—Until balance is nearly obtained, only tap K_1 for a fraction of a second.

(3) Make $r_3 = r_4 = 10$ and balance the bridge by altering r_2 so as to get no deflection on pressing K_2 and then K_1. If it is impossible to get exact balance, note the steady deflection when r_2 is just too large and too small, and calculate the correct intermediate resistance to give balance, by proportion. Thus if d_1 = steady deflection of the galvanometer to one side of zero for the adjustable arm $= R_1$, and d_2 = that to the other side of zero for the adjustable arm $= R_2$, then if R_1 is greater than R_2 we have $(R_1 - R_2)$ ohms corresponding to a deflection of $(d_1 + d_2)$ scale divisions,

and $\therefore$ d_2 corresponds to $(R_1 - R_2)\dfrac{d_2}{d_1 + d_2}$ ohms.

Hence the resistance of the arm, which would give just no deflection (the required condition)

$$= R_2 + (R_1 - R_2)\frac{d_2}{d_1 + d_2} \text{ ohms} = r_2.$$

(4) In order to obtain the true resistance (r_1) of the unknown which is being measured, without the process of interpolation mentioned in the latter part of 3 above, the value of r_3 or r_4 may be varied. Thus instead of $\dfrac{r_4}{r_3}$ being $= \frac{10}{10}$ or 1 as in 3 above, we

might have $\frac{r_4}{r_3} = \frac{1000}{10}$, $\frac{1000}{100}$, $\frac{100}{1000}$, or $\frac{10}{1000}$, depending on the value of the unknown r_1. In many cases this will be equivalent to having decimals of an ohm in the adjustable arm (r_2). Hence increase or decrease the ratio of $\frac{r_4}{r_3}$ and adjust r_2 so that on pressing K_2 and then K_1 no deflection whatever occurs on the galvanometer. Then note the values r_2, r_3 and r_4.

N.B.—If the unknown resistance r_1 is greater than 11,110 ohms, then r_4 will be greater than r_3, but if (r_1) is less than 11,110 ohms, then r_4 may be either =, or less than r_3. Tabulate as follows—

Resistance tested.	Proportional Arms.		Adjustable Arm r_2.	Unknown Resistance $r_1 = \frac{r_4}{r_3} \times r_2$.	Mean r_1.
	r_3.	r_4.			

Note.—The limits of the P.O. Bridge are $(\frac{10}{1000} \times 1) = 0{\cdot}01$ ohm and $(\frac{1000}{10} \times 11{,}110) = 1{,}111{,}000$ ohms, but measurements become less accurate as they approach these limits.

(35) Measurement of the Armature Resistance of Dynamos and Motors, and of the Copper Resistance of Transformers and Electric Light Cables. (Potential Difference Method.)

Introduction.—The Wheatstone Bridge is inapplicable for measuring very low resistances, and even if such were just within its range, the measurement would not be accurate owing to errors introduced by the variable contact resistances in the circuit. The following method, which depends directly on the definition of resistance, can be used to accurately measure very low resistances, such as are met with in large electric light cables, the armatures of dynamos and motors, and the low tension coils of transformers.

The P.D. at the terminals of each resistance can be measured relatively by a sensitive galvanometer, whose resistance is *large*

compared with that between the two points to which it is applied. Under these conditions its insertion will not lower the P.D. to be measured. If it is a reflecting instrument the scale deflections will be proportional to the P.D.

The most suitable instrument for a workshop test, which as a rule does not admit of the use of a delicate galvanometer, is a low reading voltmeter, having fairly large resistance, and reading to about 1 or 1·5 volts for a full scale deflection. Such an instrument, although not nearly as sensitive to small differences of potential as the galvanometer, has the advantage usually of being more portable, and also less easily affected by magnetic fields in the vicinity.

Apparatus.—Known standard low resistance *R* of about 0·01 ohm (Fig. 273); low resistance *r* to be tested; Pohl's commutator *C* (p. 584); secondary cell *B*; rheostat *Rh* (p. 597); fairly high resistance galvanometer (p. 569) or low reading voltmeter *G*, preferably of the moving coil type; reversing key *K* (p. 585); switch *S*.

Note.—The ends *H* and *E* of the low resistance (*r*) to be tested will of course be the terminals of the transformer coil, the ends of the cable or the brushes of the machine, the field coils being disconnected temporarily. The length of lead between *D* and *H* in the Fig. is immaterial.

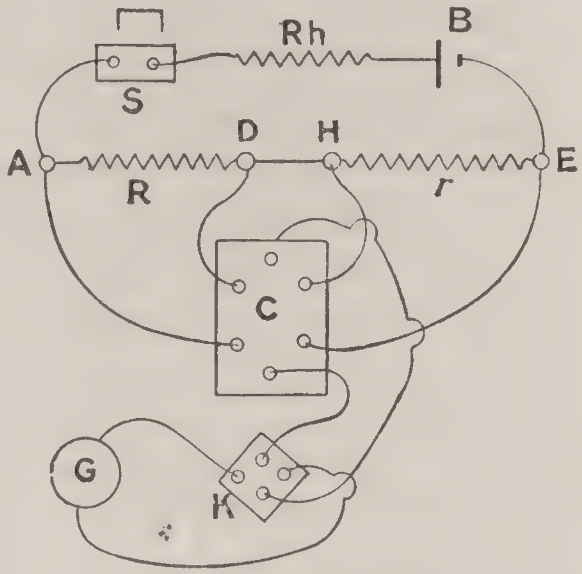


FIG. 34.

Observations.—(1) Connect up as indicated in Fig. 34, and adjust the galvanometer or voltmeter needle to zero. Clean the collecting arrangement at the part where the brushes press with fine emery cloth, assuming, for example, we are dealing with a dynamo or motor. To prevent the armature rotating, see that the *field circuit switch* is *open*, and that the brushes press on opposite ends of a diameter in the case of a direct current commutator.

(2) With *Rh* full in, close *S*, and adjust the current to give about quarter-scale deflection with the largest resistance of the

two, for then the deflection with the other is bound to be on the scale; then note the galvanometer deflection on each side of zero by turning K, when G is across each resistance in turn.

N.B.—The resistance Rh should be sufficiently high to prevent the current strength altering during any one pair of observations, and to prevent this current being strong enough to sensibly warm the resistances. The more sensitive the galvanometer the smaller this will be. After taking deflections with the second resistance, it is advisable to retake those with the first in case the current has altered. If they are not the same, take the mean of those on the respective sides of zero. For very accurate work a reversing key should be used with B to eliminate any thermo-current effects.

(3) Repeat 2 for half, three-quarter, and full scale deflections, and calculate the unknown resistance r from the formula—

$$R \div r = d_R \div d_r$$

Tabulate as follows—

NAME . . . DATE . . .

Low Resistance tested . . . Standard low Resistance = . . . Ohms

Deflection across R.			Deflection across r.			Ratio, $\frac{d_r}{d_R}$	Unknown, r ohms $= R\frac{d_r}{d_R}$
Right.	Left.	Mean d_R	Right.	Left.	Mean d_r		

Inferences.—Prove the formula given in 3, and state any assumptions made in deducing it. What sources of error is the method liable to? How can they be minimized?

(36) Measurement of Low Resistances by Voltmeter and Ammeter Method.

Introduction.—The following method, applicable to the measurement of the low resistances met with in the armatures of dynamos, motors, transformer coils and electric cables, is one of the simplest and a direct application of Ohm's law. It is not usually susceptible of the accuracy obtainable by the last method (Test No. 35) and depends on the accuracy of the ammeter and voltmeter used, and on that of observation.

Apparatus.—Low resistance (r) to be tested; accurate ammeter (A) and low reading voltmeter (V), both preferably of the moving coil type; switch S; variable current rheostat (R), the form of which will depend on the current supply (E) available. If E comprises two or three large secondary cells, then R may be a carbon rheostat (p. 597), but if E should be a 100 volt supply, then R may be a bank of lamps (p. 598).

Observations.—(1) If, as is indicated, an armature resistance is to be measured, connect up as indicated in Fig. 35. Adjust

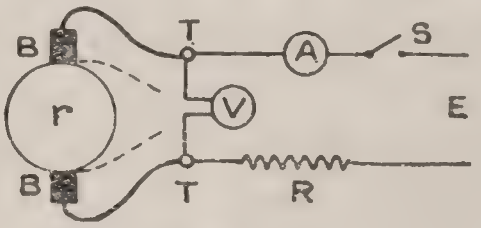


FIG. 35.

the pointers of A and V to zero, and see that the *field circuit of the machine is kept open* throughout the whole test by keeping the field switch open or otherwise.

(2) With V connected to the terminals TT of the machine, as actually shown, and with (R) *full in*, close S and take simultaneous ascending and descending readings on V and A for some five or six currents on A, differing in strength by about equal amounts between O and full-load armature current by suitably varying R—*the armature being at rest all the time.*

Note.—This measurement will give the Static "brush-contact" resistance + resistances of armature and both brush leads BT.

(3) Repeat (2) with the armature rotating (*by hand*) while taking readings.

(4) Repeat (2) with the armature at rest, but with the ends of the voltmeter wires disconnected from TT and carefully inserted under the brushes BB, so as to press against the proper commutator segments, the straight ends of the wires so inserted being parallel to the length of segment. Tabulate all your results as follows—

NAME . . . DATE . . .

Nature and Rating of Low Resistance tested . . .

V connected to	Actual Resistance being tested	Amps A	Volts V	Resistance $(r) = \frac{V}{A}$	Mean Resistance

(5) Plot, *on the same axes*, curves having values of V as ordinates, and (A) as abscissæ for tests 2, 3 and 4.

Inferences.—What can be deduced from the curves and values of (r) obtained?

(37) Measurement of the Armature Resistance of Machines and other Low Resistances by simple Potentiometer Method.

Introduction.—Since by Ohm's Law $V = I.R.$, where $V =$ the P.D. across the ends of a resistance R carrying a current I, it follows that when the same current I flows through two resistances, the P.D. (V) across each is $\propto$ to that resistance. The previous test (No. 35) was based on this fact, but since actual deflections ($\propto$ to the P.D.s) had to be compared, the accuracy depended to some extent on the current-deflection law of the instrument used, and on the instrument having a high resistance relatively to those measured. The present test, based on the Clark-Poggendorff method of comparing two E.M.F.s, and unlike the *deflection* method No. 35, is a *null* or zero method or one in which *no deflection* is the condition to be obtained. Hence the law of the instrument is immaterial, and an increase in its sensibility increases the accuracy of the test. Since also in this method the E.M.F.s to be compared are in turn placed in series with the instrument, the contact resistances of the connection to these E.M.F.s, as also the resistance of the connections, are immaterial; hence the greater accuracy with such a null method. Test No. 39 employs precisely the same principle as, but is a greater elaboration of, and a little more accurate than, the present method, in which we shall use a single or multiple metre bridge having a stretched undamaged wire of high resistance material, of *uniform cross sectional area* and size, and which will not sag due to heating by the current from one secondary cell connected direct to its extremities. Thus the E.M.F.s to be compared can be balanced against the uniform fall of potential along the wire due to a constant current flowing through it, and the ratio of the lengths so balanced will be that of the E.M.F.s across them.

Apparatus.—Armature A to be tested; known standard low resistance R; switch S; variable rheostat r; secondary cells B and E; Pohl's commutator or change over key C; metre bridge PQ with sliding contact key; sensitive galvanometer G.

Observations.—(1) Connect up as shown in Fig. 36 and adjust G to about zero. Ensure the connections being such that when C is turned so as to include the fall of potential of either R or A in the circuit of G, each P.D. opposes that along PQ due to E.

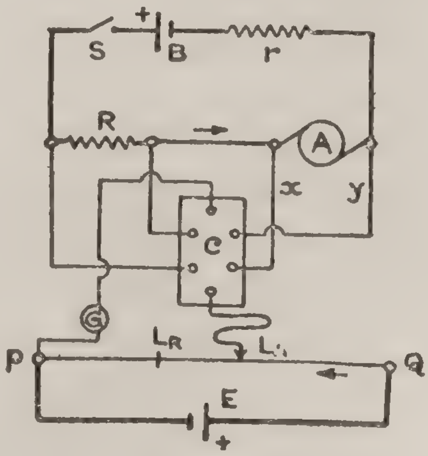


FIG. 36.

(2) With r full in, close S, and adjust r so that with C turned to the larger of the two resistances, a position, say, L_A cms. from P, is obtained at which there is no deflection on G.

(3) Now quickly turn C to R finding some position L_R cms. from P at which G does not deflect; next, again verify whether the point L_A still gives balance. If slightly different take the mean of the new position and that in obs. 2.

(4) Obtain several pairs of positions such as L_A and L_R by altering (r) and tabulate as follows—

NAME . . . DATE . . .

Nature of unknown Resistance . . .

Value of known Resistance $R =$ Ohms. Galv. No. . . .

Wires xy connected to which points of A.	Unknown Resistance measured, Hot or Cold.	Distance of Slider from P.		Unknown Resistance $R_A = \frac{L_A}{L_R} \times R$.	Mean value of R_A ohms.
		L_A	L_R		

Inferences.—On what does the accuracy of the test depend, and how can it be made more sensitive?

(33) Measurement of Metallic or Conductor Resistance by the Silvertown Portable Testing Set.

Introduction.—The method used in measuring the resistance of the conductor of the circuit under examination is that of Wheatstone's Bridge.

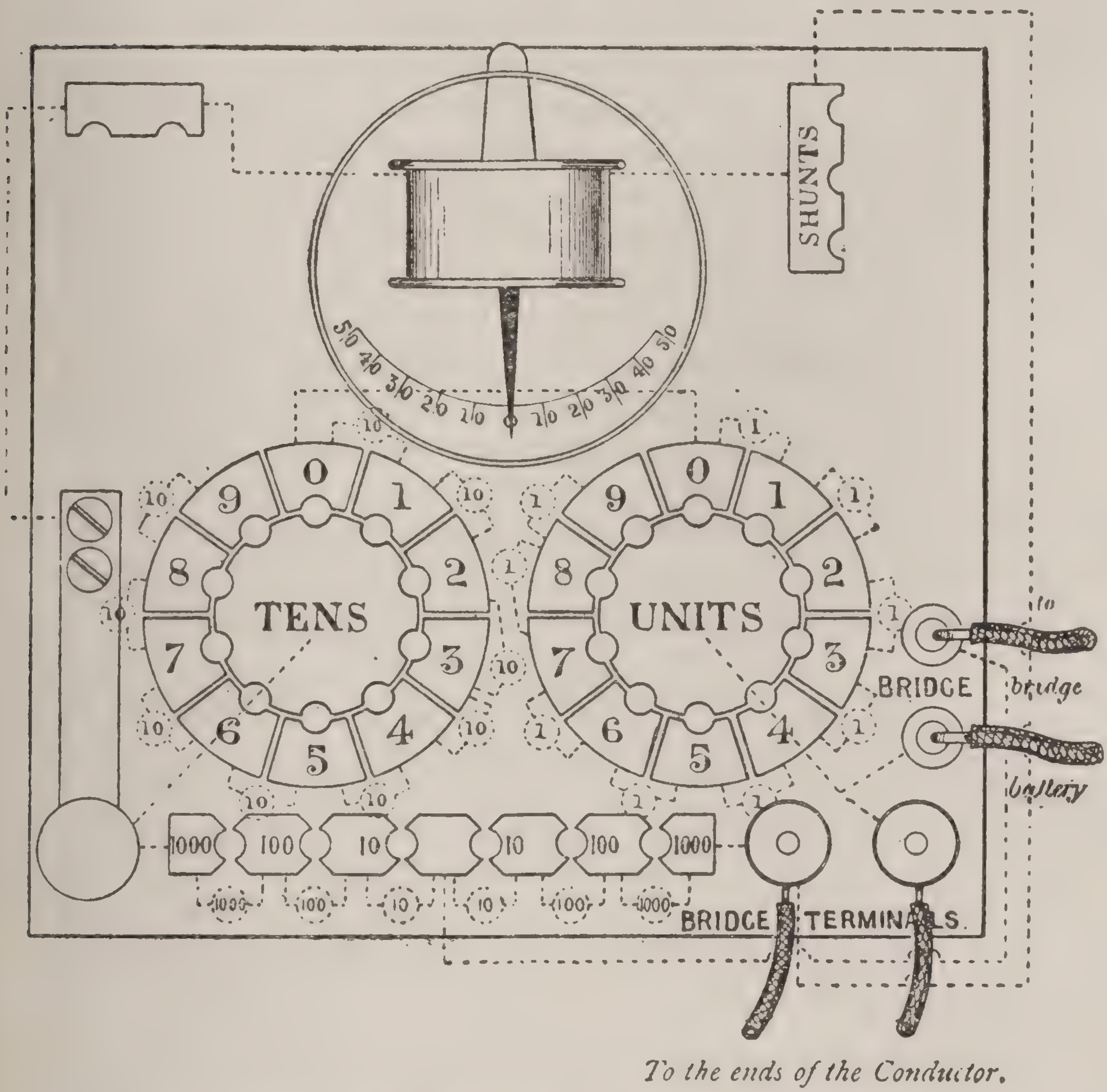
Fig. 37 below shows only those parts of the instrument which are employed in this test, and omits the parts and their connections which relate only to insulation testing. The parts employed are the following—

1. The adjustable resistance. This it will be seen consists of two sets of 9 coils, each connected to circular plug commutators or dials. One set of coils has nine resistances of ten ohms each, making ninety ohms in all, the other has nine resistances of one ohm each, making nine ohms in all. If the hole marked with any number, say 5, is plugged in the ten-ohm dial, a resistance of fifty ohms is inserted between the connecting leads entering and leading away from the dial; and a similar rule applies to the one-ohm dial. Hence if the hole 6 be plugged in the tens dial and the hole 8 be plugged in the units dial, a total resistance is inserted in the two in series of 68 ohms. The lowest resistance that can be obtained is given when both the 0 holes are plugged, when the coil resistance inserted is zero. The highest resistance is obtained by plugging the two 9 holes when the total resistance is 99 ohms. If no plug is inserted in one or both dials, the circuit is broken and the resistance is infinity.

2. The second part of the apparatus is the double set of proportional resistances, consisting of two coils of 10 ohms each, two of 100 ohms and two of 1000 ohms. Of these only one on each side of the centre is to be unplugged for any given test, and a rule is given later on for selecting the resistances to be employed to obtain the greatest possible sensitiveness; that is to say, for selecting those coils which will give the largest deflection on the galvanometer, when the resistance plugged in the dials varies by a given error from that of the circuit under test.

3. The third part is the galvanometer. Its two terminals are connected to the two ends of the Wheatstone Bridge by depressing

the contact key. It will be noticed that the shunt coils, with their plug commutator, are omitted from the diagram. This is done because they are not essential to the test, though they may be conveniently used when the balance of the bridge is not



Connections for Testing Conductor Resistance.

FIG. 37.

yet approximately correct, and very large deflections are being obtained.

4. The battery may consist of three Leclanché cells, having an electro-motive force of about 5 volts. One pole of the battery is connected in the usual way to the middle of the Wheatstone

Bridge, and the other to the point where the end of the adjustable dial coils is connected to one of the terminals, to which the conductor under test is attached. The connections are made by inserting the plugs at the end of the battery leads, in the two holes marked BRIDGE, and immediately this is done the current is established in the coils; the galvanometer circuit is of course not completed till the key is depressed.

5. The ends of the conductor to be tested are to be secured under the two terminals marked BRIDGE TERMINALS, and in measuring low resistances care must be taken that they are very securely attached. This may be done for very large or stranded conductors, either by soldering to their ends thin brass plates with holes in them of a suitable size to go under the heads of the terminals, or the connection may be made by means of finer wires soldered to the end of the main conductor. The resistance of these must be independently ascertained and subtracted from the gross result.

Providing an idea is first obtained as to the magnitude of the resistance to be measured, the following table will be found helpful in expediting any test with the "set."

TABLE III.

For Resistances tested between	Values of Proportional Arms.		No. of Significant Figures in Result.	Battery Power.	Remarks.
	Left-hand Coil.	Right-hand Coil.			
1 and 10	100	10	2	Ordinary	An extra significant figure can be obtained, calculated by proportion from the deflections.
10 and 100	100	100	,,	,,	
100 and 1000	100	1000	,,	,,	
1000 and 10,000	10	1000	,,	Increased	
0·1 and 1	100	10	,,	,,	
0·1 and 1	1000	10	,,	,,	
0·01 and 0·1	1000	10	1	,,	

A third figure can always be found in measuring resistances between one ohm and 1000 ohms, by observing the deflections of the galvanometer needles on both sides of the zero for different adjustments of the dial resistances near the balancing point.

For example, we will suppose that the 10-ohm coil in the right-band side of the bridge, and the 100-ohm coil on the left-hand side are unplugged, and that when 45 ohms are plugged in the dials, and the key depressed, a throw of three divisions of the

galvanometer needle is observed to the right; and when 46 ohms are plugged we get a throw of two divisions to the left on the galvanometer scale. It is clear that the resistance to be measured lies between 4·5 and 4·6, and is nearer to 4·6 than 4·5, as two is less than three; that is, the resistance is 4·56 ohms. As a further example, suppose 100 ohms to be unplugged on each side of the bridge, and 82 ohms to be plugged in the dials; on depressing the key, no deflection of the needle is observed. On plugging 81 ohms in the dials, a throw of six divisions to the right is obtained, and on plugging 83 ohms we get the same deflection to the left. We are then amply justified in putting the third figure in the result as 0, and the resistance to be measured is 82·0 ohms.

Except in testing at the extreme range of the instrument, *i. e.* quantities less than one ohm or greater than 1000 ohms, the galvanometer will be found amply sensitive, and it is better to place the south end of the controlling magnet uppermost, thereby reducing the time of the oscillations of the galvanometer needle.

The battery should be in circuit as short a time as possible to avoid running down the cells, and it is well to take out one of the battery lead plugs when any alterations are being made in the plug commutators, only replacing it just before pressing the galvanometer key.

Observations.—(1) Connect up as indicated in Fig. 37, using the "set" precisely as there indicated. The box should be placed on a table, or some other approximately level surface in front of the operator, he facing the magnetic east, and the controlling magnet being in a vertical position. The pointer of the galvanometer will then be found to be swinging near its zero, and may be brought exactly to it by slightly turning the controlling magnet.

(2) Find roughly the resistance to be measured by unplugging ten in each of the proportional arms, and then adjusting the dial resistance so as to give a minimum deflection on pressing the key, the dial readings will roughly be the value of the unknown.

(3) Now proceed to balance according to the foregoing table and remarks, and tabulate as follows—

NAME . . . DATE . . .

Resistance tested . . . Temperature . . .
Length of Conductor= . . . Gauge= . . .

Proportional Arms.		Adjustable dials r_3.	Deflections to the				Calculated Resistance to balance r.	Corrected dials $r_3 \pm r$.	Unknown Resistance $=\frac{r_1}{r_2}(r_3+r)$.
Left r_2.	Right r_1.		Right d_1.	with r_3'.	Left d_2.	with r_3''.			

N.B.—The resistance (r), needed to exactly balance, is calculated thus—Assuming r_3' to be greater than r_3'', then $(r_3' - r_3'')$ ohms corresponds to a deflection of $(d_1 + d_2)$ scale divisions, and d_2 corresponds to a resistance of

$$(r_3' - r_3'')\frac{d_2}{d_1 + d_2} \text{ ohms} = r,$$

$\therefore$ the correct dial resistance requisite to give no deflection

$$= r_3 + r = r_3 + (r_3' - r_3'')\frac{d_2}{d_1 + d_2} \text{ ohms.}$$

(39) Comparison of Resistances. (Crompton Potentiometer Method.)

Introduction.—This method is a valuable one for comparing two or more resistances of almost any value, within reasonable limits, and consequently of determining the actual resistance in ohms of one of them, the other being an accurately known standard, such as one of the forms described on p. 605, which are some of the accessories of the potentiometer. The method, which is very simple and susceptible of great accuracy, is more particularly applicable to low resistances such as short lengths of electric light cables and the armatures of dynamos, etc., rather than multiples of the ohm, and it can be worked in such a way that the unknown resistance can be read off by inspection directly in ohms. Thus it will be seen that the present measurement is a practical development of that known as "Measurement of Low Resistance by the Fall of Potential Method," given on p. 84, and is a direct application of Ohm's Law. The principle of it consists in comparing the relative falls of potential down the two resistances traversed by the *same* current through the

medium of the potentiometer, employing the principle of the Clark-Poggendorff method for comparing two or more E.M.F.s. The Crompton potentiometer is a specially arranged form of comparing instrument, and the operator should, prior to commencing the test, make himself acquainted with the instrument, a detailed description of which is given on p. 510, together with the method of using it. The accuracy of the results is principally dependent on the standard known resistance, and the value of the largest current sent through this and the unknown must be such that the fall of potential down either does not exceed 1·5 volts, and that neither is warmed up by that current sufficiently to alter their resistances.

The observations may be taken in one of two ways—

(α) Suppose the potentiometer has been "set" by the Clark cell in the usual way (p. 514) for E.M.F. or current measurements, and that the standard resistance $R_S = 0{\cdot}01$ ohm. Then to avoid disturbing the "setting," balance each fall of potential down the two resistances, against that down the potentiometer, and compare the two P.D.s from the relation—

$$V_S : V_R = R_S : R_R.$$

Example.—Let the standard balance with E on stud 1 and C at 95 on the scale, the P.D. across R_S is $1000 + 95 = {\cdot}1095$ volts. If now the unknown balances with E on stud 2 and C at 190 on the scale, the P.D. across it $= 2000 + 190 = {\cdot}2190$ volts,

$$\therefore\ R_R = \frac{V\ R_S}{V_S} = \frac{{\cdot}2190}{{\cdot}1095} \times 0{\cdot}01 = 0{\cdot}02 \text{ ohms.}$$

(β) Suppose the potentiometer has not been "set" by the Clark cell. Balance up on the standard resistance instead. Thus put E to stud 1 and C at 0 on the scale, and alter G and G_1 as described on p. 514, so as to "balance the potentiometer" for no deflection. Now with G and G_1 fixed, balance with the unknown resistance, which position of balance will give the resistance R_R in ohms directly. In the present instance this would be E on 2, or $R_R = 2 \times {\cdot}01 = 0{\cdot}02$.

Apparatus.—Crompton potentiometer P (Fig. 208); secondary battery B capable of easily giving the largest current suitable for sending through the low resistances; switch S; one secondary cell (b) for the "working cell" of the potentiometer; accurately known low resistance R (p. 605); unknown low resistance r to be

tested; standard Clark cell E; carbon rheostat (rh) (p. 597); sensitive aperiodic D'Arsonval or moving coil galvanometer (g) (p. 569).

Observations.—(1) As a precaution first place the levers of G and E (Fig. 208) on studs 14, and that of H on stud 1, then connect up as in Fig. 38, in which only the row of terminals on the potentiometer PP is shown symbolically.

(2) Adjust the galvanometer (g) and current indicator A to zero (roughly), levelling them if necessary, A being merely for the

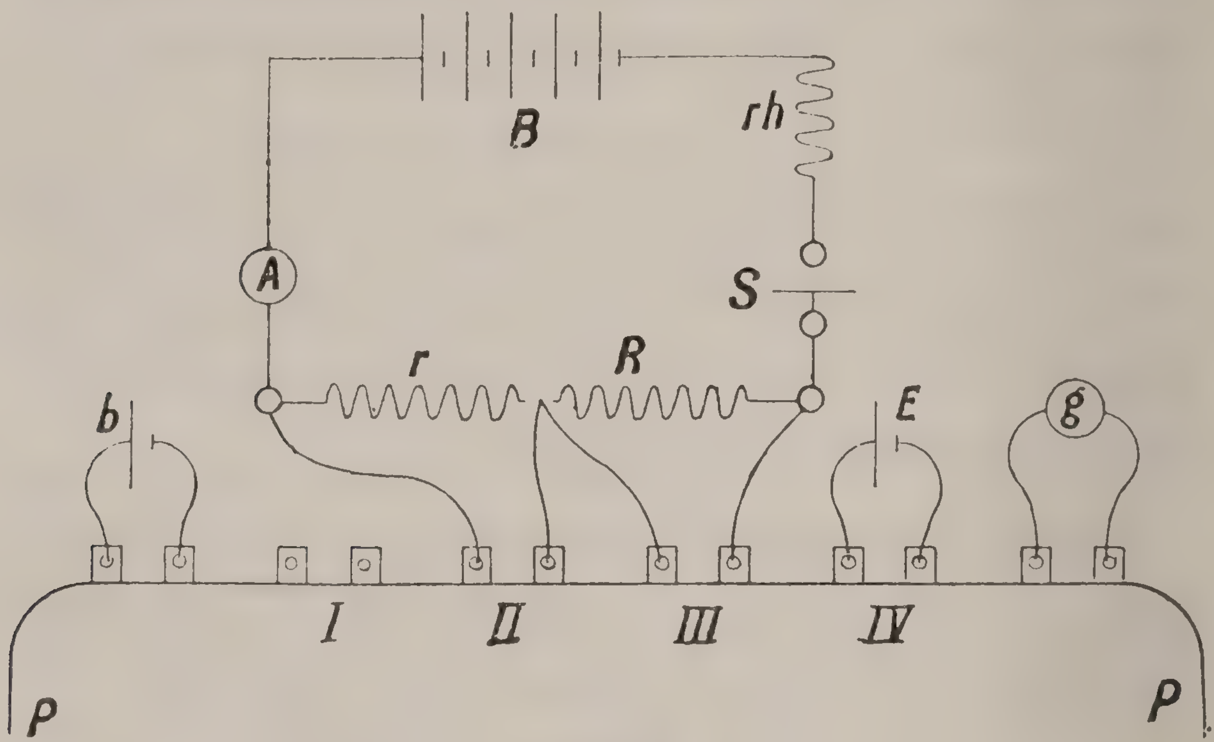


Fig. 38.

purpose of indicating roughly about what current flows in r and R. See that the standard resistance R chosen is of such a value as to be about the same as the estimated value of the unknown r.

Note.—The maximum current then to be used must not produce a fall of potential in either R or r exceeding 1·5 volts.

(3) "Set the potentiometer" as indicated in either a, or balance on the standard resistance as in β above, the contact lever H referred to above being on studs IV or III respectively, as the case may be, thus inserting E or the P.D. across R in the circuit of (g), and taking care that it *opposes* the P.D. due to (b). Now close S, and adjust (rh) so as to obtain some convenient current on A. N.B. This last-named operation is done before balancing P by method β.

(4) With the positions of the resistances G and G_1 (Fig. 207, p. 510) as found in 3, unaltered, turn H to studs II or III, according to the "setting" employed, so as to throw into circuit with (g) the P.D. across one or other of theso terminals. Then adjust E and the slider C to obtain no deflection on (g) when the latter is pressed, and note the reading of each.

Note.—If it is impossible to "balance" owing to the spot of light being deflected always to one side, the P.D. down the resistance is assisting instead of opposing (as it should be) the fall due to (b) in the stretched wire; the wires from resistance to potentiometer are then to be interchanged. Lastly, turn H again to the setting used in obs. 3 to see if balance is still obtained. If it is not, *re-set* P and repeat.

(5) Repeat 4 with H on the studs leading to the other resistance, if "setting a" is the one being used.

(6) Repeat 3–5, obtaining some six or eight distinct sets of readings by suitably altering the current through R and r, and tabulate your results as follows—

Clark Cell: No. . . . Temperature= . . . ° C. E.M.F. assumed= . . . Volts.
Potentiometer setting: E on . . . C at . . . Standard known resistance R=. . . Ohms.

Ammeter reading for reference only A	Potentiometer reading.						Unknown resistance $r=\frac{V_r}{V_R}R$ ohms.	Mean r ohms.
	R in circuit.			r in circuit.				
	Stud of E.	Slider C.	P.D. V_R.	Stud of E.	Slider C.	P.D. V_r.		

Inferences.—On what does the accuracy of the test depend?

(40) Measurement of Low Resistance by the Nalder Low Resistance Measurer.

This method has the advantage of being a null or zero one, and entails the use of a specially arranged piece of apparatus or "measurer," together with a secondary cell capable of giving a current of 5 amps and a variable rheostat to adjust this current.

The general arrangement (Fig. 214) and method of use is given on p. 521, and will not be repeated here.

Insulation Resistance.

Introduction.—Probably we shall not be straying very far from the truth when we remark that *Insulation Resistance* is one of the most important matters that an electrical engineer has to deal with. In fact, so obvious is this that the statement hardly needs qualifying; suffice it to say that a breakdown of the insulation resistance—whether of street main, appliance fed from it, or of an ordinary electrical installation supplied off it, will either cause a temporary or prolonged stoppage of the supply owing to the mere "blowing" of a protecting fuse or cut-out, or, if the circuit is over-fused, in the burning out of part of the circuit and possibly the firing of premises in which the breakdown occurs.

It is therefore of the utmost importance to be able to test the insulation resistance of a length of cable, main, or circuit, either when no current is flowing through it or when the supply is in actual progress and the main or circuit "*alive*," as it is usually termed.

A number of different methods have been devised and are in general use for measuring the insulation resistance of both "*dead*" and "*live*" cables and systems, and in the following pages devoted to this question some of the principal and common ones in use will be considered. Before, however, proceeding with actual methods of measurement it may be profitable to make some general remarks.

Electrical cables and wires are in the first place tested for their insulating qualities by the manufacturer prior to being sent out to the purchaser, but the latter should test them also himself, both *before* and *after* laying, to make sure that no faults have developed, and of course periodically during use. Such a mode of procedure is of the utmost importance if efficient working and maintenance is to be obtained, for it is quite possible for a cable to be accidentally damaged during laying and a subsequent fault to develop at this point, due to the strain of working conditions, which will finally break down the cable.

(41) Measurement of the Insulation Resistance of Electric Light Cables by the Direct Deflection Method.

Introduction.—The ordinary Post Office form of Wheatstone Bridge will measure resistances up to 1·111 megohms, though even at this maximum limit the measurements are not very accurate, owing to the resistances of the arms of the bridge being so widely different from one another; consequently it is unsuitable for measuring insulation resistance, which almost invariably amounts to much higher values, often of the order of hundreds of megohms. The present method of *direct deflections*, which is also termed the "simple substitution" method, is the most accurate in such cases and often the most convenient one to employ. The principle of it consists in comparing the deflection of a galvanometer needle caused by a given E.M.F. through a known standard high resistance in series with the galvanometer, with the deflection produced by the same E.M.F. working through the insulation of the cable to be tested substituted for the standard resistance.

Preparation of Cable for Test.—This must be carefully done and is of the *utmost* importance if the true insulation resistance of the dielectric is to be found, as the difference between the results obtained with properly and improperly prepared ends is very great. The method of doing this should be as follows—

(*a*) For vulcanized india-rubber cables, the braiding, tapes, or other covering should be removed for at least six inches from each end down to the surface of rubber covering, care being taken in doing this not to cut or otherwise injure the rubber covering still left.

(*b*) Wash this rubber surface with naphtha and scrape with a clean knife to remove any foreign material still left on the surface, and in this way so get a clean, fresh surface.

(*c*) Taper the rubber with a clean sharp knife for about 1″ to 2″ from the end, and then carefully dry the whole of the prepared end over a spirit flame *without burning the rubber.*

(*d*) Paint or coat the whole of the prepared end with three or four coatings, one after another, of clean paraffin wax, melted to a temperature *not exceeding* that of boiling water. This can be

done by placing the can of wax inside one of boiling water, whereas, if the wax is melted over a flame, it may be allowed to burn, and its insulating properties partially destroyed. As each coating of wax will have set before another can be got on, the whole cable end will be eventually sealed by a considerable thickness of the wax, which being much less hygroscopic than rubber, will not allow moisture to accumulate and so impair the prepared end. In lieu of wax the prepared end may be lapped with pure clean warm rubber tape well stretched, but this is not so good as the wax finishing.

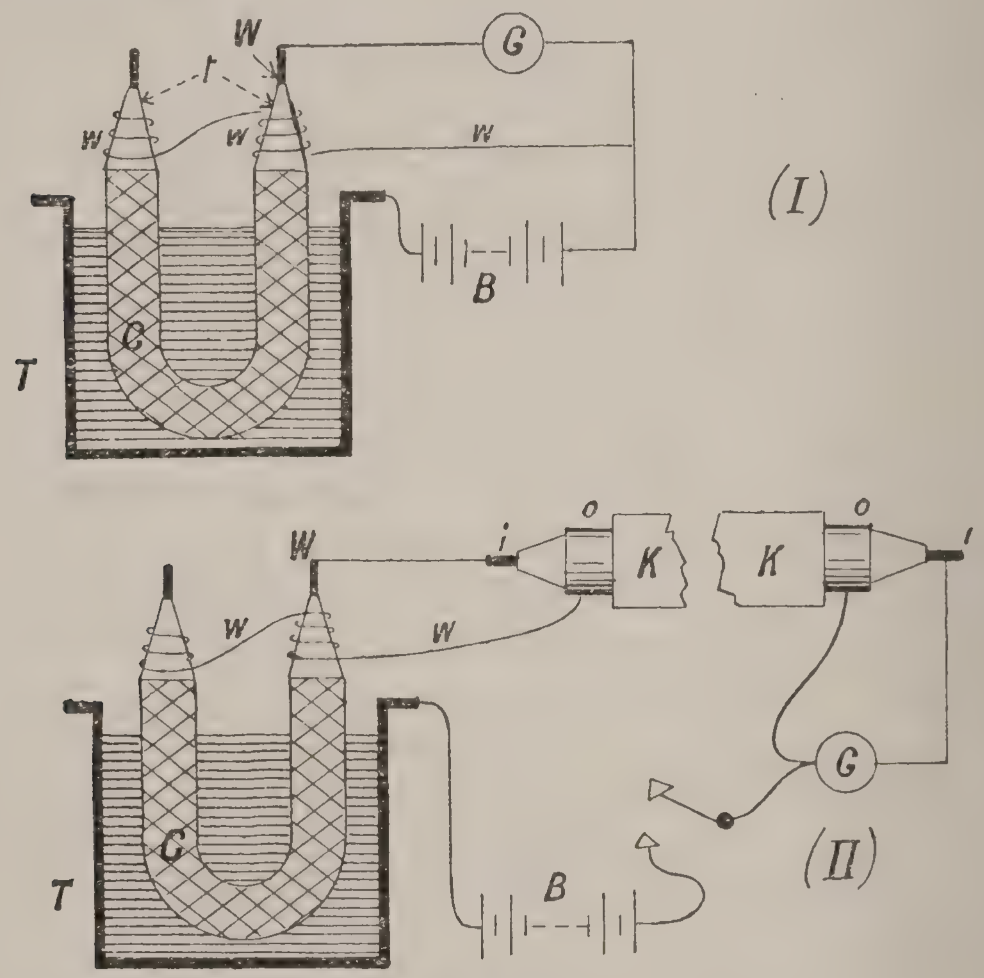


FIG. 39.

A better and much more expeditious way of eliminating errors due to surface leakage over the ends of a cable which is being tested for insulation resistance, than that just described of carefully tapering the ends and coating them with paraffin wax, is to employ an ingenious device known as Price's *guard-wire*. This, when properly applied, gives complete protection from errors due to end leakage in the *direct deflection* method, where the cable

ends are close to the galvanometer, and, consequently, the connecting wire between cable core and galvanometer is air insulated.

In the case of other methods, such as the *loss of charge* test, extra precautions are necessary to avoid errors (vide *Phil. Mag.* vol. xlix. pp. 343–7, April 1900).

If the cable ends are close to the galvanometer then Price's guard-wire device in its simplest form is shown in Fig. 39 (*I*). *T* is a lead-lined tank of water in which the cable *C* to be tested, for insulation resistance, is immersed. The ends of *C* are prepared with a long clean taper (*t*) from the core *W*, so as to give a *long clean surface* of insulation exposed to leakage. A thin copper guard-wire (*w*) is wound two or three times round the tapered part rather nearer the outer braiding than the core *W* and connected as shown to the galvanometer *G* and high voltage battery

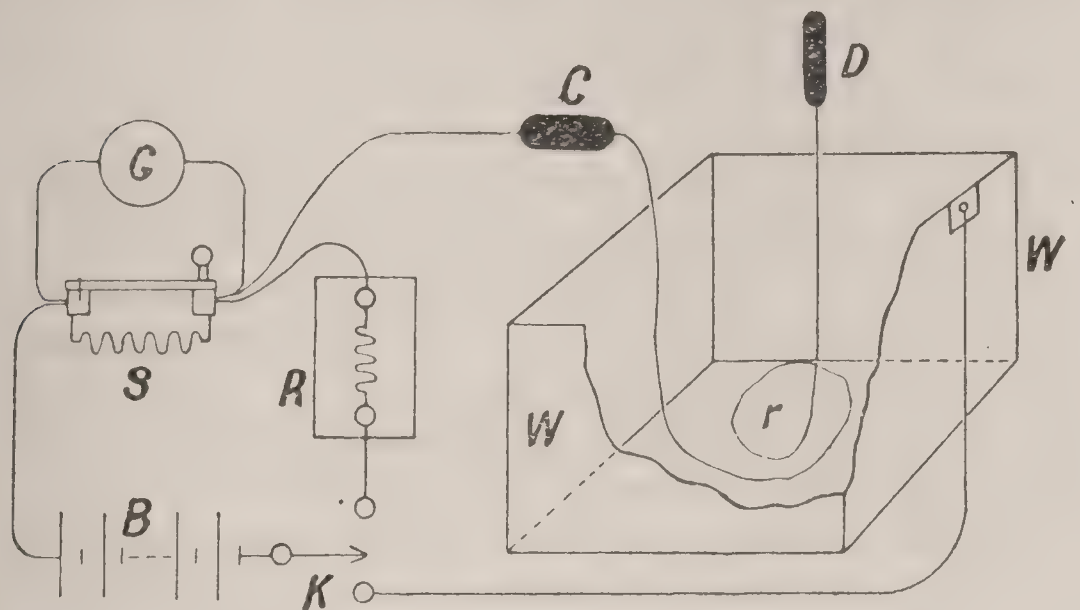


FIG. 40.

B. If now the resistances of the taper surface (*t*) are large compared with that of *G* they will all be at the same potential, and we shall have no leakage, but if any leakage exists (*w*) will tend to keep up the potential of *W*, the deflection of the galvanometer *G* being now reduced in the ratio $\frac{a}{a+b}$, where a and b represent the conductivities of *G* and *t* respectively. Consequently the correct result will be $\frac{a+b}{a} \times$ deflection.

It will, however, be evident that in some cases the cable ends

cannot be brought up close enough to the galvanometer in order to have an *air insulated* wire connecting G and W.

The simplest and best way of getting over this difficulty is that suggested by Prof. Ayrton and Mr. T. Mather and represented in Fig. 39 (II). The inner conductor (ii) of a concentric wire (K) is used to connect W and G, the outer (oo) connecting (w) with junction of G and B as before. Now if oo has a high insulation resistance compared with the internal resistance of the testing battery B, complete protection is afforded against surface leakage, even though KK is lying on the ground.

Apparatus.—Sensitive high resistance Thomson astatic reflecting galvanometer G with its box of shunts[1] S; known standard high resistance R; unknown insulation resistance (r) of cable to be tested; well-insulated battery B of either Leclanché or secondary cells capable of giving an E.M.F. of from 100 to 500 volts; two-way highly insulated spring tapping key K (p. 586); suitable lead-lined water-tank W. If the "lead" or cable to be tested is small enough, it may be run direct from the key K into the water-tank W (clear of everything) and coiled up under water, the *free end* being carefully *kept dry* and left standing upright, about 12″ out of the water, as indicated at D.

The tank should contain ordinary cold tap-water at a temperature of about 70° F., and the cable to be tested should be allowed to soak in this for 24 hours before the test, with its end trained up in mid-air above the water some 12″ or so.

If the cable is too large to be taken up to G, a short well-insulated G.P. covered wire must be tied on to it at C, and the joint insulated as at D. In all insulation tests at least the working pressure which the cable is to be subject to should be used to test it with.

The known standard high resistance may preferably consist of a metal megohm, but in lieu of this costly piece of apparatus, a carbon megohm, checked against a metal 100,000 ohm coil occasionally, will do quite well, and costs only a few pounds. It must, however, be borne in mind that such a resistance slowly alters its value with time and temperature, so that the temperature should be noted each time it is checked by the present method.

Note.—To avoid damaging the galvanometer, which is a very

[1] Or the Ayrton and Mather Universal Shunt-box.

delicate one, the shunt-box provided with it must always be used in the way indicated below.

Tests.—(1) *With the lever switch or short circuit bar of S down,* thus short circuiting the galvanometer terminals, and also with the shunt-plug in the $\frac{1}{999}$ hole, *connect S up to G first,* and *then the rest of the circuit* as indicated in Fig. 40, and adjust the spot of light to zero by means of the controlling magnet.

(2) Remove the short circuit in S, and, with the $\frac{1}{999}$ shunt plugged up, gently tap K for a fraction of a second so as to complete circuit through the standard known resistance R; if the deflection is inappreciable, release K to plug up the $\frac{1}{99}$ shunt, and again tap as before, and so on until a convenient steady deflection d_R is obtained. Note this and the shunt S_R in use at the time (if any).

N.B.—*The key K must always be released before altering the shunt S.*

(3) See that the short circuit lever of S is down so as to short circuit the galvanometer terminals, and that the $\frac{1}{999}$ shunt is in. Now close K through the insulation resistance, and after about half-a-minute open the short circuit switch.

Note.—If this method of proceedure is not followed a sudden ballistic rush of current may ensue through G, just at first, from the high battery E.M.F., into the cable, owing to this latter acting as a capacity, *i. e.* condenser, and thus damage the galvanometer. At the end of one minute note the steady deflection d_r, and with the key K still closed, again at the end of every minute up to between five or ten, say.

If without the wire connecting K and W there is leakage from battery and galvanometer which gives a deflection $d_{r'}$. Then $(d_r + d_{r'})$ or $(d_r - d_{r'})$ must be used instead of d_r simply, according as to whether $d_{r'}$ is opposite or in the same direction as d_r respectively.

(4) Repeat 2 and 3 for about 3 or 4 pairs of deflections if possible in different parts of the scale, with R and r, by altering S, and calculate the insulation resistance (r) from the formula below, and tabulate as follows—

NAME . . . DATE . . .

Cable.—Insulating material = . . . Resistance of Galvanometer G = . . . ohms @ . . . °C.
Size of copper core = . . . S.W.G. " Standard R = . . . " @ . . . "
Length immersed L = . . . Miles.
Time of immersion = . . . Hours. Temperature = . . .

E.M.F. used.	Standard Known Resistance.			Unknown Insulation Resistance.				
	In ohms R.	Deflection d_R.	Shunt. S_R.	Time in min. from closing K.	Deflection d_r.	Shunt S_r.	r megohms.	Megohms per mile $r \times L$.

(5) Plot a curve between time of electrification in minutes as abscissæ and corresponding deflections d_R as ordinates.

Note.—If $S = \frac{1}{9}$ (say), then $\frac{S}{S+G} = \frac{1}{10}$, or $\left(1 + \frac{G}{S}\right) = 10$.

Inferences.—Prove the formula mentioned in 4, *i. e.*:

$$d_R\left[R\left(1 + \frac{G}{S_R}\right) + G\right] = d_r\left[r\left(1 + \frac{G}{S_r}\right) + G\right]$$

and state what assumptions are made in deducing it.

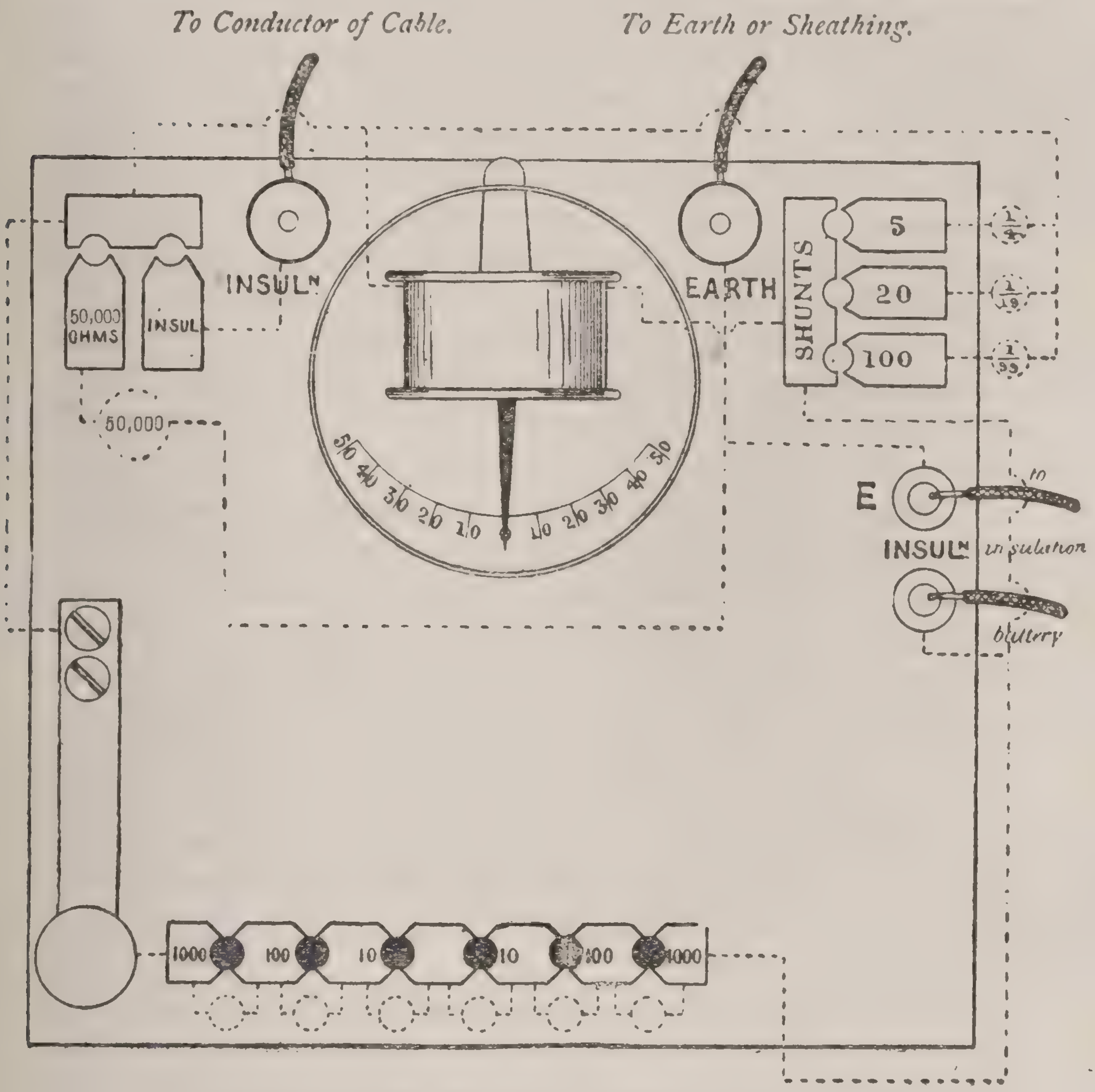
Should it be found impossible to keep the deflection on the scale with the standard known resistance in circuit and the $\frac{1}{999}$th shunt in, using the full battery E.M.F., then employ only a known fraction of this total E.M.F. (as measured by an electrostatic voltmeter) when taking a deflection d_R with the standard, whence in the above formula we must use $\frac{V}{v} \times d_R$ instead of d_R simply where V = full E.M.F. and v that used to obtain d_R. In testing short lengths of highly insulated cable at the least an E.M.F. of 300 or 400 volts should be used.

Referring to the ten 1-minute readings of deflection in observation 3 above, the deflection will fall rapidly at first and then more slowly. This is not due to increase in the insulation resistance, as it might appear to be, but to dielectric absorption in the cable through this acting as a condenser.

This particular test is a good one for developing a fault which would pass observation in a test of 1 minute's electrification, in which case there would either be an irregular—or no—crawling of the deflection in the 10 minutes.

Insulation resistance is usually specified in megohms per mile at 70° F. after 24 hours' immersion and 1 minute's electrification at some definite voltage.

It is necessary to record the temperature at the time of the test, as the insulation resistance decreases as temperature increases.



Connections for Testing Insulation Resistance.

FIG. 41.

(42) Measurement of Insulation Resistance by the Silvertown Portable Testing Set.

This is a measurement of the electrical resistance of the insulating material of a cable to the passage of a current from the inside conductor through the insulation to the lead sheathing, wet yarn, armour, or other outside conducting surface, and the inverse of the insulation resistance is generally termed the leakage.

This measurement is effected by a method known as that of direct deflections. It consists in passing a current from a battery through a galvanometer into a conductor of a cable whose farther end is free and disconnected, thence through the insulating material to the outside coating or earth, and so back along a temporary conductor to the other end of the battery, the deflection of the galvanometer needle produced by this current being noted. Replacing that part of the circuit which was formed by the insulating material of the cable by a standard resistance of known value, we obtain a new deflection of the galvanometer needle.

The diagram Fig. 41 shows only those parts of the apparatus and their connections that are used in this measurement; those which relate only to the measurement of conductor resistances being omitted.

The arrangement, it will be seen, is as follows:—One pole of the battery—the battery of Leclanché cells giving an E.M.F. of about 100—200 volts is normally employed—is connected by a conductor, ending in an ebonite-headed plug, to the lower of the two plug-holes marked INSUL^N. Thence the current passes along a connecting wire to the block marked SHUNTS, and thence through the galvanometer to the upper block on the other side. We may observe in passing that these two main blocks, one on each side, are practically the terminals of the galvanometer. If a shunt is plugged, $\frac{1}{5}$th, $\frac{1}{20}$th, or $\frac{1}{100}$th only of the current passes through the galvanometer, the remainder finding its way through the corresponding shunt coil.

From the upper block on the left-hand side the current may take two paths, according as the hole marked INSUL^N. or that marked 50,000 ohms is plugged; if neither is plugged, the circuit is broken, and no current can pass. This plug forms consequently a convenient make and break key. If the hole marked INSUL^N. is plugged, the current passes to the terminal marked INSUL^N., so through the insulating covering of the cable to the outside sheathing or earth, back to the terminal marked EARTH and the plug-hole marked *E*, and then along the lead to the other pole of the battery. If, however, the hole marked 50,000 ohms be plugged, the current will pass through the coil of 50,000 ohms, then along a connecting wire to the plug-hole *E*, and so back to the battery.

In beginning this test the conductor of the cable, or insulated wire, or a temporary lead attached to it, is connected to the terminal of the instrument marked INSUL^N., and another lead, connected to the outside sheathing of the cable, or the wet soil in which it lies, is attached to the terminal marked EARTH, care being taken that these leads are separated, and that no circuit exists between them except through the insulation of the cable.

It will be observed that when all the holes in the straight commutator near the front of the box are plugged, the key on the left-hand side, which is used in the bridge test as a galvanometer make and break key, becomes for the insulation test a short circuit key, and is useful for checking quickly the oscillations of the needle.

N.B.—Although the maximum voltage of the testing battery usually employed with this testing set is only 100 volts, the set can be used with a testing voltage of 200 volts, as required by the Board of Trade regulations. In this case, instead of using the multiplying power of 20 as described above, the multiplying power of 100 should be used in taking the constant—the deflection thus obtained will be the same as that which would be given by the unshunted galvanometer through a total resistance of five megohms, and the calculation of the resistance to be measured would then be made in exactly the same manner as described above, except that five megohms will be substituted for one megohm.

In making this test the following points may be called attention to—

(1) Too much care cannot be taken in preparing the ends of the cable. Since we are measuring a very small current of electricity passing from the conductor to the outside sheathing, through the insulated covering, it is clear that our results will be entirely misleading if any current be allowed to pass over a dirty surface at the ends where the conductor is exposed. These ends should be looked to before testing, and in the case of india-rubber or other firm material, the section of the insulator should be pared all over with a sharp and perfectly clean knife. For methods of preparing the ends see pp. 99—100.

(2) Care should be taken not to short circuit the battery, which may easily occur in two ways. One is by allowing the two battery plugs to touch one another, when the other ends of the leads are

attached to the battery terminals ; and another is by allowing the lead attached to the earth terminal to touch that attached to the insulation terminal.

In both cases the battery of small cells will be for a time much overworked, and in the second the needle may become bent or demagnetized.

(3) Another point that may be noticed is that in deducing the insulation resistance per statute mile from a test on any given length, the result obtained from a test on the latter is to be multiplied by the length of the piece in miles, and not divided by it.

For example, if the insulation of a cable three miles long be 15 megohms, the insulation per mile will be 15×3 or 45 megohms ; or again, if the insulation of a piece of cable, whose length is 350 yards, be 7520 megohms, the insulation per statute mile will be $\frac{7520 \times 350}{1760}$ megohms $= 1495$ megohms.

If the galvanometer deflections are proportional to the currents producing them, and the E.M.F. employed is constant throughout the whole test, then we have current $\propto$ deflection $\propto \dfrac{1}{\text{total resistance}}$; or if $R_I =$ insulation resistance tested and $d_I =$ deflection through it, and if $R_S =$ standard known resistance and $d_S =$ deflection through it, then

$$\frac{R_I}{R_S} = \frac{d_S}{d_I} \text{ whence } R_I = \frac{d_S}{d_I} R_S \text{ megohms,}$$

where $R_S =$ the standard resistance in megohms.

If, however, the galvanometer is shunted with, say, a $\frac{1}{5}$ shunt, for example (with the insulation), so that only $\frac{1}{5}$ of the main current goes through it, then, since without the shunt the deflection would be five times as great, we have

$$R_I = \frac{d_S}{5d_I} R_S \text{ megohms.}$$

Thus, for example, suppose that a given battery produces on the needle of a galvanometer placed in series with the insulation of a cable in the manner described, a deflection of 10·3 divisions, and that on substituting a resistance of one megohm for the insulation we get 42 divisions, we find that the insulation resistance is $\frac{42}{10 \cdot 3} = 4 \cdot 1$ megohms approximately.

Again, for example, suppose that the current from the battery

when passed through a constant resistance of 50,000 ohms gave a deflection of 42 divisions on a galvanometer shunted to $\frac{1}{20}$, and that when passed through the cable insulation it gave 23 divisions with the galvanometer shunted to $\frac{1}{5}$, the insulation resistance would be $\frac{42}{23 \times 5}$ megohms $= \cdot 37$ megohms approximately.

In cable testing the battery employed should in all cases give an E.M.F. at least = the working voltage under which the cable works. The terminal marked EARTH on the right of the galvanometer must be connected to earth (*i.e.* nearest gas or water-pipe) or to the water of the tank if the cable is being tested in such.

Tests.—(1) Connect up precisely as in Fig. 41, and adjust the galvanometer needle to zero.

(2) Now take the "constant" of the galvanometer by plugging the 50,000 ohm hole and the $\frac{1}{20}$ shunt and note the steady deflection d_S. This is the same deflection as that which would be obtained with the same E.M.F. through $(50{,}000 \times 20) =$ 1,000,000 ohms, or 1 megohm *for no shunt* at all.

(3) Plug up hole marked INSUL^{N}. instead of that in 2, and adjust the shunts (if necessary at all) to obtain a steady deflection d_I, preferably as nearly = to d_S as possible.

(4) Calculate the insulation resistance from one or other of the preceding formulæ and note for reference merely the E.M.F. used in the test.

N.B.—If the cable has been soaking in a water-tank note the time of immersion and the temperature of the water. Also the number of yards immersed.

Insulation Resistance of Electric Light Street Mains and House Installations.

Introduction.—MAINS.—Seeing the extreme importance of maintaining continuously, and without any intermission of any kind, the supply of electrical energy from a central station when once commenced, it should be the endeavour of any engineer to obtain and lay the best possible class of cables in the most efficient, thorough, and lasting manner in his power. The item of mains in the supply of electrical energy is a very serious one

at the best, and usually amounts to something like from $\frac{1}{3}$ to $\frac{2}{3}$ of the cost of the whole undertaking.

Notwithstanding this, however, the best possible main only should be laid if the system is to be a lasting one, free from perpetual worry to the engineer, of cables breaking down and the consequent temporary discontinuity of the supply. The insulation, jointing, and laying should be the best it is possible to obtain, for even in localizing a fault the accuracy of the test will greatly depend on the goodness of the joints.

There are roughly speaking three tests, which should be carried out on any new cable or main, namely—

(*a*) The insulation and copper resistances of each cable drum as soon as it arrives from the manufacturers. This can only be done satisfactorily under a pressure of at least double that which the cable will work at in practice, and with the whole cable drum wholly immersed in water at about 70° F., the ends being carefully kept dry, trained out of the water and prepared in the manner described on p. 99. Reliable results cannot be obtained from a well-wetted drum, only from one wholly immersed for 24 hours.

The insulation resistance should be obtained by the "direct deflection" method after one minute's electrification (*i. e.* application of the battery), and again at the end of every succeeding minute for some ten minutes.

The first reading will give, or should give, at least, the specified insulation resistance of the maker.

The second and subsequent readings are extremely useful in showing the existence of undeveloped faults in the cable insulation, which would in the usual course of events pass the specification in the one-minute test unnoticed. Should the insulation be faulty, the galvanometer deflection will hardly fall at all after the first minute's electrification, or may fall in irregular jumps. The resistance of the copper core should be taken and noted down, as well as the length of the cable on the drum.

(*b*) The insulation and copper resistances during laying both before and after jointing in the following manner:—A careful test should be made on the first section of the line, one end of which we will assume is in the station when laid, but before any joint is made, and with both ends carefully prepared (see p. 99).

If satisfactory, it will show that no damage has been done to it in the laying operation.

The second section is then laid and carefully but temporarily is connected to the first section by a piece of lead. These two adjacent ends and the *far* end of section 2 are carefully prepared and the two sections tested. If the insulation resistance per mile[1] is up to specification, section 2 is all right and can be now jointed to 1 and the test repeated. If not the same as before the first joint is defective and should be re-made. Now lay section 3 and again test as before, and so on; thus, finally, the whole line will have been tested section by section as the laying proceeded, and, lastly, as a whole. In this way a record of all the tests will be to hand at any future date, while any damage done in laying, or any badly-made joint, will be at once detected by fall in the insulation resistance per mile, and a rise in the copper resistance per mile.

(*c*) Daily tests (while working or otherwise), the precise method of performing which will depend on whether the main is a high or low tension one.

A fault occurring should at once be localized and remedied. If on a "feeder" it can easily be found, but if on a distributing main, sectioning off may be necessary to localize it.

The apparatus requisite for these tests is the same as that enumerated on p. 101, and with which the testing-room of every station should be provided, in addition to other instruments.

The minimum insulation resistance for low tension cables at 100 volts is about 300 megohms per mile after 24 hours' immersion and one minute's electrification. In high tension cables at 2000 volts it is about 4000 megohms per mile under similar conditions.

Ordinary Installations.—Many of the preceding remarks apply here. For example, it is much more economical in the long run to wire a building with high-class insulated wires and leads as also with good fittings having fairly high insulation.

[1] The insulation resistance per mile = measured (total) resistance × the length in miles tested. This arises from the fact that the leakage current through two miles is twice that through one mile, assuming, of course, that no appreciable fault exists anywhere along the length of cable.

The last-named condition to be aimed at is an important one when we consider that every lamp switched on brings one or more fittings, such as a lamp-holder, cut-out, switch, ceiling rose, etc., into active use, thereby adding so many additional parallel paths of surface leakage through which current can leak away to earth. This in other words means a diminution in the total insulation resistance of the whole installation, and which is considerably aggravated by damp weather, dust and dirt, etc. With respect to leakage of current, the rule given by the Institution of Electrical Engineers is that the total leakage should not exceed $\frac{1}{5000}$th part of the total working current. Numbers of different rules and regulations are given by the various Fire Insurance and Supply Companies. As an example of the latter we may cite the rules of the Edinburgh Corporation for installations tested at 115 volts with minimum insulation resistance.

TABLE IV.

For 12 lamps	5·0	megohms.		For 150 lamps	0·75	megohms.
,, 25	,, 2·5	,,		,, 200	,, 0·5	,,
,, 50	,, 1·5	,,		,, 250	,, 0·3	,,
,, 75	,, 1·25	,,		,, 300	,, 0·2	,,
,, 100	,, 1·0	,,				

The rules of the Leeds Corporation for 200 volt circuits are some 20% less than the above in the minimum insulation resistance for the same number of lamps in the respective cases.

Ordinary insulation resistance tests for installations must be taken with—*all fuses* "*in*," *all lamps removed* from their holders and *all switches* "*on*," with at least the working pressure for which the installation is intended, but preferably double this. It is then advisable to make three tests as follows—

(*a*) of the insulation resistance between $+^{ve}$ leads and earth,
(*b*) ,, ,, ,, ,, ,, $-^{ve}$,, ,, ,,
(*c*) ,, ,, ,, ,, ,, $+^{ve}$ and $-^{ve}$ leads.

The value in each case should not be less than that specified above or thereabouts for the particular number of lamps installed.

Usually the insulation resistance for alternating current circuits has to be greater than for direct currents, and in some regulations these are as 1 : 2.

The importance of testing at or even above the working pressure will be seen from the following figures by F. Uppen-

born of Berlin, which show in a marked degree the way in which the so-called insulation resistance varies with different voltages.

TABLE V.

Resistance between			
Terminals of slate cut-out.		Two twisted cotton covered wires.	
Volts.	*Megohms.*	*Volts.*	*Megohms.*
5	68	5	281
10	53	10	188
13·6	45	16·9	184
27·2	24	27·2	121

This drop in the reading of insulation resistance as the volts increase will generally be found to occur with, and is due to, moisture in or on the insulation under test. The increase of voltage may either break the insulation down, or the current, due to the voltage, may partially dry the moisture out and the reading gradually rise with the time of application.

Further, in measuring insulation resistance, sudden variations will sometimes be observed, especially will such be noticeable in direct reading testing sets. This is invariably due to metallic or other conducting particles on the surface (or, very rarely, buried in the insulation) promoting surface leakage, and the rapid fluctuations are due to intermittent sparking between such particles. Thus a direct reading testing set discriminates between low insulation due to damp, that due to dust, and that due to conducting particles, or whether the insulation is disintegrating under electric stress.

(43) Insulation Resistance of Electric Light Installations, Cables and Machinery by Evershed's Direct Reading Portable Testing Sets.

Introduction.—While there are several portable insulation testing sets on the market of both the direct and indirect reading types, we shall here consider the time-honoured form due to Mr. Sydney Evershed, which indicates the instantaneous insulation under high pressure by the direct deflection of a pointer on a scale. Thus the tests can be safely entrusted to

anyone possessing practically no technical skill who can make a report, and so enable defective work to be discovered before *covering in.*

The Evershed type is made by Messrs. Evershed and Vignoles, Ltd., in two forms, namely—

(1) *The "Megger" Insulation Testing Set,* which is the most modern development of the early form of Evershed ohmmeter and generator.

(2) *The "Bridge-Megger" Testing Set,* which combines the functions of the first-named "Megger" set with those of a Wheatstone Bridge.

Both sets consist of an ohmmeter of the moving coil type combined in one box with a hand-driven generator for providing the necessary testing pressure and current. When the handle of the generator is not being turned, the pointer is entirely free and will rest anywhere on the scale.

The internal construction and arrangement of the ohmmeter portion in both sets is the same, except for some minor additional details in the case of the Bridge-Megger set, which will be indicated later on.

We will therefore consider the use, firstly, of type (1) above, namely—

The "Megger" Insulation Testing Set.—The generator portion of this particular set may be either of the *variable pressure* or *constant pressure* kind. Unless the *electrostatic capacity* of the work to be tested exceeds one microfarad or so, a variable pressure instrument is suitable, which is the case for testing wiring, switch gear, dynamos and motors, arc lamps, instruments and accessories. Megger insulation testing sets being ohmmeters, their readings are independent of the applied pressure. If, however, the insulation under test has a large electrostatic capacity, the reading may become unsteady, due to the capacity current, caused by the variable pressure flowing through the current coil only; but even on large capacity, once the circuit is charged, the capacity current ceases and the reading becomes perfectly steady.

For work likely to have a capacity exceeding one microfarad or so, such as the wiring in metal conduits, lead-covered cable, underground mains and a modern system of house-wiring in

metal conduit, which has often a considerable capacity, the constant-pressure type of set should always be used.

The type of testing set now being considered is intended primarily for the measurement of insulation resistance, and is not available for metallic resistance tests. The low-range variable pressure sets are made in three ranges of 0–10, 0–20, and 0–100 megohms with 100, 250, and 500 volts respectively (at 100 revs. per min.), while the constant pressure low range

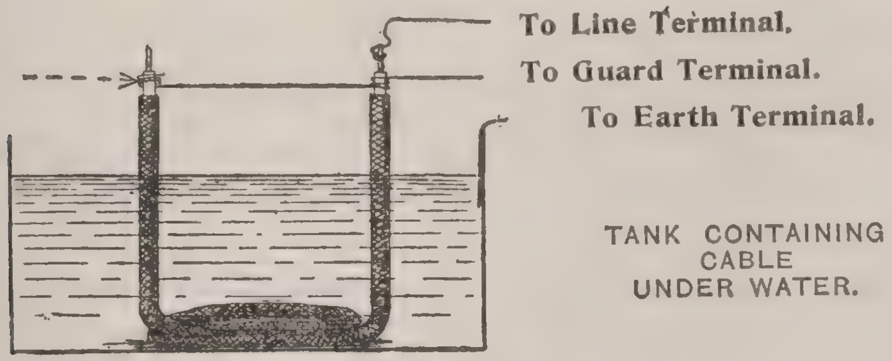


FIG. 42.

sets are made in the same three ranges together with a fourth for 0–200 megohms with 1000 volts. The high-range constant pressure sets are made in three ranges of 2–1000, 4–2000 and 4–5000 megohms with 500, 1000 and 1000 volts respectively. These last-named "Megger" Insulation Testing Sets are provided with a *guard wire* terminal, and, as explained on page 100, any error in tests of high insulation due to leakage internally or across the surface of the insulation under test can be eliminated, and hence the readings of the set remain unaffected, by tightly wrapping a so-called bare *guard wire* round the tapered insulation between conductor and earth, and connecting it to the guard wire terminal as shown in Fig. 42.

To Measure Insulation Resistance by the "Megger" Insulation Set.

Observations.—(1) For both low-range-*variable* and -*constant* pressure sets, as well as for the high-range constant pressure set: place the instrument on a steady base, but not on the bedplate of, or very close to, a dynamo or motor.

(2) Connect the terminal marked LINE to the insulated copper

core of the appliance under test, and that marked EARTH to a good earth such as a water-pipe or earthplate; or for testing between, say, two insulated wires, connect one wire core to each terminal. Then—

(3) Turn the handle in a clockwise direction at a speed of at least 100 revs. per min., at which—

In variable-pressure sets, the generator will be giving its rated or normal voltage which can be increased by increase of speed, and—

In constant-pressure sets, the clutch is felt to be slipping (for at any speed above that necessary to give slipping, the voltage will be constant). Now read the insulation resistance as given by the deflection of the pointer on the scale.

(4) High-range constant pressure sets (in addition to obs. 1–3 above) must be *levelled* by means of the spirit-level seen through the hole in the dial, and the *index must be adjusted to infinity* before connections are made to any of the terminals, by rotating the handle above the clutch-slipping speed and turning the knob of the index adjuster one way or the other until the index stands exactly on the mark at infinity.

Note.—In testing circuits of considerable electrostatic capacity it is essential to maintain full speed for at least a minute before taking the reading. Further, to eliminate errors due to surface leakage, a guard wire (see p. 100) must be used.

To Measure Insulation Resistance by the "Bridge-Megger" Testing Set.

Observations.—(1) Connect up, as in Fig. 43, and turn the change-over switch to "Megger," the instrument being on a steady base and not very close to a dynamo or motor.

Note.—In all cases the LINE terminal must be connected to the insulated conductor of the circuit or appliance under test and the EARTH terminal to a good earth, such as a water-pipe or earthplate, or the equivalent. This with machines may be the framework, with conduit wiring should be the metal conduit itself, with lead-covered street main should be the lead sheathing, and for non-sheathed cable should be the water in the immersion tank, etc.

(2) The handle is then rotated clockwise just above the speed at which the clutch is felt to slip. This occurs at about 100 revs. per min., while at any higher speed the voltage is constant, and the insulation resistance is then read off by the deflection of the pointer on the scale.

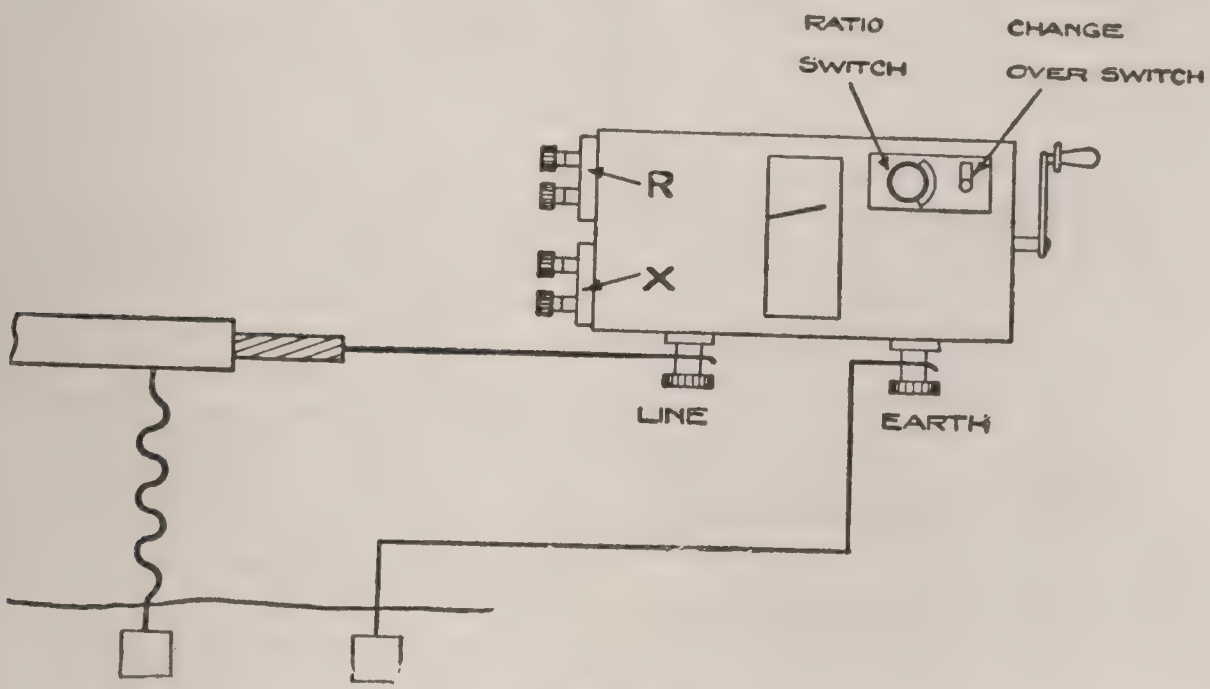


FIG. 43.

(44) To Measure Conductor Resistance by the "Bridge-Megger" Testing Set.

For use in this measurement, the adjustable standard known resistance box, supplied with this set, is required. Then to measure—

Resistances under 100 *ohms*.—(1) Stand the instrument on a steady base and with all terminals free from any outside connections. Adjust the pointer to "infinity" on the scale by turning the knob of the index adjuster one way or the other until the pointer or index stands exactly on the infinity mark.

(2) Connect up as in Fig. 44, set the change-over switch to "Bridge," the ratio switch to 10 or to 100, and all the resistance box dials to zero.

(3) Rotate the handle *slowly* clockwise with the right hand, when the pointer will float off the scale on the side marked "increase R" above the line marked G, simultaneously with the

left hand raising the value of R by turning the resistance box switches until the pointer exactly covers the line G.

(4) Now rotate at full speed to give maximum voltage and hence sensibility, readjusting the box resistance R, if necessary, to keep the pointer on the line G.

Then the resistance tested = the value of $R \div 10$, or by 100, whichever is in use.

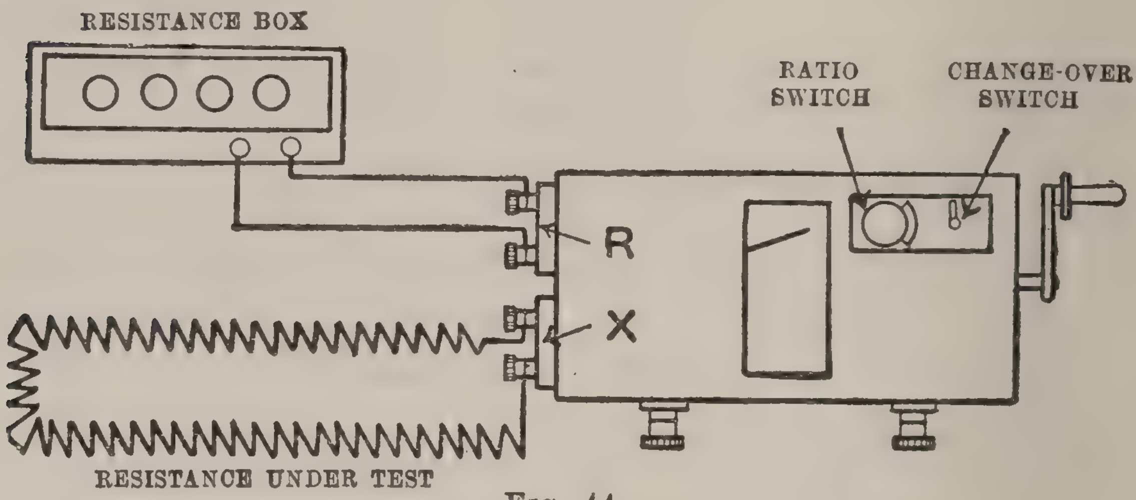


FIG. 44.

Resistances from 100 *to* 9999 *ohms.*—(5) Operate tests 1–4 above, except that in (2) the ratio switch is now set to 1 instead of to 10 or 100 as above. Then the resistance in the box required to balance the pointer exactly on the line G is now the value of that under test.

Note.—When measuring field coils of dynamos and motors, or other metallic resistances of large self induction, the generator must be driven above the speed at which the clutch slips to ensure the current being constant in the arms of the bridge.

Resistances from 10,000 *to* 999,900 *ohms* (*by* "*Bridge*" *method*).—(6) Operate test (1) above.

(7) Connect up as in Fig. 45, set the change-over switch tc "Bridge," and the ratio switch now to 10 or to 100.

Note.—It will be observed that the connections of the unknown resistance and box to the testing set in Fig. 45 are just the reverse to those of Fig. 44.

(8) Operate tests (3 and 4) above, remembering that the directions "increase R" and "decrease R" are now also

reversed, and that the unknown resistance now = box reading to balance × 10 or 100, whichever ratio is in use.

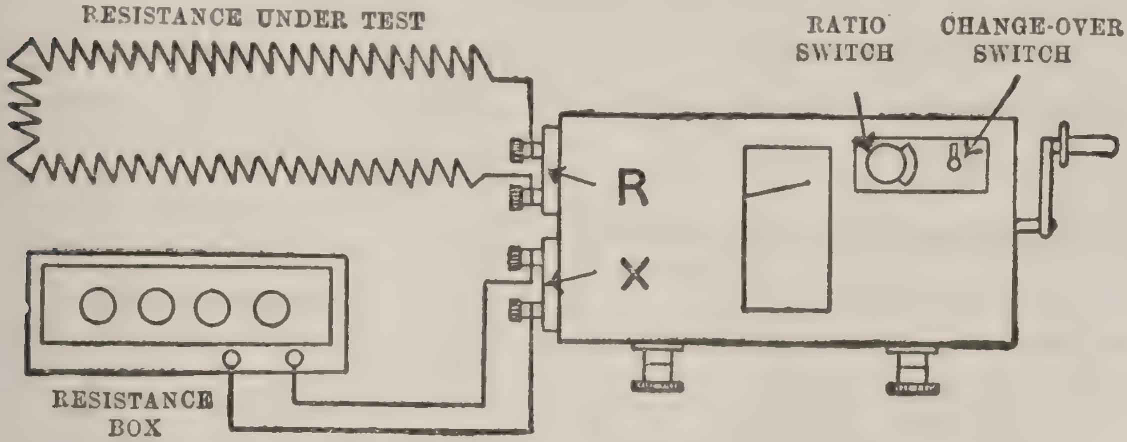


Fig. 45.

Resistances from 10,000 *ohms and upwards (by "Megger" method).*—This method is more rapid, but less accurate than the last, and is operated exactly as for Test No. 43, p. 115, the connections being as in Fig. 46.

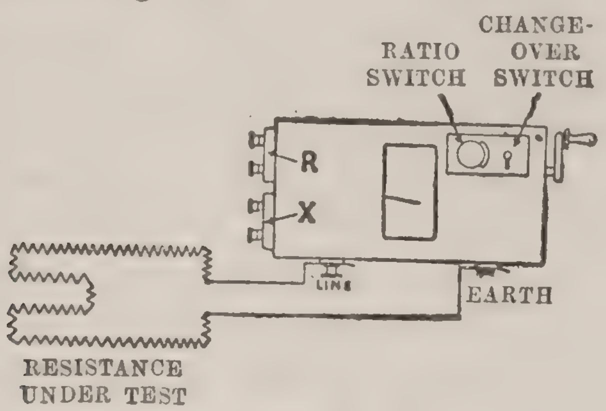


Fig. 46.

(45) Measurement of the Insulation Resistance of a Complete Electric Light Installation and plant *while working.*

Introduction.—The insulation resistance of any system of distribution of electrical energy when *not working* can be determined by one of the preceding methods. These are, however, inapplicable to systems actually running, and consequently "alive" at the full working pressure. It is most important to make frequent, if not daily, tests of the insulation resistance of any installation in order that a gradually developing fault, which

would cause the insulation resistance of the whole system to gradually fall, might be discovered in time and remedied before it perhaps burnt itself out and fired the premises.

The following method is a simple and convenient one for making such a test on any system, whether that of a country house, having its own generating plant or the main distributing network of a large town. The arrangement is shown in Fig. 47, where $+M$ and $-M$ are the positive and negative mains or wires of a two-wire system of direct current distribution.

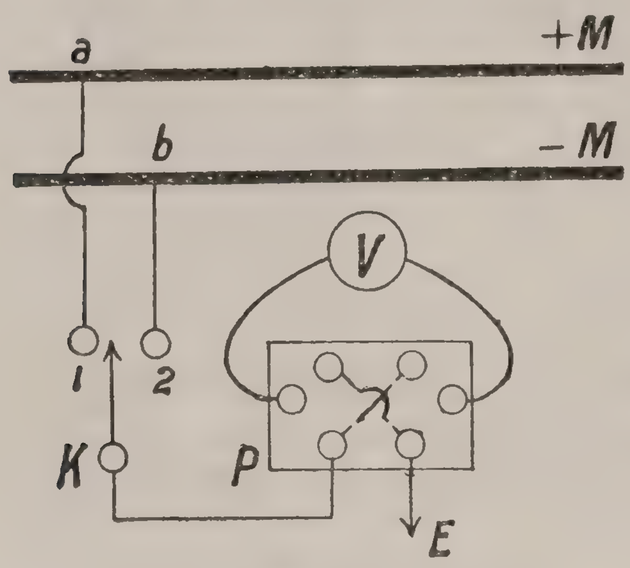


FIG. 47.

The contact studs 1 and 2, of a two-way key K (p. 586), are electrically connected (temporarily or otherwise) to any points a and b of these mains, which might be the + and − 'bus. bars on the switch-board. The common terminal of K is connected through a Pohl's commutator (p. 584) or other reversing key P to *earth* E (*i.e.* nearest gas- or water-pipe). By means of P, a voltmeter (V) connected to P has its terminals interchanged between K and E on moving P over so as to reverse. Thus a current flowing either from K to E or E to K can be made to give a deflection in the same direction on V by manipulating P. If this latter is used it can be converted into a reversing key by cross-connecting the four mercury cups, as shown by separate connectors indicated by the dotted lines; both K and P should have a good insulation resistance; V should be a + and − instrument, preferably of the moving coil D'Arsonval type. It may either be a voltmeter or ammeter, but we shall assume the former in the present case.

Observations.—(1) Connect up as shown and put K to stud 1, P being such as will allow V to deflect over the scale. Note the reading V_1 on the voltmeter.

(2) Put K to 2 and reverse at P so as to still make V read on the scale. Note the reading V_2.

(3) Calculate the insulation resistance of the whole lighting system (including dynamos, battery, leads, cut-outs, lamps, etc., etc.) from the relation—

$$R = r_v\left(\frac{V}{V_1 - V_2} - 1\right) \text{ ohms};$$

where r_v = the resistance in ohms of the voltmeter, and V = the working pressure at the time across $+M$ and $-M$.

The insulation resistance of the + main is

$$R_1 = \frac{r_v\left[V - (V_1 - V_2)\right]}{-V_2}$$

and of the − main is

$$R_2 = \frac{r_v\left[V - (V_1 - V_2)\right]}{V_1}.$$

Since V_1 and V_2 are to opposite sides of zero, or on the same side by reversing at P, they must be added; r_v should not be too large, but its value depends on the insulation resistance tested. If this be 100 ohms or so, r_v might be of the order of 1000 ohms. Since the value of R, R_1 and R_2 in ordinary small installations is usually high, *i. e.* considerably over 1000 ohms, a voltmeter may be used having a resistance (r_v) of, say, 5000 to 10,000 ohms.

Inferences.—Why is an electrostatic voltmeter unsuitable for use in the above test?

(46) Measurement of the Insulation Resistance of Complete Electrical Installations and Distributing systems while working.

Introduction.—The following method is one of the most accurate for measuring the insulation resistance of a system taken as a whole, but it does not give any idea as to which main a fault may be developing. It is really an application of Mances' method for determining the resistance of a conductor containing an E.M.F., and entails the use of preferably an ordinary Post Office pattern Wheatstone Bridge, with its detecting moving needle galvanometer G (p. 571) and an extra-resistance (r) for safety. Fig. 48 shows the sketch of connections for the test, where $+M$ and $-M$ are the mains the insulation resistance R of which it is desired to measure; E represents earth (*i. e.* the nearest gas- or

water-pipe); *FF* are protecting fuses. The P.O. Bridge is represented symbolically by the zigzag lines *A*, *C*, *S*, *D*, which indicate the rows of plugged resistance coils.

Observations.—(1) Connect up as shown, joining the bridge terminal *D* to some point (*a*) on the distributing system to be

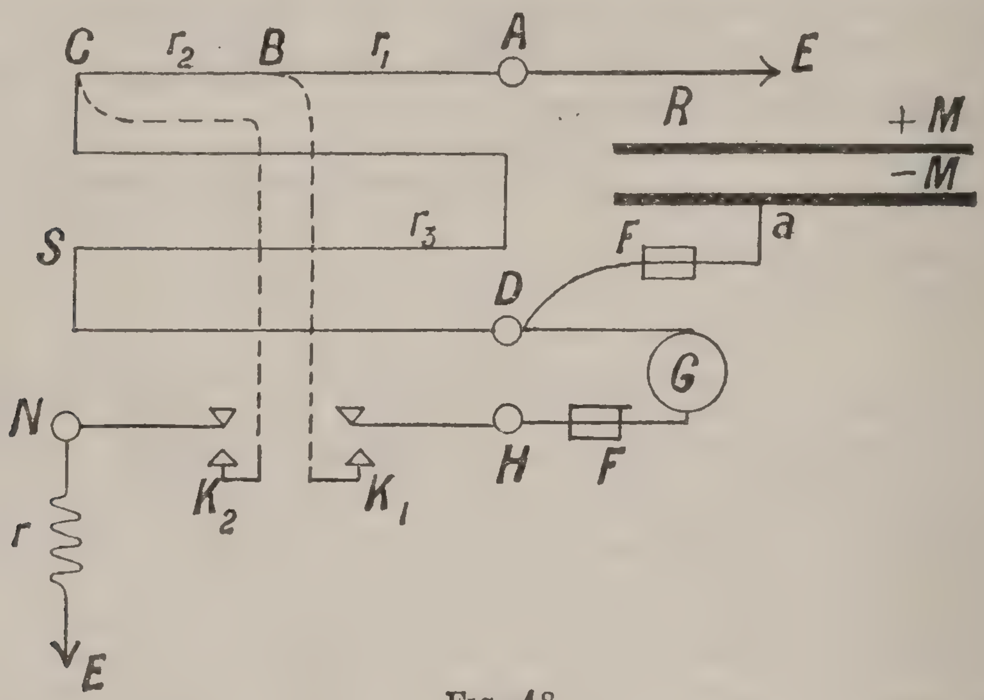


FIG. 48.

tested; *r* may be a few ohms or that offered by half-a-dozen or more glow lamps in parallel.

(2) With the proportional arms r_1 and r_2 arranged so that $r_1 : r_2$ is as small as possible in order to obtain the adjustable arm r_3 large, close K_1 and then bring back the galvanometer deflection, thus produced, to zero by means of the controlling magnet.

(3) Then with K_1 closed alter r_3 so that on opening and closing K_2 no motion of the galvanometer is observed. Then the insulation resistance of the system as a *whole* is $R = \frac{r_1}{r_2} r_3$ ohms.

Note.—This method can be employed in the case of alternating current systems at work by placing a few cells of a battery in place of (*r*) and using a galvanometer that will not indicate alternating currents. The mode of procedure is then the same as before.

(47) Measurement of Insulation Resistance and detection of faulty Telegraph Insulators.

Introduction.—It is of paramount importance that all insulators intended for use on telegraph or telephone lines should be tested prior to their erection in the circuits. This will at once be evident when it is remembered that the insulators, supporting a line having an "earth return," form so many parallel circuits between the line and earth, and though the resistance to leakage of current of each insulator may be very great, yet their parallel resistance to the same may be very considerably less, allowing a very appreciable current leakage to go on to earth continuously. This, especially in telephone circuits, is very troublesome, causing the lines to interfere with one another, and besides this, speaking generally now, it gradually wastes away the batteries of cells used in working the lines. Again, the insulation resistance of a "line," composed of all very good insulators except one, will be lowered by the one single faulty cup to a value less than that of the faulty insulator.

Hence the importance of preventing, by a suitable and timely test, the installation of a bad insulator, which is generally found to deteriorate rapidly with time. Insulators should be tested, preferably before the bolts are inserted in them, and with an E.M.F. of from 100 to 300 volts after from 24 to 48 hours' immersion in a suitable manner in water. The value of the results of such a test will depend, however, very greatly upon the exact method of preparing the insulators and of applying the test, and some precautions have to be attended to in order to obtain a result which is trustworthy and *fair to the insulator itself*. The minimum insulation resistance which each insulator ought to possess depends on the length of the line on which it is to be used. In this country, during the worst wet weather conditions, it has been found possible to maintain a minimum insulation resistance of $\frac{1}{5}$ megohm per mile on air lines, which is therefore taken as the minimum maintenance standard. Now the number of poles, and therefore insulators per mile of line, varies from 20 to 30 according to whether it is a branch or trunk

line. Hence, allowing the latter number, each insulator should have a resistance of at least 6 megohms if the line were only one mile long, 60 if 10 miles long, and 600 if 100 miles long, and so on, in the worst wet weather. As a matter of fact the resistance of a double shed or cup porcelain insulator, if a good one, may vary under testing conditions from 500,000 megohms to something like 4,000,000 megohms.

Apparatus.—Shallow trough T lead-lined inside, to which is attached the terminal N making contact with the lead; insulators to be tested I_1 I_2, etc., only these two being shown; battery B capable of giving 100 to 300 volts E.M.F. and consisting of either Daniells, secondary or other convenient cells; very sensitive high resistance reflecting galvanometer G with its shunts (S); high insulation two-way key K (p. 586); high insulation test rod t connected to K by a well-insulated length of gutta-percha covered wire; standard megohm r.

N.B.—In this and all other similar high resistance tests, the connections should be in mid air as much as possible, and all insulating material quite clean and free from dust and finger-marks.

Assuming all the insulators to be perfectly clean on the inside and outside of their sheds, there are two modes of procedure, in each of which they are immersed in the trough T with the water up to within half-an-inch of the lips of their outside walls. Thus—

(*a*) The whole of the inside of each insulator kept clean and dry, the resistance then from bolt (*b*) to water being very approximately their *insulation resistance* under working line conditions. This is usually so high that the galvanometer is not sensitive enough to indicate the extremely small current passing unless the insulator is actually deflective. If it is all right the only way usually to obtain a deflection is to test from 100 to 200 in parallel at one time.

(*b*) The inside of the shed or sheds carefully filled with water by means of a pipette to within half-inch of the lips, the resistance now from bolt (*b*) to outside water showing whether the insulator is faulty through the leakage current passing through its substance, assuming the unimmersed lips to be *clean* and *dry*. The

insulator should be discarded if under these conditions it does not

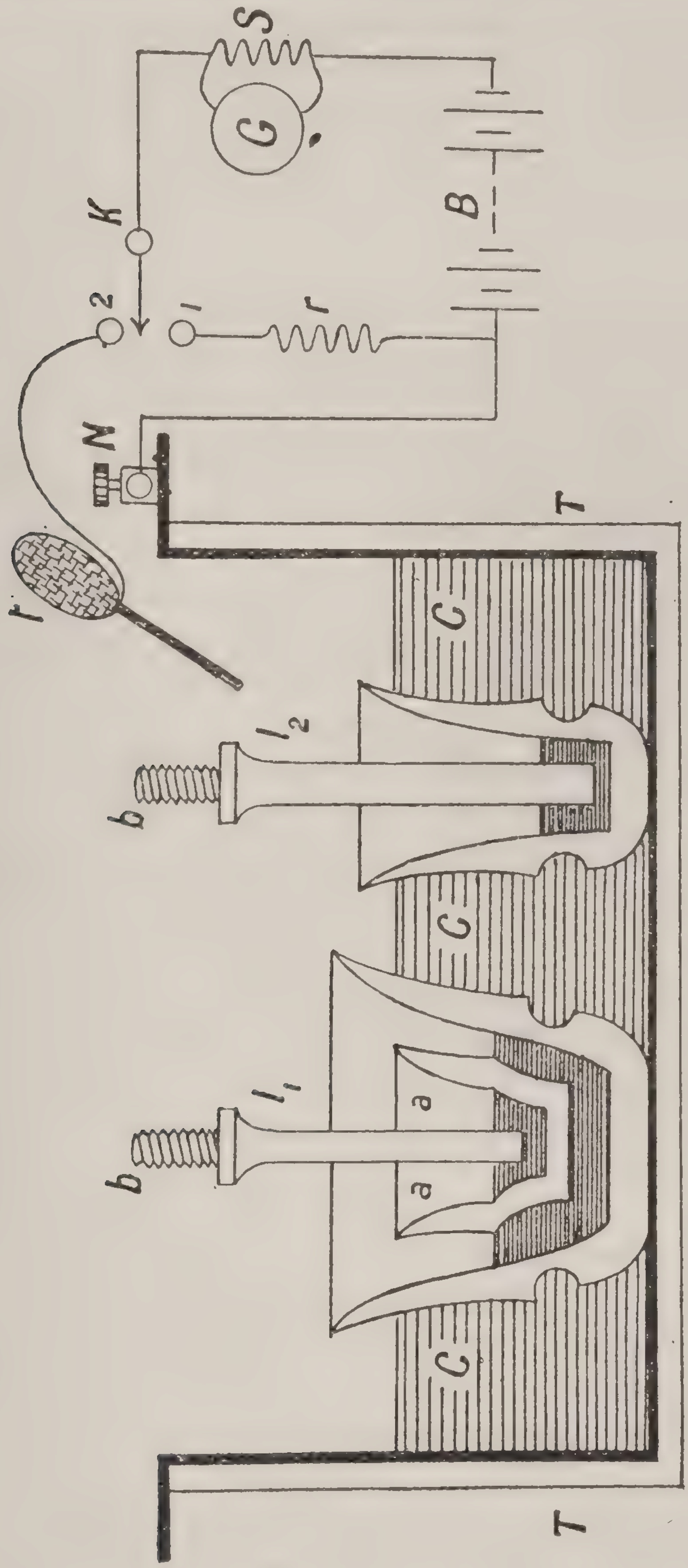


Fig. 49.

show 600 megohms at least, preferably 2000, which is the minimum in some telegraph services.

In the following test we shall adopt the latter mode of testing, since the former is in most cases impracticable.

Observations.—(1) Carefully clean the porcelain or earthenware portion of each insulator, especially the lips, with clean cold water. Then dry the lips merely in a place free from dust, and place them with their bolts pointing upwards (Fig. 49) in the tank T, packing up the smaller ones so that all the lips are at about the same level, but not touching one another.

(2) Now pour water into T to within half-inch of the lips and into the cups, by means of a pipette, to the same level. Then allow the insulators to soak for about forty-eight hours (in a place free from dust), so that the water will percolate or soak into any cracks or flaws in the mass of the earthenware.

(3) Before testing, hold a hot iron close over the rims of each insulator for a short time to dry off any moisture, as it is necessary that THESE PARTS SHOULD be QUITE DRY. For this reason avoid testing on a damp day, unless the air of the room is dry.

(4) Connect up as shown in Fig. 49. Test the insulation of the connections between t and k before each set of observations. This will be satisfactory if G does not deflect when the connecting wire is supported in mid-air when t is held in the hand.

(5) With the galvanometer at zero and the $\frac{1}{999}$th shunt in, close $K\,1$, so as to bring r into circuit, and note the steady deflection (d), then close $K\,2$ (opening $K\,1$) and touch the bolt of each insulator successively with t, noting the deflections D in each case if any.

(6) Repeat 4 and 5 for the $\frac{1}{99}$, $\frac{1}{9}$ and no shunt if possible.

(7) In the case of the double-shed insulators, such as I_1 (Fig. 49), touch the water between the sheds with t, thus obtaining the resistance of the outer shed. Next connect metallically the water between the sheds, and also that outside, and touch the bolt with t, thus getting the resistance of the inner shed.

(8) The approximate resistance of all those insulators which show an extremely high insulation can be obtained by paralleling them or joining their bolts together metallically, and then

taking a reading (as in 6), first with the $+^{ve}$ of the battery to T, and second with the $-^{ve}$ to T, the insulators being discharged in between the two reversals. Calculate the average of each insulator from the combined or parallel resistance so obtained.

(9) Employ the formula given below, or its modification, and tabulate your results as follows—

NAME . . . DATE . . .

Galvanometer Resist. G =... Ohms at... °C. Internal Battery Resist. = ... Ohms.

Insulator tested.	After hour's immersion.	Distance of Water from Lip.	Temp. of Room.	E.M.F. used.	Shunt S.	Deflection. known d.	Deflection. unknown D.	Known Resistance (r).	Insuln. Resistance R.

$$d\left\{r\left(1+\frac{G}{S_r}\right)+G\right\}=D\left\{R\left(1+\frac{G}{S_R}\right)+G\right\}.$$

$$\text{If } S=\tfrac{1}{9}\text{ (say), then } \frac{S}{S+G}=\tfrac{1}{10}\text{ or } \left(1+\frac{G}{S}\right)=10.$$

$$\text{If no shunt is used } S=\infty \text{ and } \therefore \frac{G}{S}=0.$$

(48) Measurement of the Insulation Resistance of Storage Batteries.

Introduction.—In the erection of a storage battery, provision is made for the efficient insulation of every cell composing the battery, from earth, by supporting each on suitable insulators, and keeping the cells from touching one another. Notwithstanding these precautions, leakage of current, from various causes, may go on in a greater or less degree. It may occur in different parts of the battery, or in the leads running from the battery, and the resistance opposing the leak may, and usually does, have different values at different parts of the battery; in other words, the partial "earths" at the various points are of unequal resistance, which prevents the insulation resistance being obtained by an ohmmeter or other portable testing set.

The difficulties thus met with are got over by employing the present method due to Mr. E. S. Jacob, and which at once gives the joint resistance of all the earths at whatever points of the battery they may be located.

Apparatus.—Sensitive high resistance galvanometer G; adjustable known resistance box (r); battery $B_1 \ldots B_2$ to be tested, there being any number of cells between the two B_1 and B_2 shown. Key K of very good insulation (p. 586), battery stands, or equivalent earth E.

N.B.—Earths or partial ones, *i. e.* leaks, may be assumed to

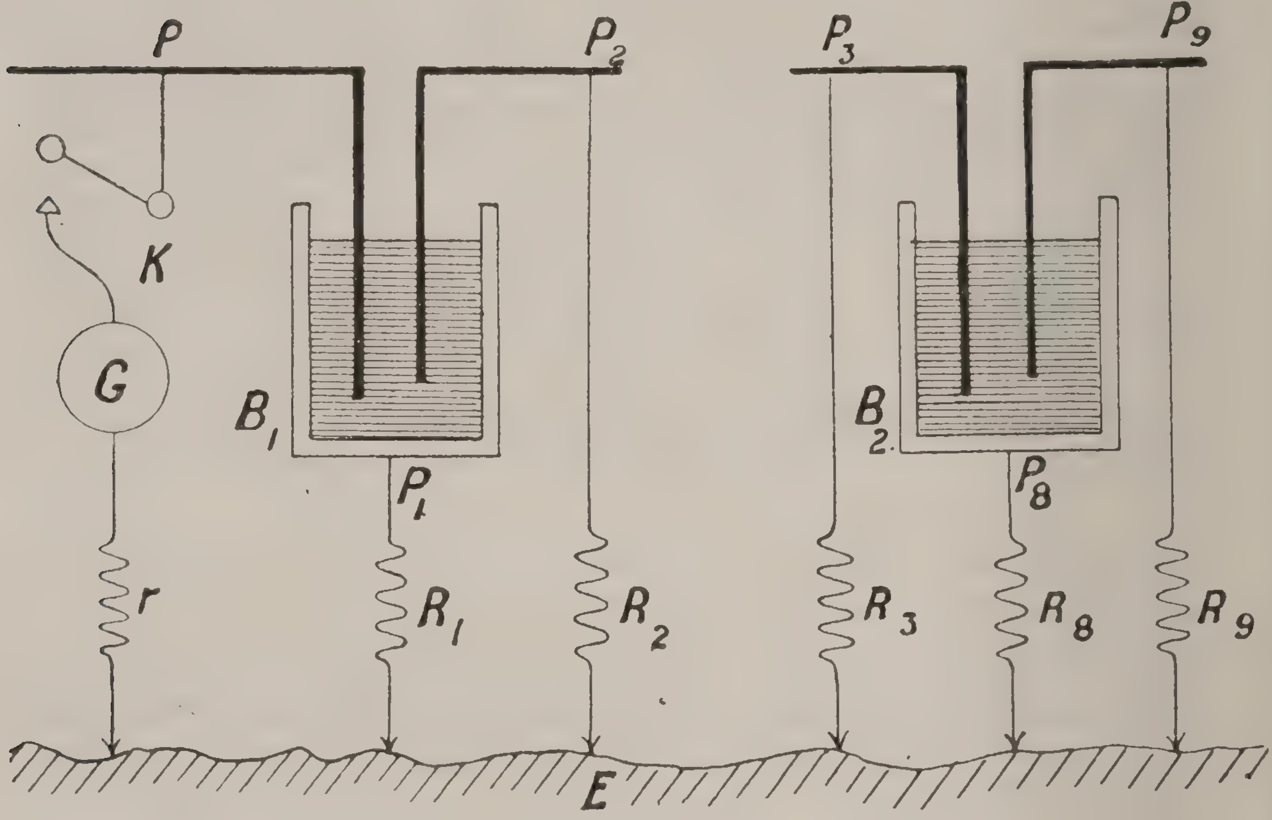


Fig. 50.

occur at points $P\,P_1 \ldots P_9$, etc., and $r\;R_1 \ldots R_9$ to be the respective resistance of such earths.

Observations.—(1) Connect the key K, galvanometer G, and resistance box (r) to the battery system at any point P, as shown in Fig. 50, E being earth, *i. e.* the nearest gas- or water-pipe, and adjust the galvanometer needle to zero.

Note.—Choose the point P, so that a conveniently large deflection is obtained on G, for it will be found that there is a certain point of minimum potential from which it increases on either side as the contact is moved one way or the other.

(2) With $r=0$, press K and note the current C_G and, for reference only, the relative position of the point of contact P on the system.

(3) With everything the same as in 2, adjust (r) so that on pressing K the new current C_r on G is about $\frac{1}{2}$ C_G. Note the resistance r and this deflection or current C_r.

(4) Repeat 1–3 for two or three different values of r.

(5) Repeat 1–4 for two or three different points of contact P.

(6) Calculate the insulation resistance of the whole battery from the relation—

$$R = \frac{C_r(g+r) - C_G g}{C_G - C_r} \text{ ohms,}$$

and tabulate as follows—

NAME . . . DATE . . .

Battery: Type . . . Containing boxes of . . . No. of cells . . .
Galvanometer Resistance g = . . . Ohms @ . . . °C.

Relative position of P.	Value of r Ohms.	Current C_G	Current C_r	Insulation Resistance R Ohms.

Inference.—Prove the relation given in 6, and state any assumptions made in obtaining it. On what does the accuracy of the test depend?

(49) Insulation Resistance of Dynamos and Motors.

Introduction.—Since the total amount of leakage from a system of electrical distribution depends on the number of appliances connected to the system, which represent so many parallel paths for leakage currents to take, it is of great importance to increase the resistance in these paths, and so diminish, as much as possible, such currents. This, in other words, means that the insulation resistance of all appliances on the system should be as high as possible. Hence it becomes of importance to carefully measure the insulation resistance of motors and dynamos, both in course of manufacture and when installed before running is commenced.

The Institution of Electrical Engineers recommends that the leakage current should not be allowed to exceed $\frac{1}{5000}$th part of the maximum current in the system at full load. As a general rule the total insulation resistance of all the insulated parts of a dynamo or motor, joined together, from the frame of the machine,

i. e. earth, should be at least equal to that of the rest of the circuit which it has to run on, but preferably much greater. The present test is a simple and convenient one for obtaining the insulation resistance of, say, a dynamo, and is as follows—

Observations.—By means of a high resistance voltmeter of resistance R_v ohms measure the P.D. V at the terminals of the machine when running at normal voltage not connected to any circuit. Also measure the P.D.s V_1 and V_2 between the + and − brushes and the frame of the dynamo. Then we shall have—

Total insulation resistance of the machine

$$R = \frac{V - (V_1 + V_2)}{V_1 + V_2} R_v \text{ ohms.}$$

(50) Localization of Faults in Electric Mains. (Murray's Loop Method.)

Introduction.—In the preceding pages, some of the best and most important methods of measuring the insulation resistance of electric mains have been dealt with, but the localization of the position of any prominent and serious fault in such is a matter of equal, if not greater, importance. The insulation of the cable may or may not have broken down at the fault, but obviously in either case, since the opening up of the duct in which the cables are laid is often a serious matter in a busy thoroughfare, it is important to be able to localize the position of the fault to within a very few feet.

There are two important methods or systems of localizing faults, each comprising modifications of the principles employed, viz.—

(*A*) "Loop" methods. (*B*) "Fall of Potential" methods.

The latter, though extremely simple, are less easy of application than the former, and must be applied so as to suit the conditions of the particular fault. *Loop methods* are much more generally applicable, and are preferable for the following reasons—

(1) They are "null" or "zero" methods.

(2) They are easier to work.

(3) The accuracy is not affected by variations of the fault resistance.

(4) The test is performed in the same way for mains of any section and whatever the fault resistance.

We shall therefore restrict ourselves to one of the best of these loop methods, but before taking it in detail there are a few remarks which must be made.

Loop methods require a continuous circuit of main or cable from the place where the testing instruments are, through the faulty portion and back again, which is termed the *loop.* The two portions of cable between fault and instruments should have good insulation resistance, but need not have the same sectional area throughout. Referring to Fig. 51, let MSN be a cable, the two ends of which cannot be brought closer to the terminals A and C of a Wheatstone Bridge than the points M and N respectively.

Let there be a fault at some point P between the core of the cable at P and the earth E, the resistance to earth being much less than that at any other point. This is called the fault resistance, and may be represented by r.

The fault might be on the "outer" of a concentric main, in which case the "inner" and "outer" conductors would be carefully joined at the further end S, so as to form a complete circuit or loop.

In fact, in practice we should either be dealing with this case of a concentric main or with a fault on one of a pair of separate mains.

In either case the loop would be formed by making the *best possible joint* between the two ends furthest from the testing point, by disconnecting them from any terminals to which they might be clamped. Now it is obvious that the cable ends M and N cannot be clamped under the terminals of the bridge, owing to their size and to them terminating in a position in which it would be impossible to have the testing apparatus. Smaller connections AM and CN must therefore be used and also allowed for in working out the results of observations, and this is done as follows—

Let D_1 D_2 = distances from M and N respectively to the looping point,

and d_1 d_2 = distances from A to M and from C to N respectively,

s = sectional area of either conductor of the main MSN,

s_1 and s_2 = sectional area of the connections AM and CN respectively.

Then, since resistance is directly $\propto \dfrac{\text{length}}{\text{section}}$, we see that AM has the same resistance as, or is *equivalent* to a length of the main $MSN = \frac{s}{s_1} . d_1$ and CN to a length $= \frac{s}{s_2} . d_2$.

Hence the bridge will behave as if the resistance between C and P (which = resistance of CN + resistance of NP) was that = to a length of the main cable $= D_2 + \frac{s}{s_2} \cdot d_2$.

The whole length of circuit L between A and C is—

$$\therefore\ L = D_1 + \frac{s}{s_1} \cdot d_1 + \frac{s}{s_2} \cdot d_2 + D_2.$$

The expression or value for L may very greatly be simplified by making $D_1 = D_2$ and also $d_1 = d_2$, which latter can easily be done, and by choosing single wires for the connections AM and CN of the *same gauges* as each wire of the strand forming the main MSN. Thus if the main was $\frac{61}{16}$ in size, use $\frac{1}{16}$ for the connections.

If this is done and n = N°. of wires in the strand of either conductor of the main,

$$\text{then } L = 2D + 2.\ n\ d = 2\ (D + n.\ d.)$$

The lengths of AM and CN, though =, should be as short as possible, so that their resistance is not large compared with that of the main cable.

Apparatus.—Post-office or other pattern Wheatstone Bridge; sensitive galvanometer G (p. 569); battery B of Leclanché cells; fuse F; resistance R; suitable connections AM and CN and main to be tested.

Note.—A few secondary or primary cells may form the battery B, and if the fault resistance r is anything like 1000 ohms, a battery giving 100 volts might be used. The fuse F is to prevent the bridge coils being fused up should r break down unexpectedly, and it can be dispensed with if a primary battery is used. It is as well to earth the battery, if not a primary one, through a resistance R of a few ohms.

The more sensitive G is, the greater the accuracy of the test, and quite small wires may be used to connect G and B up, as their resistances do not affect the accuracy of the test.

Observations.—(1) Connect up as shown in Fig. 51, and adjust the galvanometer G nearly to zero. See that a very fine fuse is inserted in F, if necessary at all, and make fairly good earths at EE. Good soldered joints at M, N and the looping point are practically indispensable, as well as good tight clean contacts at A and C. Inattention to these points will vitiate the results.

(2) Plug up $r_1 = o$ and unplug $r_2 = 1000$, with, say, the *Infinity plug* out in r_3.

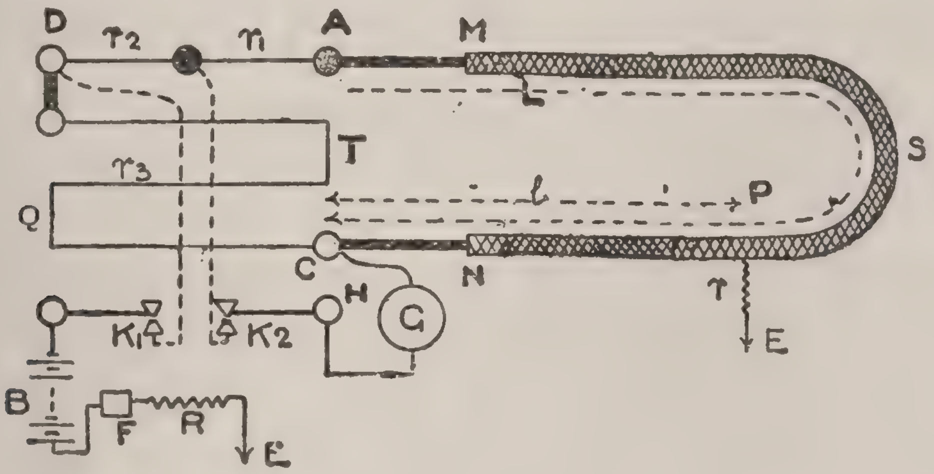


Fig. 51.

(3) Press K_1 first and then tap K_2 for an *instant*, observing which way G deflects for r_3 so great.

If on inserting "*Inf.*" G defects the other way on manipulating K_1 and K_2 as before, then a balance can be obtained by adjusting r_3 so that G does not deflect on pressing K_1 and then K_2. Note the value of r_2, r_3. If no balance can be found, the fault P is very close to M or N when the connections to A and C should be interchanged.

(4) If the balance appears to be insensitive $r_2 = 100$ or 10 may be tried and (obs. 3) above repeated.

(5) Calculate the equivalent distance of the fault P from the terminal C by means of the relation

$$l = \frac{r_3}{r_2 + r_3} \cdot L,$$

and tabulate your results in a convenient way.

N.B.—The distance of P from N therefore $= l - \frac{s}{s_2} . d_2$.

(6) Interchange the connections to A and C and repeat obs. 2–5, and if the result now given is close to the previous one, take

the mean of the two as the correct one. If they differ considerably, the contacts are bad and must be improved.

In this latter test the distance of P from A is given by

$$l = \frac{r_2}{r_2 + r_3} L.$$

(51) Calibration of Speed Indicators.

Introduction.—There are many different forms of speed indicators or tachometers, as they are frequently termed, some of which vary in accuracy with the time for which they are in use. It thus becomes of importance to standardize such instruments at frequent intervals when accuracy is required. This can be done either by comparison with an accurately calibrated tachometer specially reserved for this purpose only, or by taking simultaneous readings on the instrument to be checked and time readings on a speed-counter. This latter method, being the more generally applicable of the two, will be the one here considered.

Apparatus.—Tachometer to be tested; speed- or revolution-counter; stop-watch if available, or in lieu of this an ordinary watch with a "seconds-hand."

Observations.—(1) Arrange the tachometer so that it can be driven by some machine at a variety of speeds, each of which may be maintained constant for at least one minute. This driving machine must have a turned centre in one end of its shaft into which can be pressed the centre of the spindle of the revolution-counter.

(2) Drive the tachometer at the lowest convenient speed readable on its scale, and take the dial readings (D_1) of the revolution-counter. When this speed is constant (as will be observable by the instrument itself) insert the counter in the end of the driving-shaft *at a noted instant of time*, using a light pressure. Quickly take the counter away, the instant one minute has elapsed, and note the readings (D_2) of its dials. Then $(D_2 - D_1)$ = speed in revolutions per minute.

(3) Repeat 2 twice or three times at the same speed, and record the mean.

(4) Repeat 2 and 3 for some eight or ten readings on the tachometer, rising by about equal increments to the maximum.

(5) Check some of the readings obtained in 2–4 by taking observations over two minutes.

(6) If the tachometer is adjustable, alter it so that its indications give the correct speed in revolutions per minute. If unadjustable, plot a calibration curve having values of true speed as abscissæ and tachometer readings as ordinates.

(52) Measurement of Rotational Speed by the Stroboscopic Fork.

This method has been in use for measuring the speed of generators and motors for many years. See papers by Dr. C. V. Drysdale—(1) on "Stroboscopy"; read at the Optical Society, London, in 1905, and reprinted in *The Optician and Photographic Trades Review*, Dec. 8 and 15, 1905. (2) "Accurate Speed, Frequency, and Acceleration Measurements," *Electrical Review*, London, Sept. 7 and 14, 1906. Previous to that it had been used by scientists, *e. g.* to measure the speed of a small driving motor to within $\frac{1}{100}$ of 1% in the Lorenz apparatus for determining the absolute value of the ohm.

The stroboscope used comprises essentially—(1) either a hand vibrated, or (preferably) an electro-magnetically vibrated tuning fork fitted with two shutters, each containing a narrow slit, and (2) a discoidal target of observation.

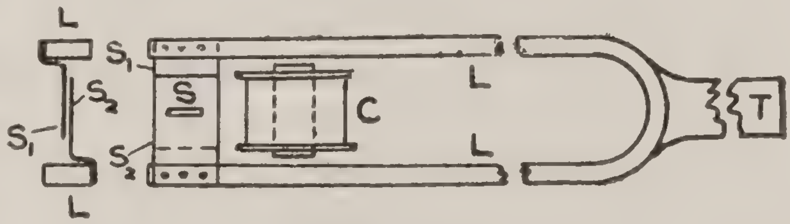


FIG. 52.

The stroboscopic fork unmounted is indicated diagrammatically in Fig. 52, and that used by Dr. Drysdale consists of a steel tuning fork, the limbs LL of which are approximately 12″ long × $\frac{17}{32}$″ wide × $\frac{7}{32}$″ thick, and are supported by a tail-rod T

clamped in a suitable frame or stand (not shown). To the extremities of LL are screwed two light and thin metal strips or flat shutters S_1 S_2 ($1\frac{1}{4}'' \times \frac{13}{16}''$) in the plane of vibration, each containing a narrow slit S ($\frac{5}{8}''$ long) parallel to the length of LL. These slits are exactly opposite to one another when the fork is at rest, so that in this condition the eye can see through both slits. When hand vibrated the fork, while being held in the observer's hand, is excited into vibration at intervals by mechanical impulses derived from a light blow on the knee.

When electro-magnetically operated, the "make and break" principle used in the electric trembling bell is adopted—an iron-cored coil C being mounted between LL on a support (not shown) adjustable, for convenience, in a direction parallel to LL.

This actuating coil C is energized from one or two dry cells through a "contact breaker," formed by two platinum-tipped contacts—one on a fixed support, the other on a very light flat spring fixed to one of the limbs L. Now when the fork is excited into free vibration, the line of vision through the slits S is interrupted by the vibrating shutters S_1S_2, except during a very brief interval, once in each half-cycle, or twice in each complete cycle of the vibration, at the moment when the slits S pass each other, moving rapidly in opposite directions. If, then, an object viewed through the slits S is rotating so that consecutive images conveyed to the eye are similar and symmetrical, that object will appear to the eye to be continuous and stationary, although actually rotating at a high speed.

Further, a certain cyclic departure from absolute similarity and symmetry in the successive images seen by the eye will make the object appear as if it was rotating. If the object appears at a standstill when seen through S, its speed must be constant, and must bear a simple proportion to that of the vibration of the fork. Therefore, since the rate of vibration of a tuning fork is always very constant at all times, and almost independent of change of temperature (varying only about 0·01% per deg. cent.), it can be determined once for all with great accuracy, and hence also any speed by comparison can be measured with like accuracy

If a fork vibrates with a frequency of 50 cycles per sec., the eye will receive 100 impressions of the object through the slit S per second, and will have 6000 peeps per minute. The geometrical form of the rotating object viewed through the slit will, in conjunction with the rate of vibration of the fork, decide the speeds at which the object will appear at a standstill. These speeds might, for example, be 100, 200, 300, etc., revs. per min., and the simple fork shown in Fig. 52 can only serve to measure speeds intermediate between these by enabling the eye to count the apparent revs. made by the object in one minute by a watch. For example, if the speed at which the object—viewed through the slit appeared stationary—was 1500 revs. per min., and afterwards the object appeared to be rotating in the direction of motion at 50 revs. per min., then the actual speed of the object would be 1550 revs. per min. If this apparent rotation of the object was in the opposite direction, the actual speed would be 1450 revs. per min.

The determination in this manner of speeds intermediate between the two nearest standstill valves, coupled with the necessity for such being constant, is a disadvantage in the stroboscopic method, for it is obviously preferable to be able to bring the object to an apparent standstill at any speed. At least two methods of doing this have been devised: one due to Dr. Drysdale employs a special arrangement of a conical roller with stroboscopic disc, as described in his paper, but which lacks portability. In another method, designed for portability by Messrs. A. E. Kennelly and S. E. Whiting, and described by them in a paper read at the Twenty-fifth Annual Convention of the Amer. Inst.E.E., Atlantic City, N.J., June 29—July 2, 1908, the stroboscopic fork is adjustable continuously in its rate of vibration through a range of about 5% above and below its mean value without sensibly disturbing its motion. This is done by a pair of sliding weights, which are moved friction tight along about $7\frac{1}{2}''$ of the limbs of the fork, gradually from one position to another, by a pair of strings, normally slack, passing over guide-pulleys, and vibrating with the fork. The form used in this design has limbs $18''$ long $\times$ $1''$ wide $\times$ $\frac{3}{16}$ thick, and weighs by itself almost 2 lbs. The object in using so long a fork was to obtain a low frequency vibration or fork-

speed comparable with the speeds ordinarily met with in rotating machinery. The fork has a mean speed giving 1900 peeps per min., and carried on its extremities a pair of thin sheet steel shutters with slits 0·59″ long × 0·008″ wide. Adjustments are provided for aligning these slits, and for tuning the fork to the required normal frequency. It is operated by an electro-magnet, the coil of which is wound with 525 turns of No. 27 S.W.G. enamel insulated copper wire (enamelled to No. 26 S.W.G.), having a resistance of about $3\frac{1}{4}$ ohms, and requiring an average current of about 0·15 amp. to operate the fork through a form of contact breaker already referred to.

The amplitude of the vibration of the fork limbs is about $\frac{1}{8}$″ at the slits, each side of their position of rest, giving a maximum cyclic velocity of about 1 foot per sec. at normal fork speed. The relative velocity of the slits is thus about 2 feet per sec., and the duration of each peep through the slits will be $\frac{1}{3000}$ of a sec. $= \frac{1}{180000}$ of a min. Hence an object rotating at 1800 revs. per min. will only move through $\frac{1800}{180000} = \frac{1}{100}$ of a revolution during each peep, and if it appears at a standstill through the slits, 1 rev. per min. more or less, *i. e.* a variation of $\frac{1}{18}$ of 1% in the speed will cause the object to make 1 rev. per min. either way.

The Target—hitherto called the rotating *object*, may most conveniently consist of a thin disc (between 9″ and 10″ in diam.) of sheet metal or thick cardboard fixed concentrically—in any convenient and satisfactory way—to the end of the shaft whose speed is to be measured. If neither end of the shaft system is available, nor any portion of the end of the machine suitable for use as a target, it may be possible to mount the special artificial target on a subsidiary spindle with pulley, and drive this by a light supple tape belt at a definite speed ratio from some part of the machine under test.

Fig. 53.

From many trials with different kinds of targets, it has been

found that a pure white pattern on a deep black grounding, and of about the size above-mentioned, is quite satisfactory. The pattern (Fig. 53) comprises: a square, a pentagon, a hexagon, a 14-point star, and an 18-point star, all concentric with one another, and with the disc and shaft, with a radial bar inside the square for enabling the apparent revolutions between intermediate standstill speeds to be counted easily.

If $P =$ number of positions of symmetry per revolution of any individual pattern on the target.

$n =$ revs. per min. of the target.

$N =$ any whole number or reciprocal of a whole number.

and $p =$ number of peeps per min. given by the fork's rate of vibration.

Then the particular pattern will stand still when viewed through the slits if

$$N = \frac{P.n}{p}.$$

For example, the square or 4-pointed star has $P = 4$ positions of symmetry per revolution.

If a given fork provides $p = 3600$ peeps per min.,

Then the square will appear to stand still for actual speeds of—

$n = 450$ revs. per min. at which $N = \dfrac{Pn}{p} = \dfrac{4 \times 450}{3600} = \frac{1}{2}$ the reciprocal of a whole number,

and $n = 1800$ revs. per min. at which $N = \dfrac{Pn}{p} = \dfrac{4 \times 1800}{3600} = 2$ a whole number.

In the first case the square will move through half of a position of symmetry, and in the second case through two positions of symmetry between successive peeps.

The useful possibilities obtainable with the above target may be conveniently seen in Table VI.

The higher the actual speed of rotation, the greater must be the rate of vibration or frequency of the slits and number of peeps per min., and the narrower must the slits be, otherwise the number of positions of symmetry P must be smaller to give an increase of pitch.

The accuracy of this method of measuring speed is very high, and of the order of about 1 part in 10,000.

TABLE VI.—Actual speeds in revs. per. min. at which each pattern of the Target (Fig. 53) will appear to stand still when viewed through the slits of a Fork giving 1800 peeps per minute.

Square.		Pentagon.		Hexagon.		14-Pointed Star.	18-Pointed Star.
$P = 4$ or $\frac{1}{4}$.	$P = 8$ or $\frac{1}{8}$.	$P = 5$ or $\frac{1}{5}$.	$P = 10$ or $\frac{1}{10}$.	$P = 6$ or $\frac{1}{6}$.	$P = 12$ or $\frac{1}{12}$.	$P = 14$.	$P = 18$.
Image stationary at every 450 r.p.m. = $\frac{1}{4}$ Synchr. Speed.	Image appearing doubled, but less clear at Intermediate Speeds or every 225 r.p.m. = $\frac{1}{8}$ Synchr. Speed.	Image stationary at every 360 r.p.m. = $\frac{1}{5}$ Synchr. Speed.	Image appearing doubled, but less clear at Intermediate Speed or every 180 r.p.m. = $\frac{1}{10}$ Synchr. Speed.	Image stationary at every 300 r.p.m. = $\frac{1}{6}$ Synchr. Speed.	Image appearing doubled, but less clear at Intermediate Speed or every 150 r.p.m. = $\frac{1}{12}$ Synchr. Speed.	Image stationary at every 128·6 r.p.m. = $\frac{1}{14}$ Synchr. Speed.	Image stationary at every 100 r.p.m. = $\frac{1}{18}$ Synchr. Speed.
—	225	—	180	—	150	128·6	100
450	450	360	360	300	300	257·2	200
—	675	—	540	—	450	385·8	300
900	900	720	720	600	600	514·4	400
—	1125	—	900	—	—	643·0	500
1350	1350	1080	1080	—	750	771·6	600
—	1575	—	1260	900	900	900·2	700
1800	1800	1440	1440	—	1050	1028·8	800
—	2025	—	1620	1200	1200	1157·4	900
2250	2250	1800	1800	—	1350	1286·0	1000
—	2475	—	1980	1500	1500	1414·6	1100
2700	2700	2160	2160	—	1650	1543·2	1200
—	2925	—	2340	1800	1800	1671·8	1300
3150	3150	2520	2520	—	1950	1800·4	1400
—	3375	—	2700	2100	2100	1929·0	1500
3600	3600	2880	2880	—	2250	2057·6	1600
—	3825	—	3060	2400	2400	2186·2	1700
4050	4050	3240	3240	—	2550	2314·8	1800
—	4275	—	3420	2700	2700	2443·4	1900
4500	4500	3600	3600	—	2850	2572	2000
—	—	—	—	3000	3000	—	—

(53) Relation between Speed and E.M.F. in "Separately Excited," "Shunt," and "Compound Wound" Direct Current Dynamos.

Introduction.—The following tests are arranged with the object of investigating the way in which the terminal E.M.F. of the various types of dynamos varies with the speed when the machine is delivering *no external current*. The mode of procedure is exactly the same for each type of machine, and consequently this will be given in one concrete instance only, and merely referred to afterwards for the others. In all cases the terminals T_1 T_2 of the machine are the ones to which the external circuit would be directly connected. Prior to starting see that all lubricating cups in use contain oil, and feed properly

but very slowly; also that the commutators are smooth and clean, and the brushes properly trimmed.

N.B.—The performance of a "magneto" machine would be exactly similar to that of (A) below, and the "series" machine approximately the same also, providing the current through the series machine be kept constant by varying the circuit resistance as the speed varies.

Apparatus.—Dynamo D to be tested; tachometer; voltmeter (V) capable of reading sufficiently high.

(A) Separately Excited Dynamo.

Observations.—(1) Connect up V to T_1 T_2, and the exciting coils to some outside source of E.M.F. through an ammeter (a), and rheostat (R), then start D. Adjust (R) so as to obtain a current which will give $\frac{1}{2}$ max. excitation (to be kept constant).

(2) Adjust the speed so as to obtain the lowest readable scale reading of V. Note this and the speed.

(3) Raise the speed so as to get about ten different values of V, rising by about = increments to the max., and note the speed at each.

(4) Repeat 3 for a similar descending set of readings.

(5) Repeat 3 and 4 for such a current through the field coils as will give max. excitation, and tabulate your results in the following general form.

Type of Dynamo tested.	Speed in Revs. per min.	Separate Exciting Currents (if any) a.	Terminal E.M.F.s.	
			Ascending V.	Descending V.

(B) Shunt Dynamo.

Observations.—(1) Connect up V to T_1 T_2, and start the dynamo.

(2) Repeat observation (2—4 A), and tabulate in the form shown above.

(C) Compound Wound Dynamo (Long Shunt).

Observations.—Repeat those for B above.

Plot curves for the tests on each type of machine in A, B and C above, having E.M.F. as ordinates and speed as abscissæ.

Inferences.—Compare the above curves and results, and state clearly all that you can infer from them.

Characteristics of Dynamo Machines.

Introductory Remarks.—There are two great classes of electrical generators for converting mechanical into electrical energy—namely, (1) those which supply *continuous* current, *i.e.* current which flows in one direction only round the circuit, and which are otherwise styled *direct current dynamos;* (2) those which supply *alternating* current, *i.e.* current that reverses its direction throughout the whole circuit many times a second, and which are usually styled *alternators.* As therefore the supply of electrical energy to any appreciable extent invariably assumes one or other of the above forms of current, a study of the behaviour of the machines that supply it becomes of paramount necessity. Restricting our considerations first of all to the former of the above-mentioned classes, it may be remarked that it is composed of a great variety of forms, the performance of which depending practically entirely on the method employed in winding their field magnets, in other words, as to whether the whole, a fraction, or a combination of this whole and the fraction of the whole current generated by the machine is utilized in magnetizing their field magnets. Direct current dynamos in general may consequently be subdivided into the following five distinctive types according to the winding of their field magnet coils—

(*a*) Magneto machine.
(*b*) Separately excited machine.
(*c*) Series machine.
(*d*) Shunt machine.
(*e*) Compound machine.

All these types are used in practice, especially the last three, which form by far the greater proportion of direct current machines in use throughout the world. Only by a minute study of the performance and action of each type can it be seen which of them is the best suited for any particular purpose.

The current that any particular dynamo will send through a given external circuit connected to its terminals will obey Ohm's Law, and will depend on the E.M.F. of the machine, as well as on this external resistance.

If therefore the machine be run at a constant speed, and the

circuit resistance R varied by suitable steps (say), both the terminal P.D. of the machine (V) and the current A will vary. Assuming that V and A are noted simultaneously for each alteration of R we can plot a curve having values of V, measured along the ordinates, and A along the abscissæ, these axes being rectangular. Such a curve is commonly called the "*external Characteristic*" of the dynamo, and by means of it many valuable and practical details can at once be deduced.

In fact, the function of a Characteristic in relation to a dynamo is extremely analogous to that of an "Indicator diagram" with an engine. In the former, not merely can the qualities and performance when working be seen, but also the H.P. at which it works or could most economically work at, and many defects in the design—such as the sufficiency of the field magnet field, the degree of saturation of the magnets, the demagnetizing action of the armature on the field, etc., etc. The Characteristic of a dynamo is therefore much more important to the electrical engineer than the author ventures to think is generally supposed.

(54) Determination of the "External Characteristic" of a Magneto Dynamo.

Introduction.—The present type of machine under consideration has a somewhat extensive field of use, two very important applications being for blasting purposes and use with the ohmmeter, and indeed in all kinds of work in which a portable E.M.F. and small current is desired.

Another application in the past on a heavier scale was in the production of lighthouse search-lights, in which kind of work heavy currents are required at a comparatively low E.M.F. Considering the machine more in detail, when the armature is delivering *no current* to the external circuit, the terminal P.D. (V) = the total E.M.F. (E) of the dynamo. When, however, a current A flows, then V is less than E by an amount depending on the armature resistance r_a; for, by Ohm's Law,

we have $E = A(R + r_a)$

where $R =$ the resistance of the external circuit; but since it is solely the P.D. (V) which drives the current through the external resistance,

$$\therefore V = AR,$$

and consequently $E = V + Ar_a$.

Thus we see that the total E.M.F. of the dynamo is = the P.D. at the terminals + that required to send the current A through the internal resistance r_a of the armature. The permanent magnetism of the steel magnets is approximately constant.

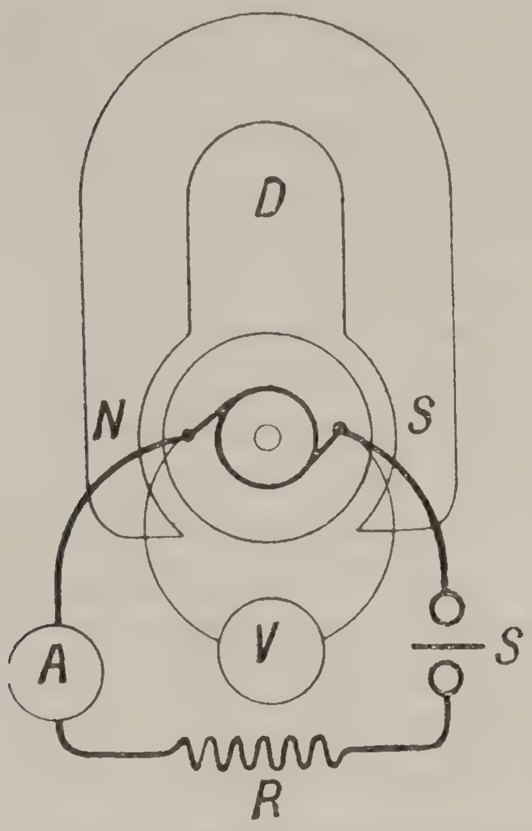


FIG. 54.

Apparatus.—Magneto dynamo D to be tested; voltmeter V; ammeter A; switch S; rheostat R (p. 606); tachometer.

Observations.—(1) Connect up as in Fig. 54, and adjust the pointers of A and V to zero, if necessary. See that all lubricating cups *in use* feed slowly and properly, and that the *commutator* is *smooth* and *clean*, and the *brushes properly trimmed.*

(2) Start D up to its normal speed, and when this is constant, note the reading of V.

(3) R being as large as possible, close S, and take a series of about ten different values of current A, rising by about equal increments from the smallest to the maximum permissible. Note the voltage V and current A for each, the *speed* being *constant* at the above value.

(4) Repeat 2 and 3 for a similar descending set of readings of V and A at the same speed.

(5) Repeat 2–4 for speeds 20% above and 50% below normal respectively.

(6) Measure the armature resistance (r_a) (while warm) by the Wheatstone Bridge, or if it is too low, by the "Potential Difference" method (p. 84), or by the ammeter and voltmeter method (p. 86), and tabulate your results as follows—

NAME . . . DATE . . .

Magneto Dynamo tested: No. . . . Type . . . Maker . . .
Normal Voltage =. . . Amps. =. . . Speed =. . . r_a =. . . Ohms. . .

Speed Revs. per min.	Ascending.		Descending.	
	Terminal Volts (V).	Amps. (A).	Terminal Volts (V).	Amps. (A).

(7) Plot the *external Characteristic curves* of the machine for both the ascending and descending readings at each speed on the same curve sheet, having values of V as ordinates and A as abscissæ.

(8) Deduce from these the *total Characteristics* of the machine by the graphical construction given below, drawing them on the same curve sheet (*vide* p. 3, obs. 8).

(9) Plot the horse-power curves (see below) to the same axes.

Inferences.—State clearly all that you can infer from your experimental results.

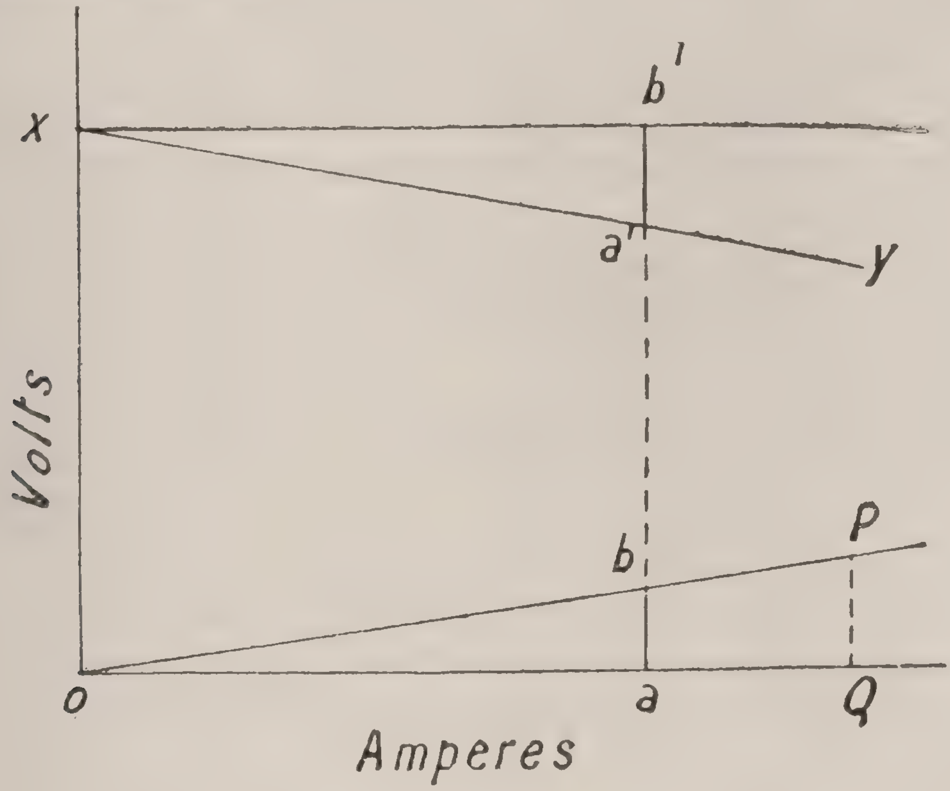


Fig. 55.

Graphical Deduction of the Total Characteristic of a Magneto Dynamo from its External Characteristic.

Referring to Fig. 55: take two co-ordinate axes having their origin in the point O. Let xy be the *external Characteristic.* Take any point Q in the abscissæ representing some definite current OQ to the scale chosen, and set off on the ordinates $QP = OQ \times r_a$ volts.

Join OP, which therefore represents the fall of Potential through the armature due to the currents flowing in it,

$$\text{or tan. } POQ = r_a.$$

Now take any point a' in xy, and through it draw an ordinate cutting OP and OQ in b and a respectively. Set off $a'b' = ab$, and b' will then be a point on the *total Characteristic.*

Repeat this construction for eight or ten points, such as a' along xy, and finally draw the total Characteristic xb' required.

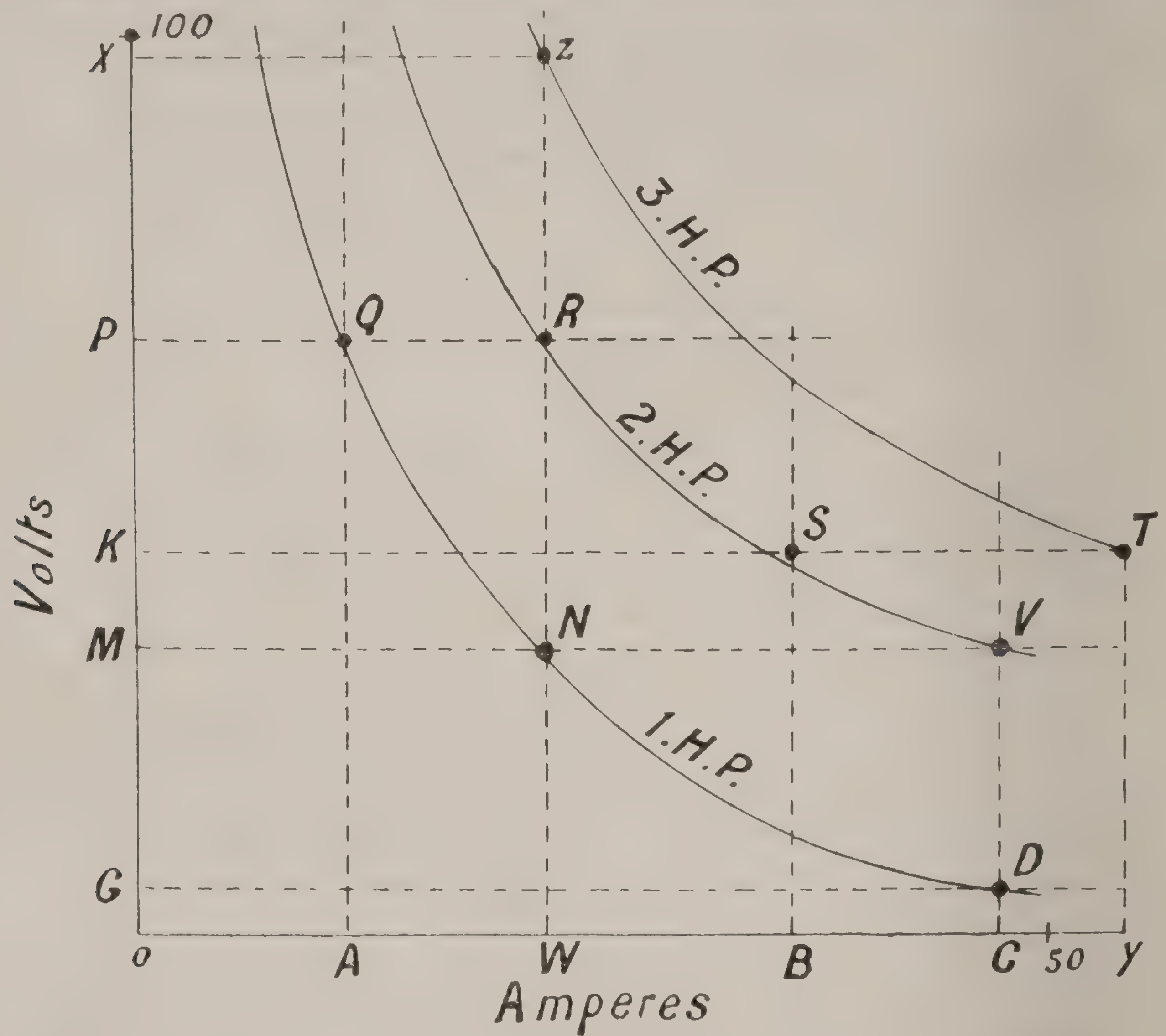


Fig. 56.

The performances of the machine, deducible from the above diagram, are roughly these—

If the curves xy and xb' are too far apart, it means that the armature resistance is excessive and is causing a large drop of terminal voltage.

Again, if the lower part of xy droops considerably, it shows that the armature current is causing a considerable demagnetizing action for the larger currents when the *angle of lead* of the brushes is greater. This downward drooping occurs more markedly as the field magnets become weaker.

Horse-Power Curves.

Since the electrical power W in Watts developed by any dynamo giving a current A amps. at a P.D. of V volts is—

$$W = AV \text{ (Watts)}$$

and since 1 E.H.P. = 746 (Watts),

$$\therefore \text{ H.P developed} = \frac{AV}{746}.$$

Now manifestly, as the Characteristics of a dynamo are plotted to definite scales of volts and amperes, the power developed at any point on such a curve will be equal to the product of the co-ordinates of that point, and can therefore at once be seen.

If, however, curves of equal H.P. are, at the onset, drawn to the axes chosen, the above calculations will be avoided, for if the 2 H.P. curve cuts the Characteristic at a given point, then the power developed by the machine corresponding to the V and A of that point will be 2 H.P.

To determine these H.P. curves : Find several points such that the products of their co-ordinates = 746 Watts = 1 H.P. Thus, in Fig. 56 we have—

$$PQ \times QA = MN \times NW = GD \times DC = 746 \text{ Watts } = 1 \text{ H.P.}$$

similarly $PR \times RW = KS \times SB = MV \times VC = 1492 \text{ Watts} = 2 \text{ H.P.}$

and $XZ \times ZW = KT \times TY = 2238 \text{ Watts} = 3 \text{ H.P.}$

Then all points on the curve QND represent 1 H.P.

and ,, ,, RSV ,, 2 H.P., and so on.

Intermediate values of H.P., such as 1·5 H.P., can be got by halving the distances between curves QND and RSV, etc.

These H.P. curves are simply rectangular hyperbolæ if equal scales are used on the axes, but are distorted hyperbolæ if the scales are unequal. Instead of H.P. they can equally well be drawn to represent kilowatts.

(55) Determination of the External Characteristic of a Separately Excited Dynamo.

Introduction.—The type of machine under consideration as representing a *direct current* dynamo has little application in practice, but as representing an alternating current

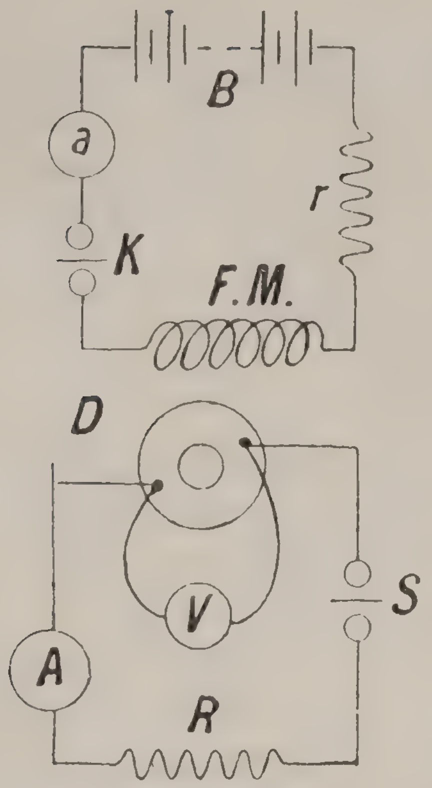


Fig. 57.

machine it has an extremely wide range of use. Such, however, we do not propose to discuss at this stage. Its utility in the first-named use lies in the fact that not merely can a far more powerful field be produced than is possible with any permanent steel magnets, which is of great value and, in fact, all-important in the generation of large E.M.F.s, but such a machine can be used with perfect safety in charging secondary cells, electroplating, etc., without the least fear of its polarity being changed should the back E.M.F. of the cells exceed that of the machine. A further advantage may be mentioned, viz. that a wide variation of E.M.F. can be obtained at any speed by suitably varying the exciting current (a) by the rheostat (r). The main disadvantage lies in having to provide an independent source of E.M.F. (B) for exciting the field magnets ($F.M.$).

The total E.M.F. is, as in the case of the magneto dynamo, given by the relation $E = V + Ar_a$, where A = the current flowing in the circuit at a terminal P.D. = V and r_a = the armature resistance.

The *external* and *total Characteristics* of the machine are found in *exactly* the same way as for the magneto dynamo, but at constant excitation as well as speed, and Fig. 57 shows the requisite apparatus symbolically and the connections of the same.

(56) Determination of the Internal Characteristic or Curve of Magnetization of a Separately Excited Dynamo.

Introduction.—In any dynamo, the P.D. across the brushes, *i.e.* armature, when no current flows in the external circuit, gives a measure, and is proportional to the magnetization of the field

magnets. If therefore the *field magnets* of any dynamo are *separately excited* by different currents from an independent source of E.M.F., the corresponding voltages across the brushes, *for constant speed*, will be approximately proportional to the inductions produced, if the machine is giving no current. The curve relating to exciting currents with E.M.F.s is termed the *internal Characteristic*, and it shows the region of saturation of the magnets, and also whether the eddy currents in the armature are producing any perceptible demagnetization of the field, and therefore shows the efficacy of the lamination of the armature core.

Apparatus.—Separately excited dynamo D to be tested; voltmeter V; exciting circuit comprising source of E.M.F. (B); ammeter a; switch K; rheostat (r) (p. 599); and field coils F.M.

Observations.—(1) Connect up as in Fig. 57, *omitting* the main current circuit there shown connected to the armature. Adjust the instruments to zero.

(2) Start D up to normal speed and with (r) large, close S, and take about ten different values of exciting current, rising by about equal increments from 0 to the maximum, and note the corresponding voltage V at each, the speed being constant throughout.

(3) Repeat 2 for a similar descending set of readings, and tabulate your results in a convenient manner.

(4) Plot the *internal Characteristic* having values of terminal voltage V ($\propto$ to field flux) as ordinates, and exciting currents (a) ($\propto$ to magnetizing force) as abscissæ.

Inferences.—State clearly the meaning of the curve, and any inferences which can be drawn from the experimental results.

(57) Relation between External Current and Exciting Current, at Constant Voltage, in a Separately Excited Dynamo, at Constant Speed.

Introduction.—If both the external and exciting currents are varied together in such a way that for a constant speed the terminal voltage is constant, then the curve showing the variation of one current with the other will show the increase of excitation that would be necessary to give constant voltage for varying external resistance at constant speed.

Apparatus.—Precisely that required for the preceding test, and in addition the main circuit comprising ammeter *A*; rheostat *R* (p. 606); and switch *S*.

Observations.—(1) Connect up as in Fig. 57, and adjust the instruments to zero. See that all lubricating cups in use feed properly.

(2) Start *D* up to the normal speed, and with a convenient excitation note the voltmeter reading *V*, which in future is to be kept constant as well as the speed.

(3) With *R* large close *S*, and take about ten different load currents *A*, rising by about equal increments from 0 to the maximum allowable, adjusting the exciting current to keep the volts constant. Note each pair of corresponding currents.

(4) Repeat 3 for a similar descending set of readings.

(5) Repeat 3 and 4 for a different speed but the same voltage by suitably altering the initial excitation, and tabulate in a convenient form.

(6) Plot curves for each speed, for both ascending and descending readings having values of exciting current as ordinates and main currents as abscissæ.

Inferences.—State clearly the practical value of the above tests, and explain the form of curve so obtained.

(58) Determination of the External Characteristic of a Series Wound Dynamo.

Introduction.—In this type of machine the whole current developed at any time is employed for magnetizing the field magnets, but these are only wound with a comparatively small number of turns of thick wire to carry this main current.

The series machine is an extremely important one in practice, and will be found throughout the world in different parts. It is essentially employed as a *constant* current dynamo, usually developing a small current at a high E.M.F. The principal application is in the lighting of arc lamps in series, and as an electro-motor in the various branches of electric traction work. Regarding it first of all from a theoretical standpoint, there is only one circuit in operation, and hence but one current, in the case of a series

dynamo running an external circuit. Thus when this is broken there is no E.M.F., and the maximum E.M.F. is only obtained when close on the full load current is flowing.

The series dynamo possesses many peculiarities, the nature and effect of which, on practical working, it is all-important to be cognizant of. This investigation is best solved by obtaining the Characteristic of the machine and studying it.

If r_s and r_a be the resistance of the series coils and armature respectively, and R that of the external circuit, then when the voltage across the terminals is V, the total E.M.F. generated

$$E = A\,(R + r_s + r_a)$$

where A = the current in the circuit; but $V = AR$; hence

$$E = V + A\,(r_s + r_a).$$

Apparatus.—Series dynamo D to be tested; voltmeter V; ammeter A; switch S; low resistance (variable) rheostat R (p. 606); tachometer. FM represents the field magnet (series) coils; TT represent the terminals of the dynamo.

Observations.—(1) Connect up as in Fig. 58, and adjust the instruments to zero, if necessary. See that all lubricating cups in use feed slowly and properly, and that the *brushes* are properly *trimmed* and the *commutator smooth* and *clean.*

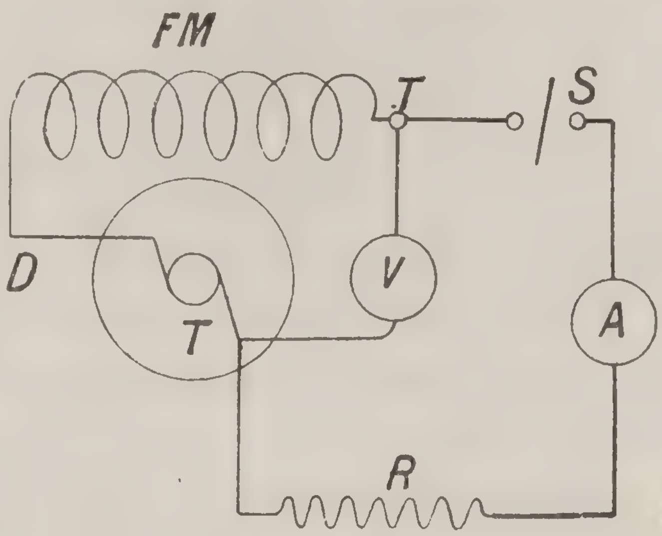


Fig. 58.

(2) Start D up to its normal speed, and with S open note the reading on V (if any).

(3) With R large, close S, and take about ten values of current A (*at constant speed*), rising by about equal increments from 0 to the maximum permissible, and note the corresponding voltage V at each.

Note.—In starting, care must be taken not to decrease R too quickly, as the machine might suddenly build up on its residual magnetism, and a large rush of current ensue.

(4) Repeat 2 and 3 for a similar descending set of readings of V and A.

(5) Repeat 2—4 for speeds 20 % above and 50 % below normal respectively.

(6) Measure, by means of the Wheatstone Bridge, the resistance r_s of the series coils, r_a of the armature, and $r_s + r_a$ of the whole machine while warm, supposing it to be not too small, and tabulate your results as follows—

NAME . . . DATE . . .

Series Dynamo tested : No. . . . Type . . . Maker . . $(r_a + r_s) = . .$
Normal Voltage = . . . Amps. = . . . Speed = . . . Resistance $r_a = . . . r_s = . .$

Speed Revs. per min.	Ascending.		Descending.		Resistance of External Circuit $= \frac{\text{mean } V}{\text{mean } A}$ ohms.
	Terminal Volts (V).	Amps. A.	Terminal Volts (V).	Amps. A.	

(7) Plot the ascending and descending *external Characteristics* for each speed to the same pair of axes, having voltage (V) as ordinates, and amps. A as abscissæ in each case.

(8) From the above curves deduce, graphically, the *total Characteristics* on the same curve sheet in the manner described below (*vide* notation, p. 3).

(9) Plot the H.P. curves on the same sheet.

(10) Determine the *critical resistance* for this machine at each speed and also the *critical current.*

(11) Determine the internal Characteristic of the machine, and plot the results, having exciting currents as abscissæ and armature E.M.F. as ordinates, to the same axes as the above curves.

(12) Plot the external resistance Characteristic having values of V as ordinates and resistance of the external circuit $\frac{V}{A}$ in ohms as abscissæ.

Inferences.—Explain the meaning of the curves very carefully. How can the effect of alteration of speed for one or more points on a Characteristic be corrected for? Why does the total Characteristic not droop so much as the external at the higher currents?

Graphical Determination of the Total Characteristic from the External One in a Series Wound Dynamo.

Take two co-ordinate axes meeting in the point O of which the ordinates represent voltage and the abscissæ current in amperes. Let x be the external Characteristic curve of the series dynamo.

Take *any* point Q on the abscissa representing *any* current OQ and set off on the ordinate a line QP to represent $OQ \times (r_s + r_a)$ volts. Join OP, which therefore gives the rate of fall of potential down the armature and series coils combined, *i.e.* down the whole machine, or we have tan. $POQ = (r_a + r_s)$. Now take *any* point a' in the curve x and through it draw an ordinate $a'ba$ cutting OP and OQ in b and a respectively; set off $a'b'$ as shown $= ab$, whence b' is a point on the total Characteristic. Repeat this operation for some ten different points along

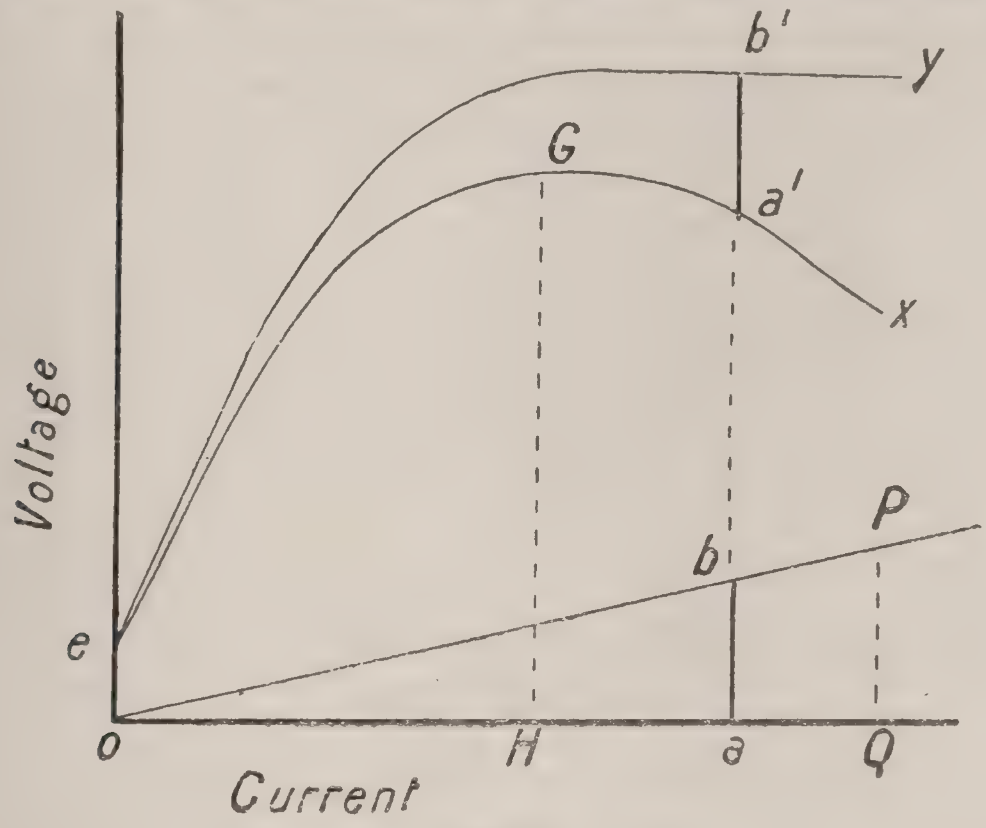


FIG. 59.

x and finally obtain the total Characteristic y of the machine at the particular speed in question, Fig. 59.

If the curves start at some point e other than the origin O, then Oe is the E.M.F. at that speed due to residual magnetism in the field magnets when the external circuit is open, as in observation 2 above; Oe will be greater the greater the hardness and retentivity of the iron in the magnets.

If the curves x and y are much separated, it shows that the resistance of the machine, *i. e.* tan. POQ, is too great, and therefore that it cannot be very efficient. Again, the summit G of the curve x shows for what output ($GH \times HO$ Watts) and current OH the armature is magnetically saturated, and the drooping of the part Gx gives an idea as to the demagnetizing action of the

armature on the field. It is greater in dynamos in which the magnets are relatively less powerful than the armature, and is greatest in those machines in which the armature core is more nearly saturated than those of the field magnets.

Graphical Determination of Critical Resistance at a Given Speed for a Series Wound Dynamo.

Let the curve OCx be, say, the total characteristic of the machine for a given speed. Then the total circuit resistance

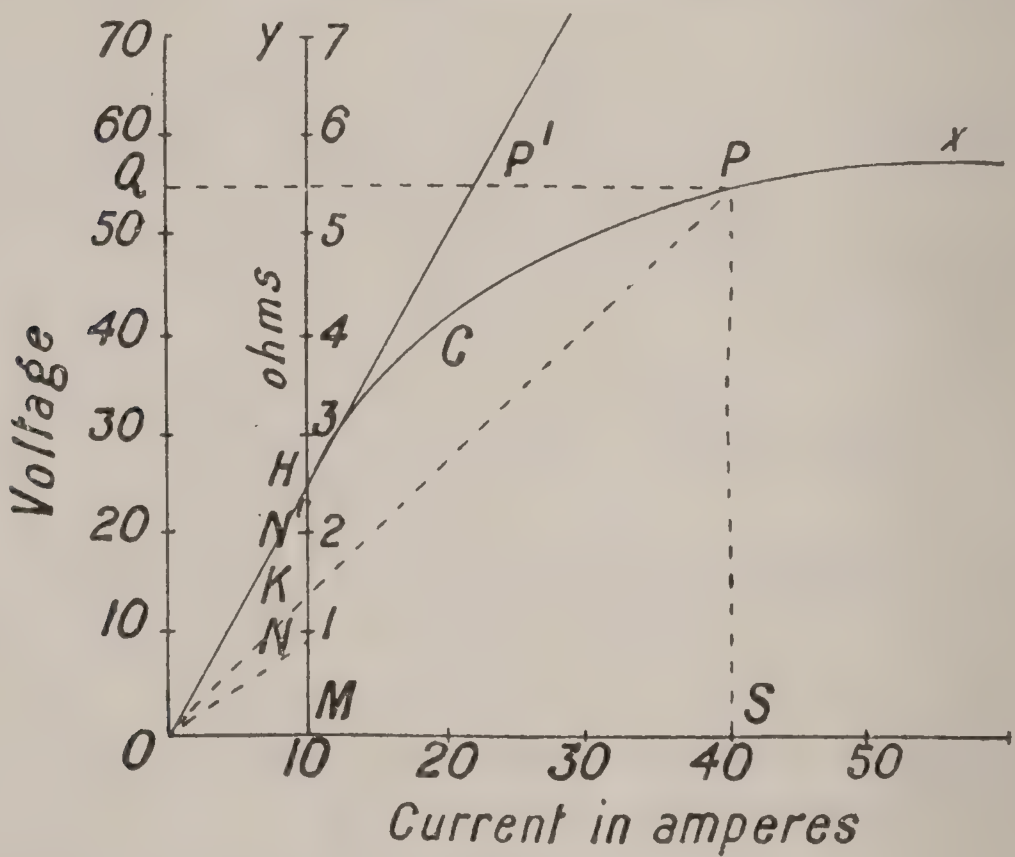


FIG. 60.

$(R+r_a+r_s)$ corresponding to any point P can at once be found if the *resistance line* y is known, and which is found as follows—

Let N = point of intersection of the two axes 10, Fig. 60.

Then joining ON we have tan. $NOM = \frac{NM}{OM} = \frac{V}{A} = R = \frac{10}{10} = 1.$

Hence N is a point representing 1 ohm.

Similarly N' being the point of intersection of the 20 volt axis and 10 ampere one, will represent $\frac{V}{A} = \frac{20}{10} = 2$ ohms, and so on.

Thus the whole *resistance line yM* is obtained.

Now to obtain the circuit resistance for the point P, draw its co-ordinates PQ and PS and join OP.

Then $$\tan. POS = \frac{PS}{OS} = \frac{V}{A} = \frac{55 \cdot 0}{40} = 1 \cdot 375 \text{ ohms},$$

but $MK = 1 \cdot 375$. Hence the resistance of the circuit $(R + r_a + r_s)$ is given by the point of intersection K of the *join* of P to O, and the resistance line My.

If then P approaches the origin O just so close that OP coincides with OP' and is practically a tangent to the curve (x), then its point of intersection with My gives the *critical circuit resistance*, above which the series dynamo will not excite.

This point is H and corresponds to 2·60 ohms on the resistance line. Thus for this speed the machine will not work if $(R + r_a + r_s)$ is greater than 2·6 ohms. For this instance the ordinates represent *total E.M.F.* of course, but if they were terminal volts and Ox the external Characteristic, then MH would = the critical *external* resistance R.

(59) Determination of the Internal Characteristic or Curve of Magnetization of a Series Wound Dynamo.

This is obtained in precisely the same manner as that in the case of the separately excited machine (p. 148), and consequently the mode of procedure will not be repeated. The curve should be plotted to the same axes as the ordinary Characteristics when the relative positions will form a measure of the armature reaction on the field.

(60) Determination of the External Characteristic of a Shunt Wound Dynamo.

Introduction.—In the type of machine under consideration only a fraction of the full load current generated is employed for the purpose of magnetizing the field magnets, the winding of the coils of which consists of a large number of turns of small

gauge wire, possessing a comparatively large resistance. This shunt winding is connected directly across the brushes of the dynamo. Shunt dynamos possess a region of approximately constant potential, the falling off in this latter being mainly due to armature resistance.

This type of generator is an extremely important one and its sphere of application very large.

The performance and qualities of such a machine can best be investigated by means of its Characteristics. Regarding the shunt dynamo from a theoretical standpoint, suppose that A_a, A_s and A are the currents flowing through the armature, shunt, and external circuit whose resistances are respectively r_a, r_s and R, then we have $A_a = A + A_s$, since the armature current splits up through shunt and external circuits.

Now since we are dealing with two parallel circuits and one common potential difference, we have, that the terminal voltage $V = AR = A_s r_s$ and the parallel resistance of shunt and external circuit is

$$\frac{Rr_s}{R + r_s}.$$

Hence
$$E = A_a\left(r_a + \frac{Rr_s}{R + r_s}\right) = A_a r_a + V,$$

or by substituting for A_a its value $A + A_s$ and reducing we get

$$E = Vr_a\left\{\frac{1}{R} + \frac{1}{r_a} + \frac{1}{r_s}\right\}.$$

Apparatus.—Shunt dynamo D to be tested, of which FM represents its field coils in series with a high resistance rheostat r (Fig. 269); voltmeter V; ammeter A; main variable rheostat R (p. 606); switch S; tachometer.

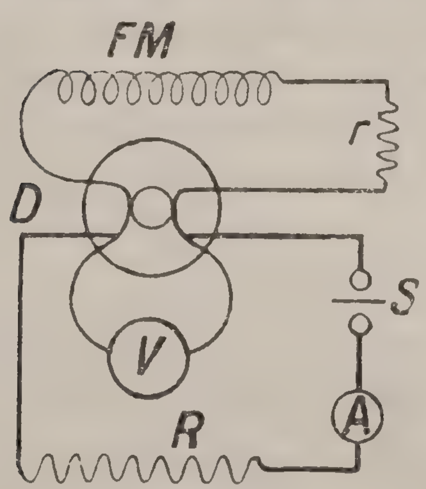


Fig. 61.

Observations.—(1) Connect up as in Fig. 61, and adjust the instruments to zero if necessary. See that all lubricating cups in use feed properly.

(2) Start D up to its normal speed, and with S open vary r so as to produce normal excitation, then note the value of V.

(3) With R large, close S and take about ten different values

of current rising by about equal increments from 0 to the maximum. Note the voltage V at each and the current A, the speed being kept constant at the above value.

(4) Repeat 2 and 3 for a similar descending set of readings.

(5) Repeat 2–4 for speeds 20% above and 50% below normal respectively.

(6) Measure, by means of the Wheatstone Bridge, the resistance r_s of the shunt coils (warm), and by the "Potential Difference" method, the resistance r_a of the armature if very low, and tabulate your results as follows—

NAME . . . DATE . . .

Shunt Dynamo tested: No. . . . Type . . . Maker . . .
Normal: volts . . . Amps. . . . Speed . . . Resistance r_a = . . . r_s= . . .

Speed Revs. per Minute.	Ascending.		Descending.		Resistance of External Circuit $= \frac{\text{mean } V}{\text{mean } A}$ ohms.
	Terminal Volts V.	Amps. A.	Terminal Volts (V.)	Amps. (A).	

(7) Plot the ascending and descending *external Characteristics* for each speed on the same curve sheet, having values of V as ordinates and A as abscissæ.

(8) From these deduce, by means of the following graphical construction, the *total Characteristics* on the same curve sheet.

(9) Plot the H.P. curves to the same axes also.

(10) Plot the external resistance Characteristic, having values of V as ordinates and resistance of the external circuit $\frac{V}{A}$ ohms as abscissæ.

(11) Determine the *critical resistance* of the machine for each speed and also the *critical current.*

Inferences.—State clearly all that you can deduce from your experimental results, explaining carefully the meaning of the shape of the curves.

Graphical Determination of the Total Characteristic from the External Curve in a Shunt Wound Dynamo.

Referring to Fig. 62: let x be the external Characteristic. Take *any* point Q in the axis of abscissæ representing any

current OQ, and from it set off $QR = OQ \times r_a$ volts; join OR, which therefore gives the fall of potential in the armature, we also have

$$\tan. ROQ = \frac{RQ}{OQ} = r_a.$$

Again, take *any* point T on the ordinates representing any voltage OT at the shunt terminals, and from T set off $TS = \frac{OT}{r_s}$

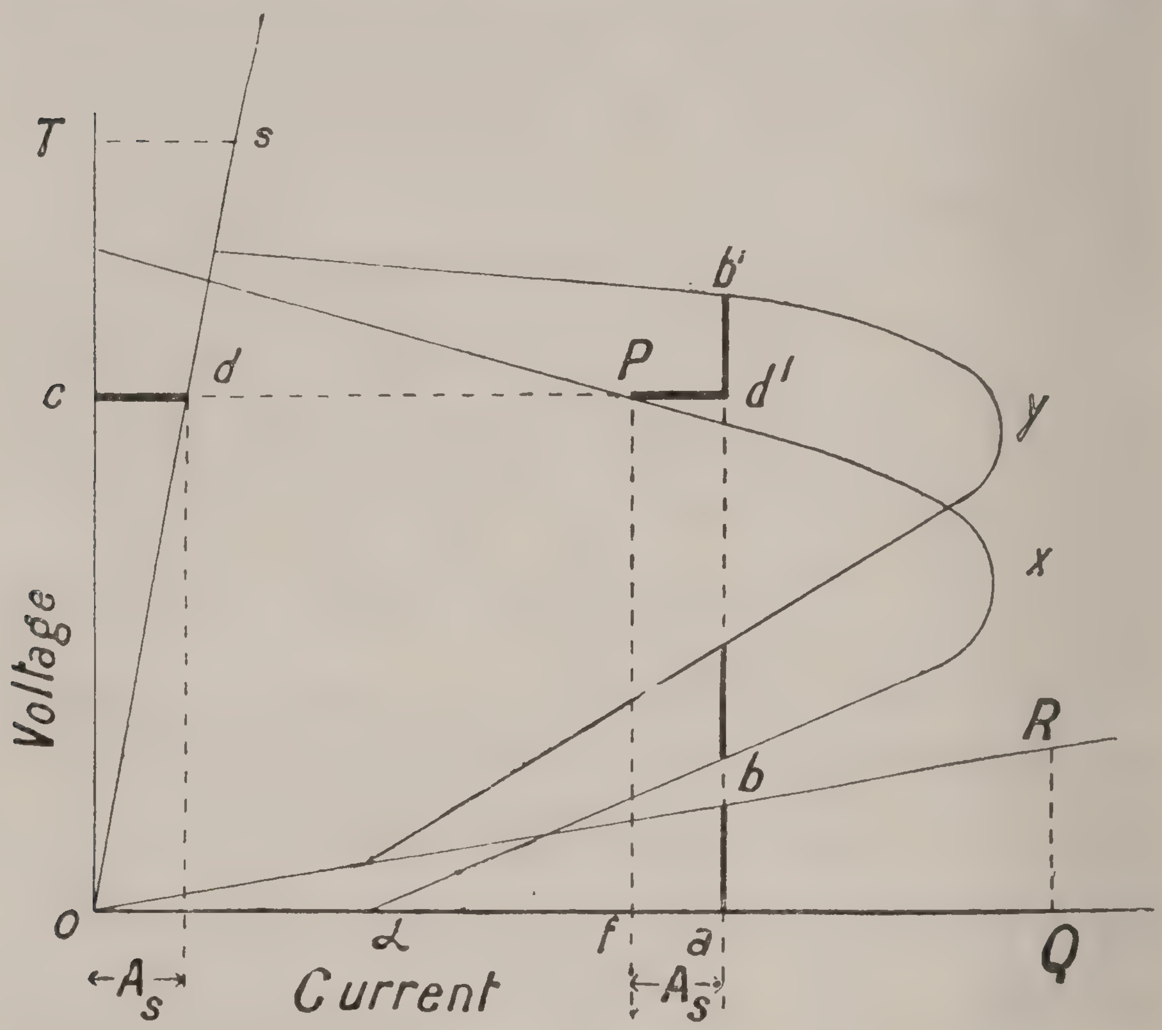


FIG. 62.

amperes; join OS, which therefore gives the current taken by the shunt coils at different voltages, and we have

$$\tan. SOQ = \frac{OT}{ST} = r_s$$

Now take any point P in x, and draw its co-ordinates Pc and Pf of which Pc cuts OS in d. Then cd = shunt current $= A_s$ and cP = external current A. If now cP is produced to d' so that $Pd' = cd$, then the total armature current $A_a = A + A_s = cP + Pd'$.

Now draw an ordinate through d' cutting OR and OQ in b and a respectively. Then ab = loss of volts in armature due to total

current $A_a = cd'$, consequently setting off $d'b' = ab$ we get b' a point in the total Characteristic which in a shunt dynamo gives the relation between total armature current $A_a = (A + A_s)$ and total E.M.F. $E = (V + A_a r_a)$. On repeating this operation for a number of points such as P on x we are finally able to draw the total Characteristic (y).

The working part of the curve is that at the top down to the sharp bend on the extreme right.

The critical external resistance $R = \tan. \alpha$ for this speed, and the machine will only work providing R is greater than $\tan. \alpha$.

The lower or straight portion of the curves is the unstable part. Thus it will be seen that a shunt dynamo would give a nearly constant voltage for a given speed and variable external resistance, if it were not for the armature resistance.

(61) Determination of the Internal Characteristic or Curve of Magnetization of a Shunt Wound Dynamo.

Introduction.—The internal Characteristic of a shunt dynamo is similar in shape to the total Characteristic of a series dynamo, and is a curve showing the relation between exciting or shunt current and the E.M.F. across the brushes for open external circuit. Since, however, the shunt current is so small that its passage through the armature would not cause any appreciable reaction or demagnetization, the shunt may be excited from the brushes and the armature allowed to give this small current, instead of disconnecting the shunt and separately exciting it from an independent source. When the dimensions of the magnetic circuit of the machine are known, scales of the axes can be marked in air-gap flux density as ordinates, and amp. turns per pole as abscissæ.

Apparatus.—Shunt dynamo; variable high resistance r capable of carrying the shunt current (p. 599); low reading ammeter (a); switch S; voltmeter V.

Observations.—(1) Connect the shunt coils of the dynamo in series with r, a and s across the brushes and V also across them, and adjust the instruments to zero.

(2) Take an ascending and also a descending set of readings

of V and a (at constant speed, say the normal) by varying r, and tabulate the results in a convenient manner.

(3) Plot the internal Characteristic having voltages V ($\propto$ to magnetic field flux) as ordinates and shunt currents (a) ($\propto$ to magnetizing force) as abscissæ in both ascending and descending readings.

Inferences.—Carefully point out the meaning of the curves so obtained.

(62) Determination of the External Characteristic of a Compound Wound Dynamo.

Introduction.—The type of machine under consideration is a combination of a series and shunt dynamo, so far as the field arrangements go, *i. e.* its field magnets are wound with both series and shunt coils.

The compound dynamo is a *self-regulating* machine for *constant terminal voltage* (at *constant speed*), independent of variations of external current. The principle of the self-regulating property is as follows:—As the external current increases in the case of a shunt dynamo the lost volts due to armature resistance and demagnetizing reaction of the armature in the field reduces the effective magnetism of the field magnets, but this increase of main current causes an increase in the field magnetism due to the series coils, which can be made to just counteract the diminution due to the lost volts, at *one definite constant speed*, thereby producing constant voltage at the terminals of the machine.

There are two possible methods of connecting the shunt coils to the machine, and which are shown in Fig. 63 symbolically, where I. represents what is called "*Long Shunt*," and II. that termed "*Short Shunt.*" $T_1 T_2$ are in each case the terminals of the dynamo, and as seen in the first case the shunt (*sh*) is across the extreme ends of armature and series coils (*se*), *i. e.* across $T_1 T_2$, while in the second case it is across the brushes alone. A careful study of the dynamo with each method of connection will show which is the most desirable arrangement to use in any particular case. This, together with the investigation of the performance, etc., while working, can best be obtained by means of the Characteristics of the dynamo.

Apparatus.—Compound wound dynamo D to be tested; switch S; voltmeter V; ammeter A; rheostat R (p. 606); tachometer.

Long Shunt.

Observations.—(1) Connect up as in Fig. 63 (I.), and adjust the instruments to zero if necessary. See that all lubricating cups feed slowly and properly.

(2) Start D up to its normal speed, and when excited note the reading on V.

(3) With R large close S, and take about ten different values of current A rising by about = increments from 0 to the

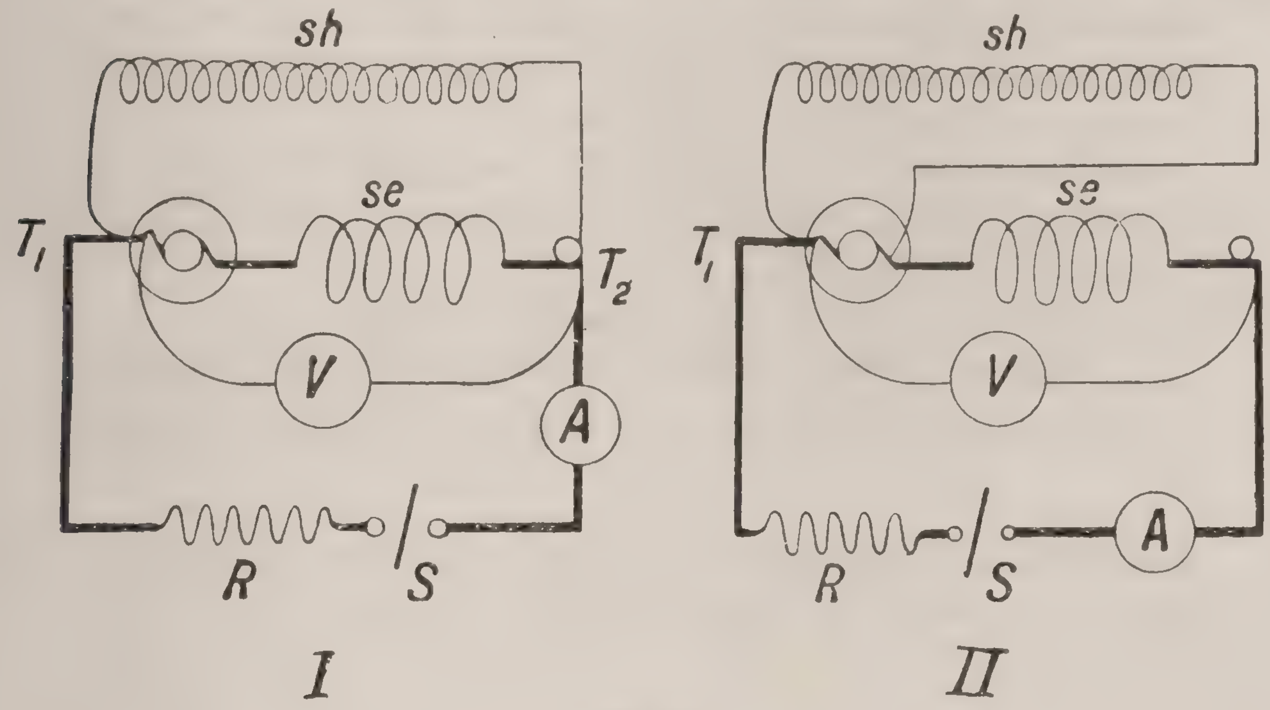


Fig. 63.

maximum allowable. Note the voltage V at each, the speed being constant at the above normal value.

(4) Repeat 2 and 3 for a similar descending set of observations.

(5) Repeat 2–4 for speeds 20% above and 50% under normal respectively for constant excitation in each case in the shunt coils.

(6) Measure by means of a Wheatstone Bridge the resistance r_{sh} of the shunt, and by the "Potential Difference" method (p. 84) that of the armature r_a and series coils r_{se}.

(7) Disconnect either coil in turn and repeat the above obs. 2–6 with the remaining coil, and so obtain the external Characteristic of the dynamo for each winding separately.

Note.—In doing this with *sh* connected alone, obtain the *same* initial voltage V as when running compound.

Tabulate all your results as follows—

NAME . . . DATE . . .

Compound Dynamo tested : No. . . . Type . . . Maker . .
Normal Volts = . . . Amps. = . . . Speed = . . . Resistances r_a = . . . r_{se} = . . . r_{sh} = . .

Winding used.	Speed Revs. per min.	Ascending.		Descending.		Resistance of External Circuit $= \frac{\text{mean } V}{\text{mean } A}$ ohms.
		Terminal Volts (V).	Amps. (A).	Terminal Volts (V).	Amps. (A).	

(8) Plot the *external Characteristics* for each speed, and both ascending and descending observations, having voltages V as ordinates and currents A as abscissæ.

(9) From these deduce graphically, as described below, the *total Characteristics*, drawing them on the same sheet of paper. (For notation, see p. 3.)

(10) Plot the H.P. curves to the same axes also.

(11) Plot the external resistance Characteristic having values of V as ordinates and resistance of the external circuit $\frac{V}{A}$ in ohms as abscissæ.

Inferences.—State carefully all that can be inferred from your results, and explain the meaning of the various curves.

Graphical Determination of the Total Characteristic from the External one, in a Compound Wound Dynamo.

LONG SHUNT.

Referring to Fig. 64, let x be the external Characteristic. Take *any* point Q in the abscissæ representing *any* external current OQ, and set off from Q on the ordinates $QR = OQ (r_a + r_{se})$, whence tan. $ROQ = (r_a + r_{se})$. Again, taking any point T representing any voltage OT at the shunt terminals, set off from T a line $TS = \frac{OT}{r_{sh}}$ whence tan. $SOQ = r_{sh}$.

Now take any point P in x and draw its co-ordinates PC and Pn, then the intercept Cd between OT and OS = shunt current at this voltage Pn. As in the case of the pure shunt machine make $Pd' = Cd$, and draw an ordinate $d'a$ through d' cutting OR and OQ in b and a.

Then making $d'b' = ab$, we get b' to be a point on the total Characteristic (y). Repeating this process for several points, such as P along (x), we finally get the curve y showing the relation between *total armature current* A_a, which $= A + A_s$, and total E.M.F. E, which $= V + A_a\,(r_a + r_{Se})$, where in Fig. 64 $On = A$, $na = A_s = cd$,

$$\therefore\ Oa = On + na = A + A_s$$

$$\therefore\ ab = Oa\,(r_a + r_{Se}) = A_a\,(r_a + r_{Se}).$$

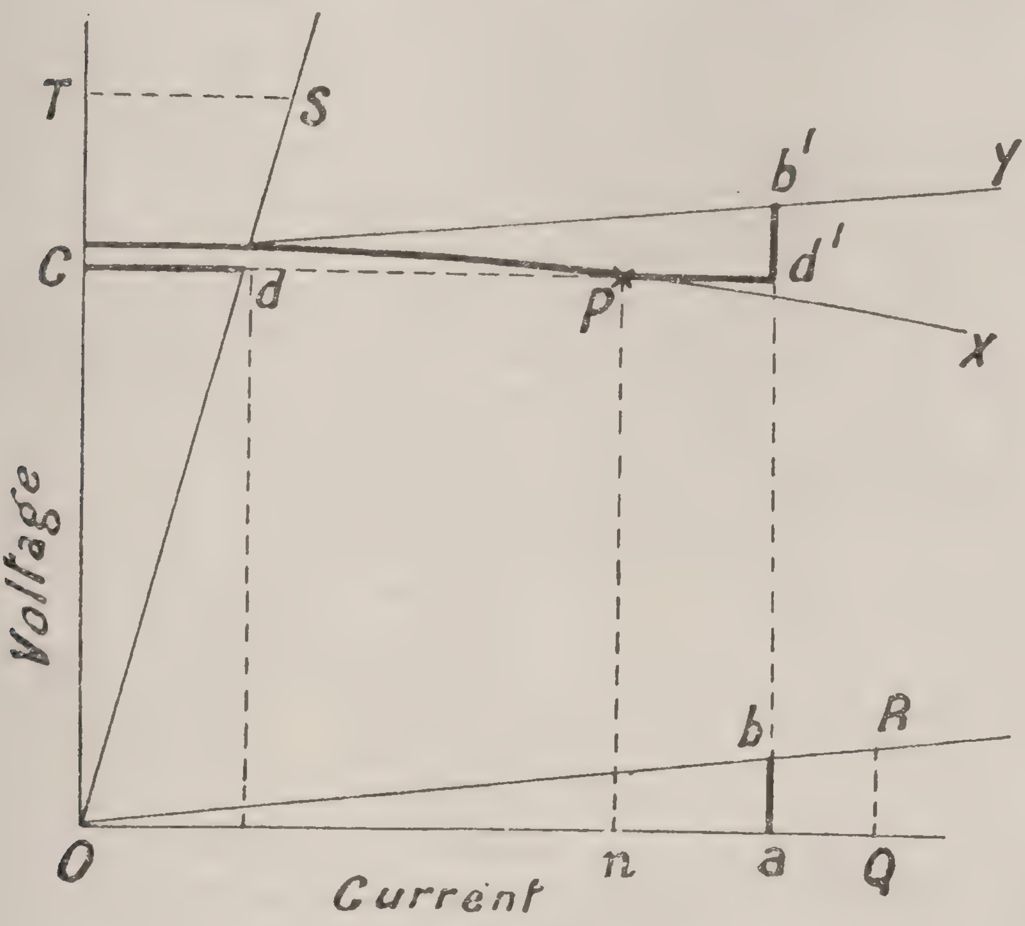


FIG. 64.

(63) Determination of the External Characteristic of a Compound Wound Dynamo.

SHORT SHUNT.

As the mode of procedure is precisely the same as in the case of the corresponding long shunt test, this latter will not be

repeated here, and must be referred to both for apparatus, which is the same in both cases, and for the diagram of connections which is shown in Fig. 63 (II.).

Graphical Determination of the Total Characteristic from the Internal one in a Compound Wound Dynamo.

SHORT SHUNT.

Referring to Fig. 65, let x be the external Characteristic. As

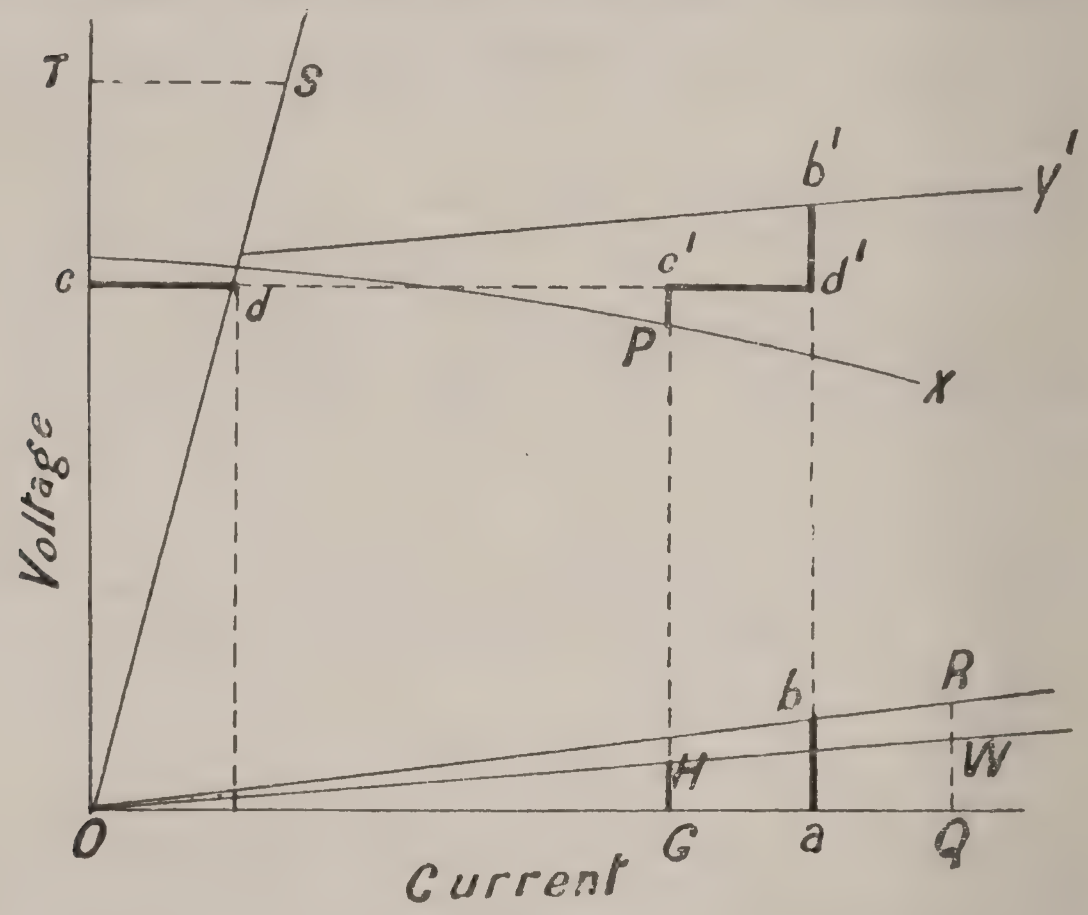


FIG. 65.

in the last case take any current OQ and set off $QW = OQ \times r_{Se}$ and $QR = OQ \times r_a$ and join OR, OW.

Obtain OS as before, and let P = any point on the curve (x). From P draw $Pc' = GH$, and from c' draw $c'd' = cd$; lastly, make $d'b' = ab$ and b' is a point on the total Characteristic y, thus y is easily obtained. By reference to Fig. 65 it will be seen that the total E.M.F. $E = V + A_a r_a + A r_{Se}$, while the total current in the armature $A_a = A + A_{Se}$, or in this Fig.—

$$ab' = PG + GH + ab = V + Ar_{Se} + A_a r_a$$

and $$Oa = OG + Ga \qquad = A + A_{Sh}.$$

(64) Determination of the Separate Field Magnet Windings for Truly Compounding a Dynamo.

Introduction.—This is a practical method of experimentally determining the number of amp.-turns of excitation to be supplied by both the series and the shunt coils in a compound machine without having to calculate them in the ordinary way from data pertaining to the magnetic circuit. The method forms an exceedingly instructive one for use in a laboratory, in illustrating theoretical principles, but it is essentially a *works* method, and has the advantages that the amp.-turns are determined under full load *working* conditions, *i.e.* when the armature is exerting its maximum demagnetizing influence on the field, and furthermore that since the whole machine is already constructed except for these coils, any errors in the lengths and sections of the various parts of the magnetic circuit and in their estimated permeabilities are nullified.

The conditions for automatic self-regulation by compound winding are, firstly, that with the external circuit *open*, i. e. no current through the series coils, the shunt winding must alone produce, at the given speed, the full specified voltage of the machine. Secondly, that on full load the series or winding must supply such an extra amount of magnetization, due to the full-load current in its coils, as will maintain the same specified voltage of the dynamo. There are obviously two modes of procedure differing slightly from one another and depending on circumstances at hand, namely, (1) to compound a machine *given* its carcase ready built up and the armature ready wound; (2) to compound a shunt machine by finding the necessary series turns to be added. Consider the former conditions first.

Apparatus. — In addition to the machine in question—a voltmeter V; switches S and S_1; ammeters A and a; rheostats R (p. 606) and r (p. 599); source of E.M.F. (B) for excitation, either battery or other dynamo, etc.

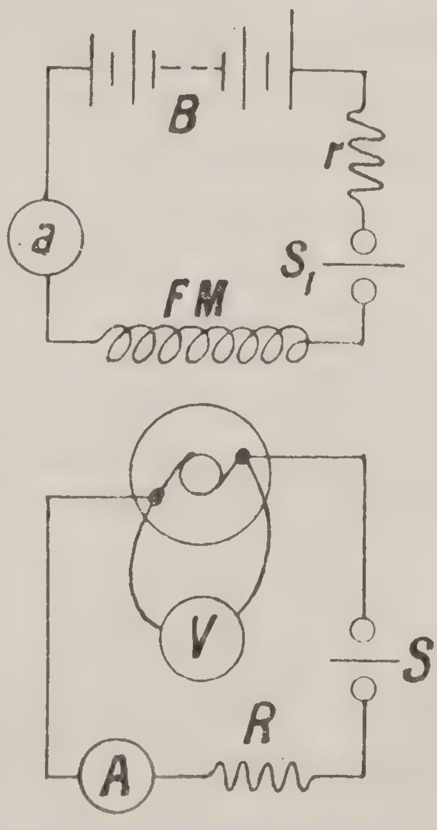


Fig. 66.

Determinations.—(1) Take a *suitable* field frame of the type of machine specified, *i.e.* one that previous experience shows to be of sufficient size for the work, also a *suitable* ready-wound armature, wound with a suitable number of turns of a gauge sufficient to take the specified full-load current of the machine at an orthodox current density in the coils.

(2) Wind the field magnet limbs uniformly with a few temporary turns F (known) of thick wire or lead, and connect up as in Fig. 66, adjusting the instruments to zero.

(3) Run the armature at the specified speed (n) revolutions per minute, and with S open close S_1, adjusting the exciting current (a) by means of r until V reads the specified voltage. Then aF = amp.-turns to be supplied by the shunt coils alone.

(4) Now close S, and obtain the full-load current A, of specification, at the same speed (n) above; again adjust r so that V again reads the same specified voltage as before. If a_1 now = the exciting current, then a_1F = amp.-turns to be supplied by (series + shunt) together; hence $(a_1 - a)F$ = amp.-turns to be supplied by series coils alone at that voltage V and speed (n) or number of *series turns* $= \frac{(a_1 - a)F}{A}$.

Such a gauge of wire is then chosen for the shunt coils, that with a shunt current of, say, 2% of A (the main one) the shunt resistance $= \frac{V}{\frac{2A}{100}} = 50\frac{V}{A}$ ohms and the amp.-turns $= aF$.

If we are dealing with case 2, in which it is desired to convert a shunt machine into a compound one, it should be remembered that such an alteration is only possible when the field magnets are not magnetically saturated.

The mode of procedure is then practically the same as before, and is as follows—

Disconnect the shunt and separately excite it, so as to reproduce the same terminal voltage (at full load) as the shunt (by itself) gave with the armature on open circuit.

If then the total number of shunt coil turns = T and the shunt current rose from the original value (a) to the new value (a_1) in order to keep up the same voltage at full load as on open circuit, then $(a_1 - a)T$ = amp.-turns to be supplied by series coils;

and if the full-load armature current is A, then number of series turns $= \frac{(a_1 - a)\,T}{A}$, the speed being constant all the time.

It will be obvious that the method serves to determine the windings necessary to *over compound*, the exciting current at full load being raised until the excess over the normal voltage = the drop of volts in the mains at that current.

(65) Determination of the Speed and E.M.F. at which a Dynamo truly Compounds.

Introduction.—The present test of course relates to a compound wound dynamo which is already built and finished. It has been previously mentioned that a given machine can only compound truly and exactly to give *constant voltage* for wide variations of external current, at one particular definite speed. This is owing to the different alterations which a definite variation of speed produces on the series and shunt Characteristics of the machine. Thus, if after a machine is built and completely finished it is found that the compounding is not quite correct at the speed used in the calculations, which could easily be the case, then the speed would form a means of final adjustment and the mode of procedure would be as follows—

(*a*) With the machine connected so as to self-excite in the usual way, place an external circuit consisting of an ammeter A, rheostat R, and switch S in series with the machine terminals and a voltmeter V across them.

(*b*) Run the dynamo at the speed employed in the design, and take first the open-circuit volts V and then that at some 4 or 5 loads between 0 and the maximum.

(*c*) If on plotting these observations the Characteristic thus formed droops, the series coils are too weak in their effect and the speed should be raised a little, another set of readings being taken.

In this way a speed will be found by trial such that the curve is a *horizontal* straight line or nearly straight between 0 and full load. The speed and volts at the terminals now are the values required for exact compounding.

(66) Variation of the E.M.F. of an Alternator with Speed at Constant Excitation.

Introduction.—The present test is an important one, as showing not merely the effect of alteration of speed on the E.M.F. developed by an alternator for constant exciting current, but also whether the demagnetizing effect of the armature on the field, due to eddy currents generated in it, is producing a perceptible effect, and if so the speed at which this effect begins to assert itself most forcibly. The eddy current loss varies as the square of the speed for constant magnetic field, so that the results of the test will give a measure of the adequacy of the lamination of the iron parts of the machine.

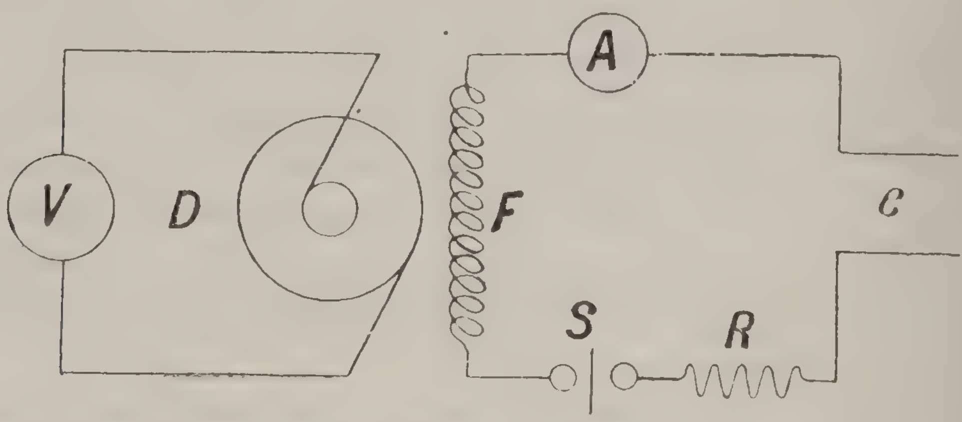


Fig. 67.

Apparatus.—Alternator D to be tested; alternating current voltmeter V; speed indicator and means of driving D at any suitable speed. For the exciting circuit, in addition to the field coils F of the alternator—a variable rheostat R (p. 599); switch S; ammeter A; exciting E.M.F. e consisting either of the requisite E.M.F. from a secondary battery or auxiliary direct current supply, etc.

N.B.—Since in this test the speed is variable, and consequently also the periodicity of the current, the voltmeter V should be either a "hot wire" or "electrostatic" instrument, these being the only two types of meters which are unaffected by the variation of periodicity.

Observation.—(1) Connect up as in Fig. 67, and adjust the pointers of the instruments to zero. See that all lubricating cups in use feed properly and slowly, and then start the alternator.

(2) Close *S*, and adjust the exciting current on *A* to the normal for the machine by altering *R*, and keep it constant; then adjust the speed so as to obtain the lowest readable voltage on *V*; note simultaneously the speed and voltage *V*.

(3) Repeat 2 for about ten different speeds, rising by about equal increments from the above value to the highest allowable (*a*) at the same constant normal excitation, (*b*) at a constant value 50% less.

(4) Repeat 2 and 3 for the same constant exciting currents with constant full-load armature current in each case respectively, and tabulate your results as follows—

NAME . . . DATE . . .

Alternator tested: No. . . . Type . . . Maker . . .
Normal Full-Load Volts = . . . Amperes = . . . Speed = . . .
Normal Exciting Current = . . .

Speed Revs. per min.	Voltage (terminal) *V*	Exciting Current *A* Amps.

(5) Plot curves having E.M.F. (*i.e.* terminal voltage) as ordinates and speed as abscissæ for each excitation both on no-load and full-load to the same pair of axes.

Inferences.—Carefully state all that can be inferred from the results of the above test, and point out their bearing on the design of alternators.

(67) Variation of the E.M.F. of an Alternator with Excitation at Constant Speed (Magnetization or Open-Circuit Characteristic).

Introduction.—This test is an important one, in that it shows the degree of approximate magnetic saturation of the field magnets at any excitation, and in conjunction with the short-

circuit characteristic will give a predetermination of the regulation of an alternator for terminal E.M.F., on load. If the magnetization of its field magnets is carried too close to the point of saturation, then manifestly it will require large variations of exciting current to produce comparatively small variations in the E.M.F. of the machine. This, it need hardly be pointed out, is undesirable, and would cause a most insensitive form of regulation. Again, it will be apparent that, if no demagnetizing action of the armature on the field is going on, the exciting (*i. e.* magnetizing) current will represent the magnetizing force and form a measure of it. This in turn will create a certain magnetic field and magnetization, and in this field the armature rotates, or as in the inductor form of alternator this field is made to change from zero to full and back to zero, and in so doing cuts the stationary armature conductors. But the E.M.F. generated is $\propto$ rate of change of this field, and at constant speed to the field strength itself. Hence the E.M.F. is a measure of the magnetic field or induction excited, providing there is no armature reaction. Thus the present test will give us approximately the curve of magnetization of the alternator field magnets, but it must be carefully remembered that this is only approximate and provided there is no armature reaction on the field due to eddy currents in the armature, which should not occur when the machine is running on no load.

Apparatus.—Precisely the same as that used in the preceding test.

Observation.—(1) Connect up exactly as indicated in Fig. 67, and adjust the instruments to zero. See that all lubricating cups feed properly and slowly, and then start the alternator.

(2) Adjust the speed of the alternator to the normal value for that machine, and keep it constant. Close S and adjust the exciting current (by varying R), so as to obtain the lowest readable voltage on V. Note this simultaneously with the exciting current and speed.

(3) Repeat 2 for about ten different exciting currents, rising from the preceding amount to the maximum allowable, the speed being constant at the above value throughout.

(4) Take a similar descending set of observations as in 3.

(5) Repeat 2 and 3 for (constant) speeds 50% below, and if possible 20% above *normal*, and tabulate your results as follows—

NAME . . . DATE . . .

Alternator tested: No. . . . Type . . . Maker . . .
Normal Voltage = . . . Current . . . Speed . . .
Normal Exciting Current = . . .

Speed Revs. per min.	Exciting Current A Amps.		Terminal Voltage V.	
	Ascending	Descending	Ascending	Descending

(6) Plot curves for each speed to the same axes having volts as ordinates and exciting currents as abscissæ, both for ascending and descending readings.

Inferences.—State very clearly all that can be inferred from your results, and point out their bearing on the design of alternators.

(68) Magnetization Curve of an Alternator on Full Load—Non-Inductive and Inductive.

Introduction.—While the "no-load magnetization" curve or "open circuit" Characteristic of both an alternator and D.C. dynamo take the same general form, the curve which, as we have seen, relates terminal voltage of armature with excitation current is different on full load, and in the case of an alternator depends on the nature of the external load, *i. e.* whether inductive or non-inductive. This difference is due to the loss of terminal voltage on load caused by the *ohmic resistance, reactance,* and *reaction* of the armature when delivering current; though only resistance causes loss of power. Armature reaction is responsible for distorting and either strengthening or weakening the main magnetic field in the air gap, and is due to armature current and depends on the power factor of the external circuit, while armature reactance, due to the self-induction of the armature conductors,

causes the current to lag behind the induced voltage, though in phase with the terminal voltage, and is unaffected by the power factor of the external circuit. The magnetization curve on full load, when compared with that on no load, affords valuable information needed for the design of a field regulator for the alternator to give any particular degree of sensitiveness.

Apparatus.—Precisely that given for Test No. 70.

Observations.—(1) Connect up as in Fig. 69, levelling and adjusting such instruments as need it to zero. See that the lubricating arrangements are working properly on starting up the motor alternator, which should be the same as that used in obtaining the "open circuit" Characteristic of Test No. 67.

(2) With a variable *non-inductive* resistance R connected for absorbing the output, and with R and r full in, adjust the speed of the alternator to its normal value and maintain this constant throughout by field regulation on the driving motor. Now close S_1 and S and reduce R until the alternator gives full-load current on A, then note the readings of V, A, and (a) at *constant normal speed* for this and a series of exciting currents (a) decreasing by about equal amounts to the lowest possible, the armature current A being kept constant by reducing R.

(3) With a variable *inductive* resistance for R—composed either of an adjustable choking coil in series or parallel with adjustable non-inductive resistance, or of a synchronous motor, the excitation and loading of which can be varied (*vide* p. 305), and the addition of a wattmeter for measuring the power absorbed in R, Fig. 69. Repeat obs. 2 for constant power factors $\cos\phi$ of, say, 0·9, 0·8, etc., leading and lagging if possible, and tabulate all your results as follows—

Alternator tested : No. = . . . Type . . Maker . . .
Full load : Volts = . . . Amps. = . . . Speed = . . .

Nature of load :— Ind. or Non-Ind.	Speed.	Armature Amps. A.	Exciting Amps. a.	Volts V.	Watts W.	Power Factor $\cos\phi = \frac{W}{AV}$.

(4) Plot (from obs. 2 and 3) curves of full-load magnetization to the same axes, having volts V as ordinates with exciting currents (a) as abscissæ.

(5) For comparison replot the "no-load" magnetization curve obtained in Test No. 67 for the same alternator on the above curve sheet.

Inferences.—What can you deduce from the results of the above test, and how can the range of field regulating resistance be obtained for maintaining constant voltage between 0 and full load on any particular power factor of circuit? What excitation is necessary to send full-load short-circuit current through the armature at normal speed?

(69) Determination of the Short-Circuit Characteristic of an Alternator.

Introduction.—This characteristic, which is really the magnetization curve of the alternator on short circuit, differs from that obtained in Experiment 70, in that in conjunction with the "open" circuit characteristic of the machine, p. 169, it forms a means of predetermining the "voltage drop," *i.e.* the regulation of any alternator at different external loads and power factors. In this way large alternators may be tested while giving full-load current although requiring very little power to drive them.

Apparatus.—Precisely that detailed for Experiment 70, the resistance R being capable of variation and of being short circuited.

Observations.—(1) Connect up precisely as in Fig. 69, and adjust the pointers of such instruments as require it, to zero. See that all lubricators in use feed slowly and properly.

(2) Start the alternator up to *normal speed* and close S, noting the readings of V and A (if any) with R cut out to short circuit.

(3) With (r) full in, close (s), and adjust the exciting current (a) (by varying r) to some small value that will cause A to read about $\frac{1}{10}$th of the full load current of the alternator. Now *read* all the instruments. Next open S and again take readings, the *speed* being kept constant at the *normal value* throughout.

(4) Close S again, and increase A by about another $\frac{1}{10}$th, by suitably increasing the exciting current (a). Then note the

readings of all the instruments. Next open S and again take readings, the speed being constant at normal value throughout.

(5) Repeat (4) for a series of values of A up to 20% above full load value.

(6) To determine the effect of speed variation on the short-circuit current, run the alternator up to max. safe speed and raise its excitation until full load, or 20% over full load, current is obtained. Note this current, the speed, excitation and voltage and also for some ten different speeds, decreasing by about equal amounts to the lowest convenient at *constant excitation*, and tabulate in the following manner.

Note.—To avoid the risk of damaging the armature of the machine, especially if a large one, the series resistance R should all be cut into circuit before closing and again before opening S, and should be cut out to short-circuit just before taking instrument readings.

Alternator: No. . . . Type . . . Maker . . .
Full Load Output . . . . Amps. . . . . Volts . . . revs. per min.
" " Excitation = . . . Amps. Frequency = . . . ~ per sec.
Resistance (warm) of Armature + Ammeter and Leads R = . . . ohms.
Percentage Allowance K (if any) = . . .
Total Equivalent Resistance $R_T = (R + KR)$ = . . . ohms.

Speed (constant).	Terminal Armature Volts.		Short Circuit Armature Current A.	Exciting Current a.	Effective Volts for Ohmic Resist. AR_T.	Idle or Inductive Voltage Drop $\sqrt{E^2-(AR_T)^2}=E_I$.
	Short Circuit E_A.	Open Circuit E.				

(7) Plot the "short-circuit characteristic" having values of (A) as ordinates and (a) as abscissæ, and also the curve having A as ordinates with speed as abscissæ.

Inferences.—State all that can be deduced from the results of the test.

Determination of the "Voltage Drop" of an Alternator for any given load and circuit Power Factor.

From the results of the above test and with the aid of a simple graphical construction, it is easy to find approximately the voltage drop corresponding to any load current and power factor. Thus—

From any point O in any straight line XY set off OE, to a convenient scale, to represent any desired "open circuit" voltage of the alternator and at such an angle θ to the current line XY, that the power factor (cos. θ) of the circuit has the desired value for the particular load current (= short circuit current A) assumed.

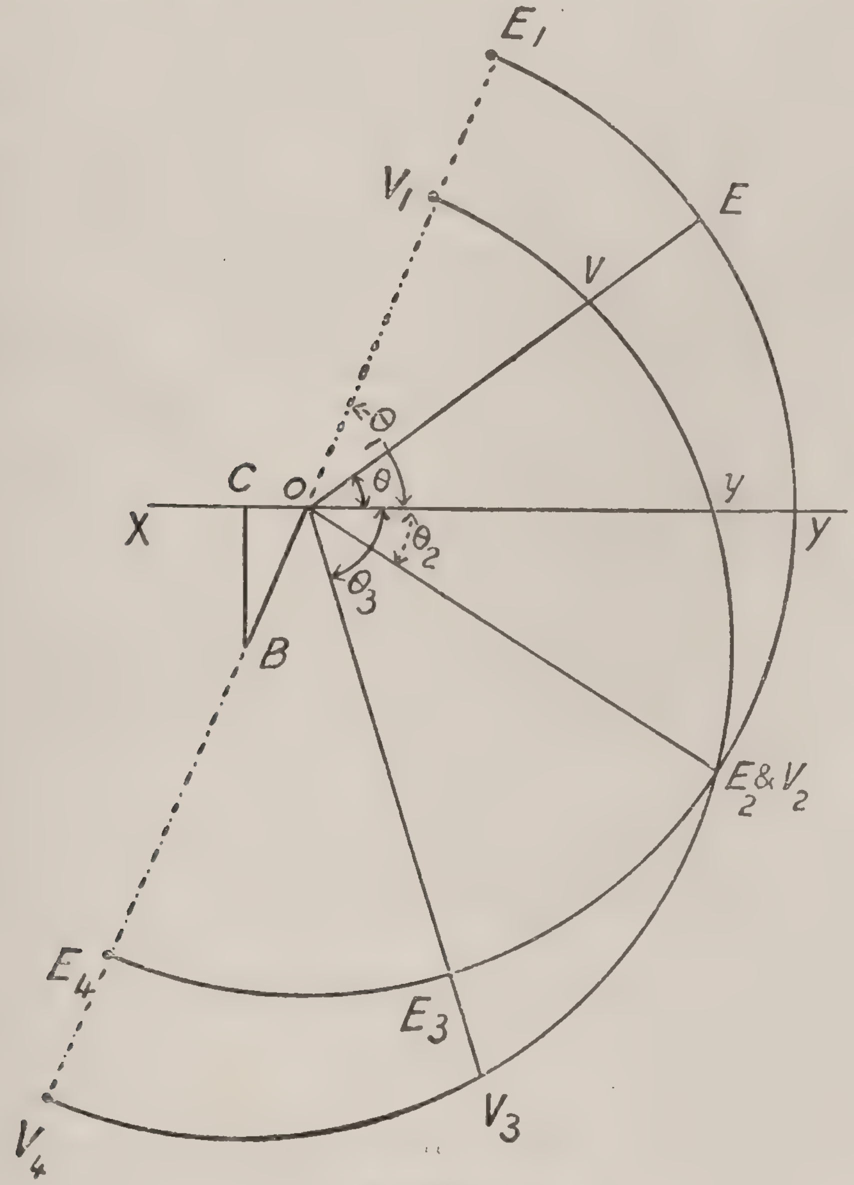


Fig. 68.

With centre O and radius OE draw the semi-circle E_1YE_4. Next set off along XY, the line $OC = AR_T$ the armature drop and allowance (K) for eddy current loss, corresponding to the same main current A.

Note.—In small, slow speed machines, the power lost due to eddy currents in armature core and pole pieces is small, and the effect of this on the voltage drop is also small compared with the term AR and can be neglected. When, however, the effect is too

large to neglect as in large machines the term AR can be increased by a certain percentage (K) depending on the form of alternator and the speed, (as dictated by experience with this type of machine or by experiment) to allow for the eddy current effect. Since the term AR_T is small compared with E, the error introduced in the value of the inductive drop E_I, through estimating the percentage increase of AR wrongly, is very small.

From C draw CB perpendicular to XY and = the short-circuit inductive or idle voltage drop E_I due to the self-induction of the armature and loss of voltage due to armature reaction. Join OB which therefore = the open circuit voltage E, which is necessary to overcome the self-induction and resistance of the armature and leads, for the short-circuit current A assumed. OCB is therefore the right-angled triangle of E.M.F.'s on short circuit and the angle COB = the angle of lag between current and voltage on short circuit. Now with centre B and radius OE draw the semi-circle V_1VV_4. Then OV = terminal voltage of the machine and VE the voltage drop for the main circuit current A and circuit power factor cos. θ. OV and VE can similarly at once be found for any other value of power factor with the *same main current* A from the *same diagram.* A new diagram must, however, be constructed in a similar manner for a different current A in order to obtain the terminal volts such as OV and drop such as VE for this new current. Some interesting cases are now obvious, namely—

For main current lagging behind the voltage, i. e. self-induction predominating in the main circuit.

(1) For main circuit power factors corresponding to an angle $\theta_1 = COB$, when OE takes the position OE_1, in line with BO, the terminal voltage OV_1 = minimum and the inductive drop V_1E_1 = maximum.

(2) For a non-inductive main circuit, $\theta = O$ and the terminal voltage OY is in phase, and coincides in sense, with the main current, but will differ from the resultant voltage OB by a little more than 90°.

For main current leading in front of voltage, i. e. capacity predominating in the main circuit.

(3) For the negative angle θ_2, terminal voltage OV_2 = open circuit voltage OE_2 and there is no inductive drop.

(4) For greater "leads," the terminal voltage is greater than the open circuit voltage, *e.g.* for a $-^{ve}$ angle θ_3 the terminal voltage $= OV_3$ and open circuit voltage OE_3, maximum values of these quantities being reached at OV_4 and OE_4 where the maximum $-^{ve}$ inductive drop $= E_4 V_4$.

(70) Determination of the External Characteristic of an Alternating Current Generator.

Introduction.—This test is for the purpose of experimentally determining, for different speeds and excitations, the relation

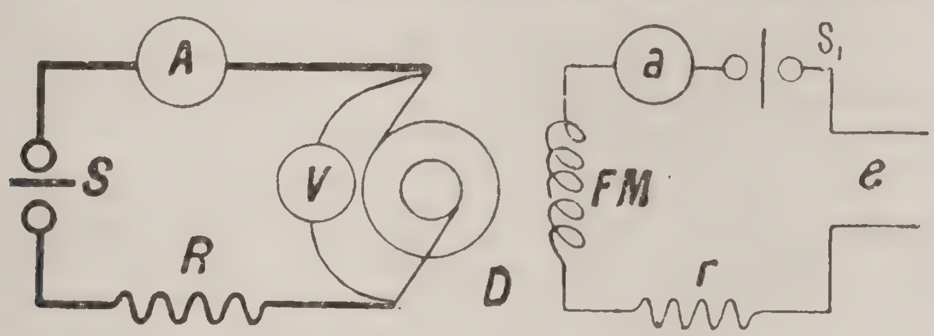


FIG. 69.

between external current and the terminal voltage producing it. By means of the *Characteristic curve*, as it is called, so obtained by plotting the observations, it can be seen at a glance whether the alternator possesses any series defects. Its performance when running on some particular circuit at some particular speed and excitation is also easily discernible therefrom.

Apparatus.—Alternator D to be tested; switch S; alternating ammeter A; and voltmeter V; non-inductive resistance R, which should preferably consist of either a bank of glow-lamps (p. 598), carbon (p. 597) or water rheostat, all of which are non-inductive; in circuit with the exciting coils F—a rheostat r (p. 599); switch S_1, ammeter (a); and source of direct current e from secondary cells or otherwise; speed indicator. Wattmeter W (not shown) for measuring the true watts absorbed in R when this is inductive.

Observations.—(1) Connect up as in Fig. 69, and adjust the pointers of all the instruments to zero, if necessary.

(2) Start the alternator up, seeing that all lubricating arrangements in use feed properly. Close S_1, adjusting both the speed and excitation to the normal for that machine.

(3) With this *excitation* and *speed constant*, take, by varying R, a set of readings for about ten different values of external current between 0 and the maximum permissible, differing by about equal amounts, and note the simultaneous readings of V and A.

(4) Repeat (3) with the same normal excitation, but for constant speed 50% below normal.

(5) Repeat (3) and (4) for a different excitation, say, 50% lower than normal, and tabulate all results in the accompanying form.

(6) Measure the resistance of the armature, *while warm*, by the "Potential Difference" method (see p. 84), or by the ammeter voltmeter method, p. 86.

NAME . . . DATE . . .

Alternator tested: No. . . . Type . . . Maker . . . Periods per Revol. $K =$ . . .
Normal Volts . . . Amps. . . . Speed . . . $p = 2\pi \times$ Frequency.
Armature: Self-induction $L =$ . . . Henry: Ohmic Resistance Warm r_a . . .

Speed Revs. per min.	Exciting Current (*a*) Amps.	Volts. *V*.	Amps. *A*.	Watts *W*.	App. Watts $A \times V$.	Power Factor $\cos\phi = \frac{W}{AV}$.	Induction. Resistance. *Lp*.	Induction. E.M.F. *LpA*.

(7) Plot the *external Characteristic* curves for each speed and excitation having terminal volts V as ordinates and current A through armature and external circuit as abscissæ, in each case (see p. 3 on curve plotting) using the same pair of axes and curve sheet.

(8) Deduce the *total Characteristic* curves of the machine from the "external" ones in 7 above by means of the graphical method given below.

Inferences.—Very carefully state all that you can infer from your experimental results, and show in what way the above curves indicate the performance of the alternator, and give an idea as to the goodness of the efficiency.

Graphical Determination of the Total Characteristic of an Alternator from the External Characteristic.

The *total Characteristic* of an electrical generator is the curve showing the relation between the *total E.M.F.* in volts generated by the machine and the *total armature current* in amperes.

Analytical Treatment.

Referring now to the preceding diagram (Fig. 69), let L = self induction of the armature in henries—

r_a = ohmic resistance of the armature in ohms.

R = ohmic resistance of the external circuit, assumed to be non-inductive.

p = angular velocity of the alternating current $= 2\pi n$.

n = frequency or periodicity in $\sim$ per sec.

If then a current A flows in the external circuit at a terminal voltage V and E = total E.M.F. of the generator,

$$\text{then } A = \frac{E}{\sqrt{L^2p^2 + (R + r_a)^2}} \qquad \text{but } V = AR\,;$$

$$\text{hence } E = \sqrt{L^2p^2A^2 + (R + r_a)^2A^2} \quad = \sqrt{L^2p^2A^2 + (Ar_a + V)^2}.$$

Consequently we see that the total E.M.F. in volts generated by the alternator is represented by the hypothenuse of a right-angled triangle, of which the other two sides represent LpA—the inductive E.M.F. necessary to overcome that of self-induction, and $(Ar_a + V)$—the effective E.M.F. necessary to send the current A through the ohmic resistance $(R + r_a)$.

If K = number of periods per revolution of the armature, then $\frac{KN}{60} = (n)$, the frequency in $\sim$ per sec.; whence $p = 2\pi\frac{KN}{60}$ where N = number of revolutions per minute the armature is making.

Having thus obtained an expression for the total E.M.F. of the machine analytically, we will now proceed to deduce it graphically from our external Characteristic.

GRAPHICAL TREATMENT.

Let xy represent the external Characteristic plotted to the rectangular axes OT and OS, of which the former represents voltage and the latter current, respectively. (See Fig. 70.)

From the point O or origin draw the straight line OQ such that tan. $QOS = r_a$, and also draw the straight line OP such that tan. $POS = Lp$ for the particular speed at which xy is taken.

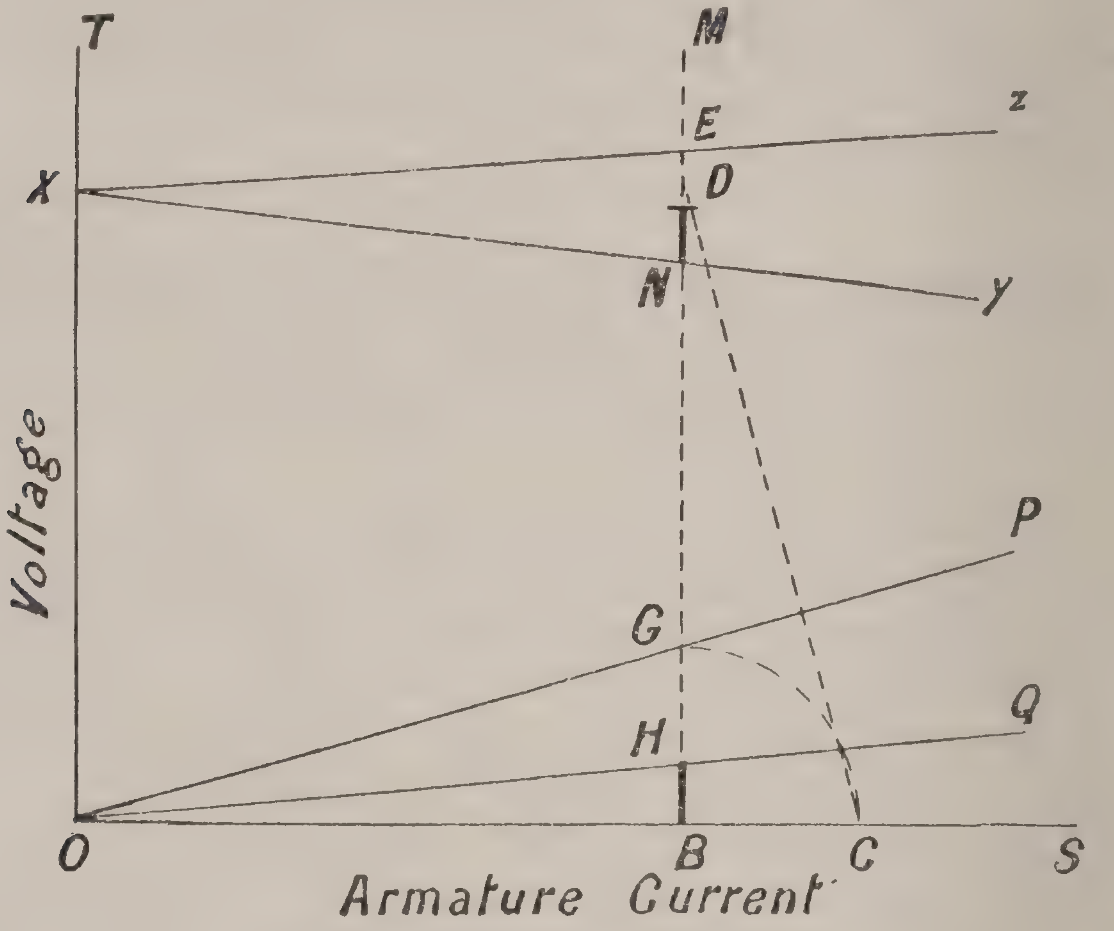


FIG. 70.

Take any point N on the external Characteristic xy and through it draw an ordinate MNB cutting OP, OQ and OS in the points G, H and B, respectively.

Set off $ND = BH$ and $BC = BG$ and join DC, which therefore $= E$. Lastly, make $BE = CD$.

Then E is a point on the total Characteristic required. If this construction is repeated for several more points, such as N on xy, we finally obtain a series of points such as E, and on drawing a *mean curve* through them all we obtain the required total Characteristic xz. In this construction it will be noticed that—

$BN =$ the terminal voltage (V).

$BG = BC$ = the inductive E.M.F. (LpA).

$BH = ND$ = the effective E.M.F. (Ar_a).

DBC is consequently a right-angled triangle, and

hence $DC = E = \sqrt{BC^2 + BD^2} = \sqrt{(LpA)^2 + (Ar_a + V)^2}$.

Probably a better mode of procedure and one *not* requiring the use of a protractor in obtaining the correct angle at which to set off OP and OQ is to first take any point N on xy, obtain the ordinate MNB and set off BG = the current $OB \times Lp$, also $BH = OB \times r_a$, then joining these points H and G to O we get the required lines OHQ and OGP respectively, after which proceed as above.

(71) Determination of the External Characteristic of an Alternator for different circuit Power Factors.

Introduction.—The circuits which have to be supplied from alternators, in ordinary commercial work, are invariably inductive, possessing either self-induction or capacity or both. As the inductiveness of the circuit affects the terminal voltage of an alternator,—self-induction causing the characteristic to fall and capacity causing it to rise, it is important to be able to determine the characteristic for any power factor of the circuit, whether the current lags or leads with respect to the voltage.

Lagging currents can easily be obtained with any power factor by suitable combinations of self-induction and ohmic resistance, as for example by carbon rheostats and continuously variable choking coils or dimmers.

Leading currents can be obtained by inserting along with, say carbon rheostats (1) a number of condensers possessing considerable capacity, (2) a long length of concentric cable, (3) a *synchronous* a.c. motor with fields excited so that the back *E.M.F. of the motor exceeds the voltage of the a ternator*. This is by far the most convenient method, for any load can easily be taken from the alternator by either braking the motor pulley or making it drive a d.c. dynamo at various loads. It can be shown that over-excited synchronous motors cause a leading current to flow in the circuit supplying them, and hence produce the same effect as 1 and 2 above. By adjusting the excitation of such a motor any lead can be obtained within limits.

Where the only means available for obtaining *lagging* power factors is by varying combinations of self-induction and ohmic resistance, or for obtaining *leading* P.F.s is by varying combinations of capacity and ohmic resistance, it is a very tedious process to try and take a series of readings of output current with terminal voltage *at constant P.F.* This is obvious since it entails varying the inductive and non-inductive sections of the load circuit so that they always bear the same ratio to one another, as only this will keep cos ϕ constant in value, since the P.F. or $\cos\phi = \dfrac{\text{ohmic resistance}}{\text{impedance}}$. In other words, trial combinations would have to be repeatedly made and the value of $\dfrac{W}{AV}$ worked out for each, to see if it had the desired value for cos ϕ. For this reason the following method is better, and is applicable to any appliance capable of giving an electrical output which has to be absorbed in an inductive circuit. Since the output obviously comprises (amps $A \times$ volts V), both of which are variable simultaneously between 0 and full load, and $A = \dfrac{V}{\text{impedance}}$ or $V = A \times$ impedance, it follows that A will be constant if V and the impedance vary in the same proportion, while V will be constant if A and the impedance vary in inverse proportion. Thus if the inductive portion of the load circuit is adjusted to max. value, and the non-inductive portion in series or parallel with it (whichever is found most suitable) is varied, it is possible to obtain different values of A corresponding to different values of cos ϕ at constant V, or different values of V corresponding to different values of cos ϕ at constant A, and a curve can be plotted between cos ϕ as ordinates and the variable as abscissæ. If, now, the inductive portion is made less inductive and the non-inductive part again varied, a new value will be obtained for the constant and a new series of values of cos ϕ and the variable obtained, which will give a second curve with cos ϕ as ordinate and the variable as abscissæ.

Repeating the process for some five or six degrees of inductiveness, giving therefore as many values of the constant and corresponding curves between cos ϕ and the variable, we can utilize these *auxiliary curves* to obtain the same number of

curves between V and A each *at any desired constant power factor* ($\cos \phi$) within the limits produced by the range in degree of inductiveness used. As an example—Suppose A is the constant and V the variable with $\cos \phi$, and that six auxiliary curves, each obtained for, and marked with, the corresponding constant, are plotted on the same sheet. Now to obtain the curve between V and A at, say, 0·9 power factor: note the six points or voltages on the volt scale cut by the ordinates through the six points of intersection between the single horizontal line through the 0·9 mark on the $\cos \phi$ scale and the six curves. Then the six voltage values so found, plotted against the six constant values of A marked on the respective curves, will give the desired relation between A and V at a constant power factor $\cos \phi = 0{\cdot}9$.

Similarly, a curve between A and V could be drawn for $\cos \phi = 0{\cdot}8$, 0·7, 0·6, etc.

Apparatus.—The same as that detailed for Test No. 70, except that the load resistance R must now comprise a variable self-induction and variable non-inductive resistance, and that a wattmeter W must be inserted so as to give the true watts absorbed in the whole circuit.

Observations.—(1) Connect up as in Fig. 69 with the slight modifications just mentioned. Level and adjust the pointers of any instruments needing it to zero, and see that the lubricating arrangements are in operation on starting up the machine.

(2) With S closed and with both *excitation* and *speed* adjusted to the normal value and maintained *constant*, vary the proportions of heavy self-induction and large ohmic resistance in the main circuit, so as to obtain some six different voltmeter and corresponding wattmeter readings for each of five different but constant alternator currents A, differing by about equal amounts between 0 and full load, the same value of main current to be obtained for each of the six readings of a set.

(3) Plot five curves one for each of the five constant values of main current, each having the terminal voltage of the alternator as abscissæ and power factor (obtained by indicator or wattmeter and volt-amperes) as ordinates. Mark on each curve the constant main current at which it was obtained.

(4) Repeat obs. 2 and 3 so as to obtain a similar set of five curves for *leading currents* of the same magnitude as previously

used by varying the load on the synchronous motor and its excitation, and tabulate as in Test No. 70.

(5) Plot the external characteristic curves having volts as ordinates and currents as abscissæ for power factors differing by 0·1 at a time between 1·0 and the lowest obtained in the curves between voltage and power factor (3) to (4) above, each curve in (3) and (4) supplying one point only in each of the characteristic curves corresponding to the current and voltage, and the power factor for which the particular characteristic is plotted.

Inferences.—State clearly all that you can deduce from the results of your tests.

(72) Variation of Exciting Current with the Armature Current of an Alternator to maintain Constant Terminal Voltage on Inductive and non-Inductive Loads.

Introduction.—This test is a direct measurement of the range of variation in both the resistance and current of the field regulator required to maintain constant terminal voltage for any range of load, and which can otherwise be deduced from a reference to the external and magnetization characteristics of the alternator when available.

Apparatus.—That detailed for Test No. 70, with the addition of an adjustable inductive resistance for combination with R.

Observations.—(1) Connect up, as in Fig. 69, *with R non-inductive* only at first and the wattmeter inserted as a check on the product $A \times V$. Level and adjust all instruments, which require it, to zero, and on starting up the motor alternator see that all lubricating arrangements are working properly.

(2) With the speed and terminal voltage each adjusted to the normal value and kept constant—the former by regulation at the driving source, and the latter by field regulation of the alternator—first note the value of exciting current (a), on open main circuit, and then with S closed for each of a series of 8 or 10 armature currents, A rising by about equal increments to full load by adjusting the non-inductive resistance R, the

field regulation of the alternator being adjusted so as to keep V constant with the speed constant.

(3) *With R now inductive*, and the same value of speed and voltage as in obs. 2 maintained constant, adjust the inductive part to its maximum value and vary the non-inductive part so as to maintain the main current A *constant* at about quarter full-load value, the field current being correspondingly varied to keep V constant. Note the readings of A, V, W, and (a).

(4) Repeat (3) for the same speed and voltage, but for constant values of A of about half, three-quarters, or full load, and tabulate as in Test No. 70.

(5) Plot the four *auxiliary curves* (vide p. 182), having power factors, cos. $\phi\left(=\frac{W}{AV}\right)$, as ordinates, with exciting current (a) as abscissæ, for each of the four constant values of A respectively.

(6) By interpolation and transference, plot the desired relations between exciting currents (a) as ordinates, with main currents (A) as abscissæ for non-inductive load (obs. 2), and for inductive load (obs. 3 and 4) at constant power factors of 0·9, 0·8, etc.

Inferences.—State clearly all that can be deduced from a study of the shape and relative dispositions of the curves in (6) above.

(73) Determination of the Efficiency of an Alternator without running it on load.

Introduction.—The preceding method, although a direct one for obtaining the efficiency of any generator by measuring the H.P. absorbed in driving it and the output in the usual way, requires some type of transmission dynamometer and has therefore a limited application on account of the difficulty of measuring the large H.P.s in the case of large generators. While small alternators can be tested in this way, the method of driving the generator by an electromotor (preferably direct coupled) having a known efficiency-load curve, is more accurate, and also has a much more extended application in range of power. The power supplied to the motor × by its efficiency at that load = the power taken to drive the generator.

Both methods are however costly of application in the case of the larger generators and an arrangement similar to Swinburne's Test No. 82 would obviously be preferable in many ways.

Further, if while absorbing (from an outside supply) only a small fraction of its rated full-load output, the armature of an alternator carries full-load current at normal excitation, both the losses and temperature rise can be determined at an economical cost of energy consumed in the test.

From the fundamental principle that, in any transforming device

$$\text{output} = \text{input} - \text{total internal loss}$$

we see that if the losses can be obtained in the various portions of the alternator, the input and efficiency are at once deducible and that the method is applicable to any size of machine, however large.

The total internal loss in an alternator is made up of—

(*a*) The total copper loss W_C in the armature windings, thus—

If C_a = the *current per phase* of armature winding

and r_a = the *resistance per phase* " " "

the total copper loss W_C in the armature of a—single-phase alternator $= C_a^2 r_a$; of a two-phase alternator $= 2C_a^2 r_a$; and of a three-phase alternator $= 3C_a^2 r_a$.

(*b*) The total power absorbed in excitation W_E. Thus if C_E = the exciting current employed at a pressure of V_E volts, then $W_E = C_E, V_E$.

(*c*) The total power absorbed in mechanical friction W_{MF} and made up of—windage, bearing friction, brush friction.

(*d*) The total power spent in magnetic friction made up of—hysteresis and eddy currents in field and armature core W_{IF}.

Then the total internal loss $W_L = W_C + W_E + W_{MF} + W_{IF}$.

The last two losses, which may be termed the stray power, may be determined experimentally at no load by running the alternator as a synchronous motor, light and unloaded, and measuring the power absorbed by wattmeter.

Apparatus.—Alternator *D* to be tested, capable of being driven at no load and normal speed by some outside source of power. This might preferably be its direct-coupled exciter, if

there is one, and provided that, when used as a motor, it is powerful enough, or, in the absence of this, a direct-coupled motor M of known efficiency. The necessary switches; a watt-meter having a range up to say 5% of the full load of the alternator; an ammeter A; and a voltmeter V in the case of single-phase alternators and of two- and three-phase alternators with equally-balanced phases. Two of each type of instrument will be needed for two- and three-phase machines out of balance.

Observations.—(1) Connect up as shown in Fig. 71, adjusting such of the instruments as need it and assuming the alternator D to be a three-phase machine for instance. See that all lubricators feed properly.

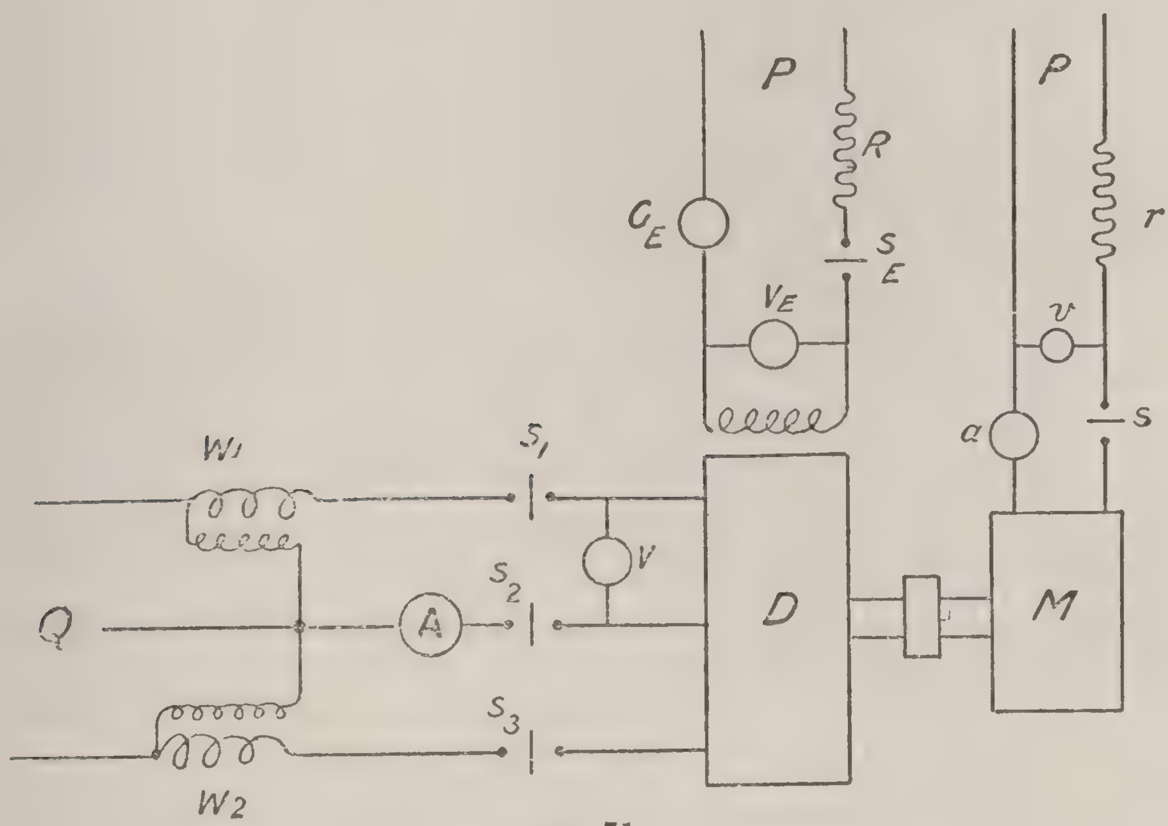


FIG. 71.

(2) To run D as a synchronous motor from some a.c. supply Q of the normal voltage V and frequency developed by D. First start up D to its normal speed by means of M and adjust its exciting current C_E so that its terminal voltage $V =$ that of the supply Q. S_1, S_2, S_3 being open, but S_2 shorted by a wire and S_3 by two synchronizing lamps (see p. 441) not shown, close this three-throw switch at the instant *when the lamps are out* and at

once open S. The alternator D will now continue to run as a synchronous motor at normal speed, frequency and voltage.

(3) With D running as just mentioned, adjust its éxciting current C_E by the rheostat R so that a minimum intake current A is obtained and hence maximum power factor, and note the wattmeter readings W_1 and W_2 and all the other instruments and speed.

Then $W_1 + W_2 - 3A^2r_a = W_{MF} + W_{IF}$.

These losses are approximately constant for all loads, slightly increasing as the load increases due to change of induction through increase of excitation for voltage regulation, and to armature reaction in the normal operation of the alternator.

(4) With the *same normal supply frequency* as in obs. 2, and *same excitation* as found in obs. (3), increase the supply voltage V (by means of the field excitation of the supply Q) so as to obtain a series of supply currents A rising by about equal increments from the value found in obs. (3) to full load, and note the readings of A V and W_1 W_2 at each.

Note.—The synchronous motor D will run throughout obs. (4) at constant normal speed but at a decreasing power factor, as was shown in Test No. 106, and since the excitation is constant at normal value, the losses $(W_C + W_{MF} + W_{IF})$ as measured by W_1 and W_2 will be the same as for the machine used as an alternator when giving the same current loads from its armature. The excitation loss W_E is also known, and the total internal loss at all loads can therefore at once be found and the efficiency Σ obtained from the relation—

$$\Sigma = \frac{\text{output}}{\text{output} + \text{losses}}.$$

The accuracy of the values of efficiency (Σ) thus found will depend on the degree of accuracy with which the losses, in obs. 4, can be measured; this latter may not be high on account of the increasing inaccuracy of wattmeters on the lower power factors.

(5) If necessary, find the rise of temperature of the machine after a six hours' run on full-load or other desired condition.

Tabulate your results for the preceding test as follows—

Alternator: No. . . . Type . . . Maker . . .
Full Load: Amps. = . . . Volts = . . . Speed = . . . Frequency = . . .
Resistances: Armature per phase r_a = . . . Field = . . .
Wattmeter Constants K_1 = . . . K_2 = . . .

Speed of D.	Supply Frequency.	Exciting			Current A.	Volts V.	Apparent Watts $\sqrt{3}AV$.	Wattmeter Readings.		Total True Watts. $W = K_1W_1 + K_2W_2$.	Power Factor $\cos\theta = \frac{W}{\sqrt{3}AV}$	Armature loss $3A^2r_a$.	Stray Power loss $W - 3A^2r_a = W_{MF} + W_{IF}$.	Total loss W_L.	Efficiency Σ.
		Current C_E.	Volts V.	Watts C_EV_E.				W_1.	W_2						

Plot the efficiency-load curve of the machine considered as an alternator having values of efficiency Σ as ordinates, and values of $\sqrt{3}AV$ (calculated for each value of A in the table but at normal voltage) as output at P.F. = 1.

The separation of the losses can be obtained in much the same way as that indicated in Test No. 77 for direct current machines by the use of the motor M in the following way—

(4) With S_1, S_2, S_3 and S_E both open, run the alternator at its normal speed and note the readings (a_1v_1) of a and v. If y_1 = the efficiency of M at this speed and load, then $y_1a_1v_1 = W_{MF}$ the power absorbed in mechanical frictions since with the fields of D unexcited there will be practically no iron losses.

(5) Next close S_E and adjust R so as to give normal full load exciting current. Vary r so as to obtain the same speed as before and note the readings a_2v_2 of a and v. If y_2 = the efficiency of M at this load, then $y_2a_2v_2 = W_{MF} + W_{IF}$ and hence the iron losses $W_{IF} = y_2a_2v_2 - y_1a_1v_1$.

Having now obtained the iron and friction losses the efficiency can at once be deduced for all assumed loads.

Tabulate your results for Tests 4 and 5 as follows—

Efficiencies of Motor M at Speed used: (at load a_1v_1)y_1 = . . . ; (at load a_2v_2)y_2 = . . .

Alternator Speed.	Alternator unexcited.			Alternator Excited.			Iron Loss $W_{IF} = y_2a_2v_2 - y_1a_1v_1$.
	Reading of		Friction Loss	Reading of		Friction + Iron Losses	
	Ammeter (a) a_1.	Voltmeter (v) v_1.	$W_{MF} = y_1a_1v_1$.	Ammeter (a) a_2.	Voltmeter (v) v_2.	$W_{MF} + W_{IF} = y_2a_2v_2$.	

The determination of the iron losses at any speed by the retardation method can be undertaken in alternators having sufficient inertia or momentum in their moving portion to prevent them slowing down to rest too quickly for readings of speed to be taken at intervals.

(6) With S_1, S_2, S_3 and S_E both open, run D at normal speed by means of M. Then at a noted interval of time, the speed being at normal value, open S and note the speed by tachometer at, say, $\frac{1}{4}$ minute intervals as the alternator D slows down to rest, the motor M being of course unexcited. The retardation in this test is due to W_{MF}.

(7) Repeat (6) with $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ and full load exciting currents C_E by varying R. The more rapid slowing down in this test is due to $W_{MF} + W_{IF}$. Tabulate your results as follows—

Value of $W_1 + W_2 - 3A^2r_a$ (from Exp. 3).

Alternator Unexcited.		Alternator Excited			Values of		Actual mean Power in Watts absorbed.
Times t, t_1, t_2 . .	Speeds n, n_1, n_2 . .	Exciting Current C_E . .	Times t, t_1, t_2 . .	Speeds n, n_1, n_2 . .	$\frac{W_K}{I}$	Average Speed $N = \frac{n + n_1 \ldots}{2}$	

Plot curves for 6 and 7 between speeds as ordinates and times as abscissæ, and between $\frac{W_K}{I}$ as ordinates and average speeds N as abscissæ.

The rationale of the retardation method is as follows—

Let I = the moment of inertia of the rotating system in C.G.S. units, or, gramme—cm.^{2} about the axis of rotation, and let W be its angular velocity in radius per sec. Then the kinetic energy of the whole system or energy of rotation $K_E = \frac{1}{2}\omega^2 I$ ergs $= \frac{1}{2}(2\pi n)^2 I$ where (n) = speed in revs. per second. Hence the kinetic energy $= \frac{1}{2}\left(\frac{2\pi n}{60}\right)^2 I \times 10^{-7}$

$$= 548\ 10^{-12}\ n^2 I \text{ Joules or Watt seconds,}$$

where (n) is now in revs. per min.

If now (n) = the normal speed of the alternator at the instant t of opening s and n_1, n_2, n_3 . . . the successive speeds noted at times t_1, t_2, t_3 . . . from the instant (t) when slowing down commences.

Then the energy expended or work done in the first interval of time in overcoming resistance is proportional to

$$548 \times 10^{-12} I(n^2 - n_1^2)$$

and the mean power absorbed $W_K = \dfrac{548 \times 10^{-12} I(n^2 - n_1^2)}{t - t_1}$ watts at an average speed $N = \dfrac{n + n_1}{2}$ revs. per min.

By plotting a curve between values of $\dfrac{W_K}{I}$ for successive speeds and intervals as ordinates and the corresponding average speeds N for successive pairs of speeds as abscissæ, we can get the loss at any speed, and that at *normal speed* by the point of intersection of the curve (produced backwards) with the full speed ordinate.

Note.—Since I is a constant but unknown quantity, the ordinates of the curve are the values of $\dfrac{W_K}{I}$ and the ordinates do not therefore represent actual watts, but = watts ÷ a constant (I).

From Observation 3, however, we know that this normal speed ordinate $= W_1 + W_2 - 3A^2 r_a$, the total friction and iron losses. Hence the value in actual watts of any other ordinate corresponding to any other speed is at once found by simple proportion.

From the above data compile the following general table—

Alternator: No. . . . Type . . . Maker . . .
Full Load: Amps. = . . . Volts V = . . . Speed = . . . Frequency = . . .
Resistances: Armature per phase r_a = . . . Field r_s = . . .

Main Output Current Assumed C.	Watts Output $W_o = VC$.	Losses.				Input Watts $W_o + W_L$.	Efficiency $100 \dfrac{W_o}{W_o + W_L}$ %
		Excitation $C_E V_E = W_E$.	Armature $3 C_a^2 r_a = W_C$.	Iron and Friction $W_{MF} + W_{IF} = w$.	Total $W_L = W_C + W_E + w$.		

Plot the load-efficiency curve having efficiency as ordinate and W_o as abscissæ.

(74) Efficiency and Internal Loss Test of a Pair of Alternators (by the Hopkinson Principle).

Introduction.—The following method is available when two similar alternators, as nearly alike in output as possible, are obtainable. It is analogous to the Hopkinson test of a pair of D.C. dynamos, and has the double advantage that all the measurements are electrical ones; and also that while both alternators run under load conditions, so far as field and armature current is concerned, each is running at only a fraction of its full K.W. capacity, and consequently the power taken from the necessary outside supply is small, even in the case of the testing of large alternators. The method further lends itself most conveniently to the determination of the temperature rise of each machine under the same heating conditions as would obtain if each was run for, say, six hours at full-load

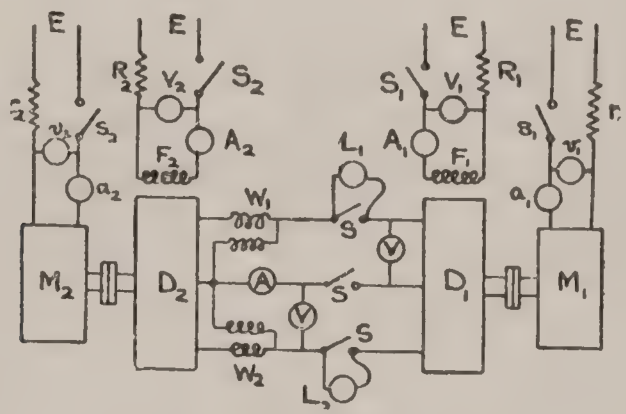


FIG. 72.

output, but with far greater economy in the cost of energy consumed from the outside supply.

The test can be carried out with the two alternators under one or other of two conditions, viz. (1) when they are not mechanically coupled together, or (2) when their shafts are in accurate alignment and rigidly coupled. In each case (*a*) the alternator to be used as a generator (say D_1) must be either belted or (preferably) direct coupled to a small direct-current motor M_1 having a known "efficiency-load" curve at the speed to be employed; (*b*) the second alternator D_2 must be electrically connected to D_1 and run as a synchronous motor from its supply; (*c*) an

additional driving source of power (which might be another D.C. motor M_2) will be needed to run D_2 into synchronism, and afterwards to be disconnected if possible. Now, although under condition (1) above, the power factor of the circuit between D_1 and D_2 will decrease as the circulating current increases, any error that might be introduced from this cause, and referred to in test No. 73, p. 188, is eliminated in the present test, as the losses are now measured in the D.C. circuit of M_1 instead of being obtained from the readings of W_1 and W_2.

Under condition (2) above, however, in which the shafts of D_1 and D_2 are rigidly coupled, the P.F. of the circulating circuit remains constant for all currents with any particular bolting of the half couplings. If this bolting can be varied, then each constant value of the P.F. can be varied *from unity*, when the half couplings are bolted so as to make the E.M.F.s of D_1 and D differ by 180° in phase (*i.e.* when in direct opposition of phase), *to zero*, when one half coupling is bolted with an angular difference relatively to the other half coupling equal to the angular pitch of the alternator field. Throughout this range the alternator with the greater excitation will be acting as generator.

Obviously with the rigid coupling of condition (2), the two alternators must not only be similar in output and voltage, but must also give the *same frequency at the same speed*, whereas in condition (1) their frequencies can be different if necessity arises. Further, the general applicability of the present method is questionable, *e.g.* in condition (2) the alternator D_2 must be coupled either (on the left) up to D_1, or (on the right) up to the other end of M_1, so that either D_1 or M_1 must have a shaft extension each end. On the other hand, the test under condition (1) will need a second driving-motor M_2, which for the highest accuracy should be capable of disconnection from D_2 after this is synchronized. These facilities may be obtainable in certain works, but seldom exist in a college laboratory.

Apparatus. That depicted in Fig. 72, where R_1R_2 are field regulators for adjusting the exciting currents in the fields F_1F_2 of D_1D_2.

r_1r_2 are starters or main circuit adjustable rheostats for the motors M_1M_2.

$L_1 L_2$ are synchronizing lamps made for voltages equal that per phase of the supply.

E is a D.C. supply, and three-phase alternators are assumed for test as presenting slightly greater complication in connection and test than single or two-phase machines.

Observations.—(1) Connect up similarly to Fig. 72, but with any modifications which the facilities available in machines necessitate. Level and adjust to zero all instruments needing it, and on starting up see that all lubricating arrangements feed properly.

(2) With D_1 running at normal voltage V and frequency, synchronize D_2 by obtaining equal voltages as V.V., and closing SSS at the moment when $L_1 L_2$ are definitely out. M_2 (if used) being disconnected electrically and mechanically, if the latter is possible.

(3) Next adjust R_2 so as to *make A a minimum* for *constant normal values* of both V and the frequency.

Note the readings of all the instruments and the speeds of D_1 and D_2.

Then the output of M_1, or power required to drive D_1 $= \frac{a_1 v_1}{e} =$ the total internal *running losses* $(W_C + W_{MF} + W_{IF})$ (see p. 186) in D_1 and D_2 together (excluding excitation losses in $D_1 D_2$),

Where $e =$ the efficiency of M_1 at this load and speed (from curve),

$$\frac{a_1 v_1}{2e} = \text{the losses in either alternator.}$$

(4) Now reduce the excitation of D_2 by the same amount as that of D_1 is increased, so as to obtain a series of main circulating currents A, rising by about equal increments up to the full load current of the machines, and note the readings of all instruments at each value of current A. Then the increased value of $\frac{a_1 v_1}{2e} =$ the losses in either alternator at the respective current values A, where (e) has an increasing value at each load as taken from the efficiency-load curve of M_1. Adding half the total excitation loss, viz. $\frac{1}{2}(A_1 V_1 + A_2 V_2)$, to the above loss, we get the total internal loss W_L (p. 186) corresponding to each

of the current A, and which will be practically those which would exist if either machine was supplying those currents as an alternator at the same speed and voltage. Tabulate all your results as follows :—

Alternator D_1 : No. = . . . Type . . . Maker . . . Armat. Res. per phase r_a = . . .
Full Load : Amps. = . . . Volts V_s = . . . Speed = . . . Frequency = . . .
Field Res. r_s = . . .
Alternator D_2 : No. = . . . Type . . . Maker . . . Armat. Res. per phase r_a = . . .
Full Load : Amps. = . . . Volts V_s = . . . Speed = . . . Field Res. r_s = . . .
Wattmeter Constants K_1 = . . . K_2 = . . .
Driving Motor M_1 : No. = . . . Full Load : Amps. = . . . Volts. = . . . Speed = . . .

Speed of D_2.	Frequency f.	Intake of M_1.			Output of M_1 = Intake of D_1 $= a_1v_1 \div e$ Watts.	Excitation of D_1.			Excitation of D_2.			Amps. A.	Volts V.	Apparent Watts $\sqrt{3}.A.V.$	Watts W_1.	Watts W_2.	Watts intake by D_2 $W = W_1 + W_2$.	Power Factor $\cos.\theta = \frac{W}{\sqrt{3}AV}$.	Armature loss $3A^2r_a = W_C$.	Stray Power loss $W - 3A^2r_a$ $= W_{MF} + W_{IF}$.	Total loss W_L in D_2 $= W_C + A_2V_2 + W_{MF} + W_{IF}$.	Total loss in D_1 or D_2 $W_T = \frac{a_1v_1}{2e} + \frac{1}{2}(w_1 + w_2)$.	Output as an Alternator $(PF = 1)$ $= \sqrt{3}.A.V_S = W_O$.	Efficiency as Alternator $\Sigma = \frac{W_O}{W_O + W_T}$.
		Amps. a_1.	Volts v_1.	Watts a_1v_1.		Amps. A_1.	Volts V_1.	Watts $w_1 = A_1V_1$.	Amps. A_2.	Volts V_2.	Watts $w_2 = A_2V_2$.													

(5) Plot the efficiency-load curve of either machine considered as an alternator having values of Σ as ordinates and values of load $\sqrt{3}AV_S$ as abscissæ.

Inferences.—Clearly state all that can be deduced from the results of the test.

Note.—If the wattmeters W_1 and W_2 (which are not really essential to this test, and only useful, if available, for observing and comparing certain quantities) are omitted, the eight columns in the table, necessitated by their use, can also be omitted.

Determination of the Distribution of Potential round the Commutator of a Dynamo.

General Remarks.—On considering the action which occurs with a single turn of wire on a coreless armature as it rotates at

a uniform rate through one revolution we find that, starting from a position 0°, which may be termed the zero position, when its plane is perpendicular to the direction of the lines of force due to the fixed field *NS* (Fig. 73), its E.M.F. is 0, because it is slipping through and not cutting these lines. When it gets to 90°, the rate at which it cuts the lines is a maximum, and this decreases round to 180° again, when the E.M.F. is 0, and after then the effect is simply repeated. The zero position *nn* is the neutral axis or diameter of commutation for *no current* in the coil or armature, while *RR* is the line of resultant magnetization at right angles to

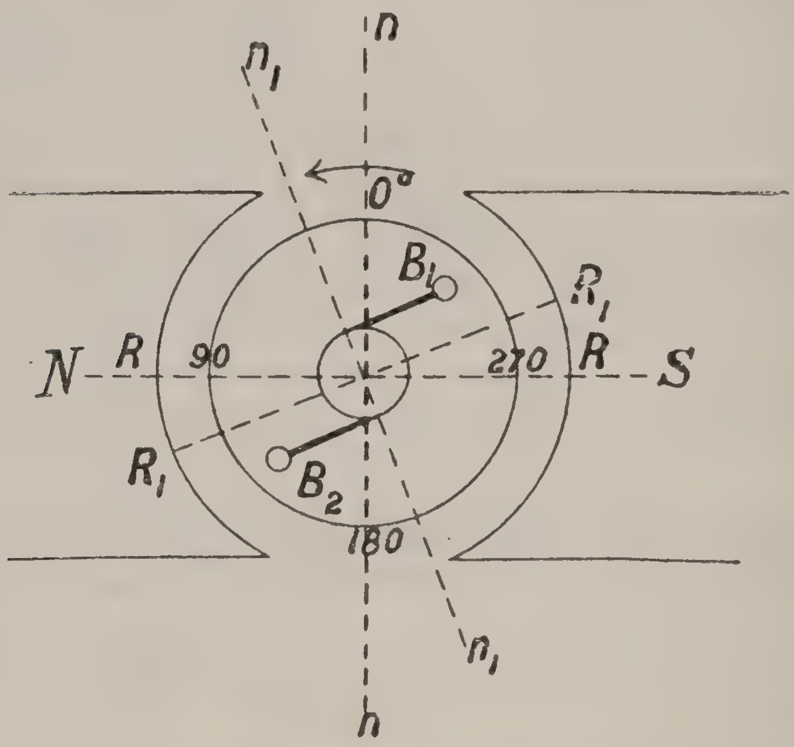


FIG. 73.

nn. In fact, the E.M.F. generated in the coil at any position is approximately ∝ sine of the angle of rotation from *nn*, and, as we have seen, is zero at 0° and 180°, and a maximum at 90° and 270°. If the coil be wound on an iron core, carries a current, and is made to rotate, it will react on the fixed field *NS*, causing a distortion of this latter, so that n_1n_1 will now be the neutral axis or diameter of commutation and R_1R_1 the line of resultant magnetization. In other words, the resultant field produced by that due to the armature and field *NS* will be forced round through an angle ROR_1 in the direction of motion, and will cause the brushes to advance through an equal angle non_1 to the

position $n_1 n_1$, which angle is called the "*angle of Lead*" of the brushes.

If now, the circular path of the coil, which we will assume for the moment not wound on an iron core, is developed out into a straight line *AC*, and the sine of the angular position from 0 (*i.e.* *A*), Fig. 74 I., plotted on the ordinates at each such position, the curve *APBQC* will be obtained, showing the variation of E.M.F. with angular position in one revolution. Thus *A* and *C* correspond to 0° or position *nn* (Fig. 73) when the

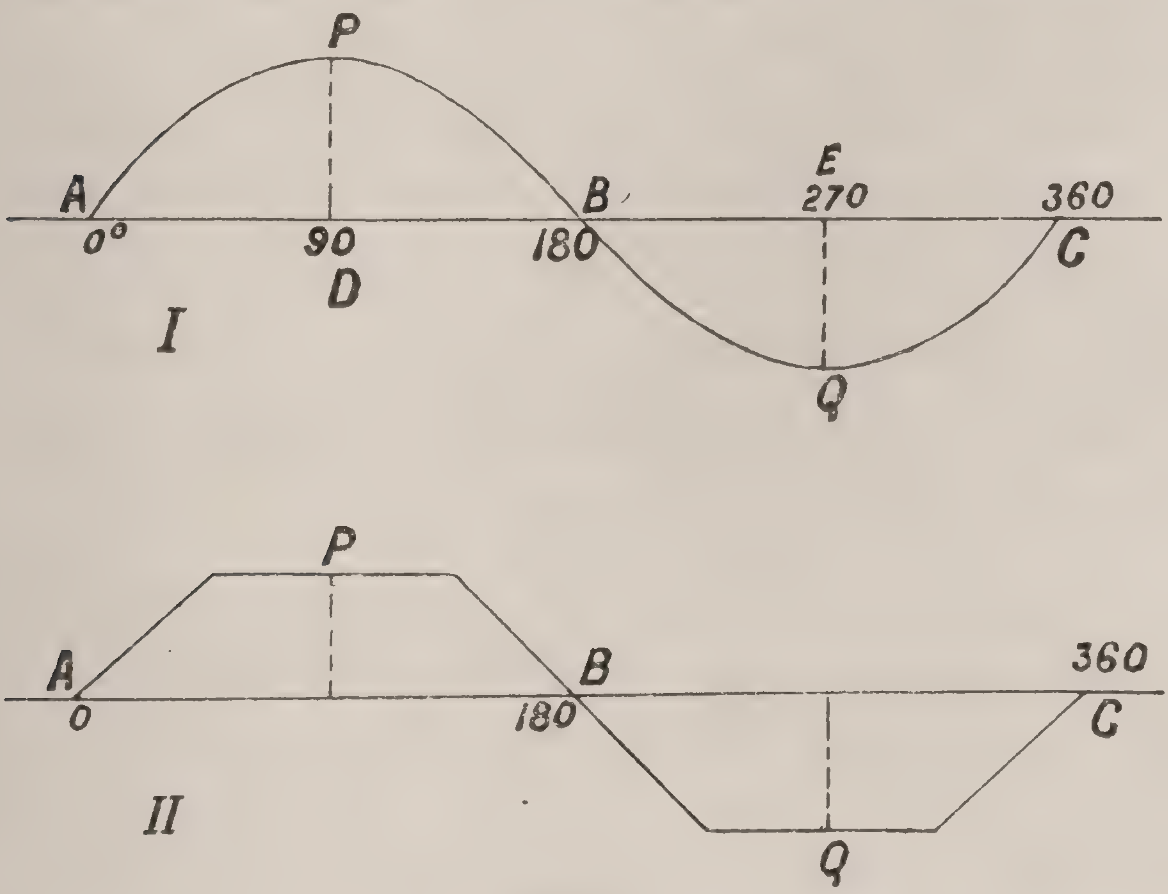


FIG. 74.

E.M.F. is nought, while *PD* and *QE* correspond to 90° and 270°, or position *RR* when E.M.F. = maximum. This curve is called a sine curve, and it possesses the uniform shape shown in Fig. 74 I.

Now in an ordinary iron core armature the E.M.F. of each coil fluctuates in a manner similar to that shown in Fig. 74 I. in a bipolar machine, and to that shown in Fig. 74 II. in a multipolar machine, but the commutator commutes such E.M.F.s so as to develop an E.M.F. at the brushes perfectly continuous in direction.

Considering the armature as a whole, the line *nn* or $n_1 n_1$, *i.e.* the brushes of the machine divide the armature coils into two

halves, which are in parallel with one another, now each half consists of separate coils in series with one another, each giving a certain but different E.M.F. depending on their position relatively to 0° or *nn*. These E.M.F.s being in series are added together in each half and the two summational E.M.F.s put in parallel. Thus the E.M.F. between the brushes = sum of E.M.F.s round one half of armature between those brushes. Consequently, as we proceed from, say, the negative main brush, the E.M.F. (if we could sample it) round either half increases up to the other main brush, first slowly, then rapidly, and finally slowly again when nearing the maximum point.

From the preceding remarks it will be evident that two investigations can be made on the E.M.F. of armature coils—(*a*) that of any one coil in different positions of a revolution; (*b*) the way in which the E.M.F. varies as we proceed from one brush right round the armature.

There are many methods of performing these investigations, and amongst those most easy of application in practice may be mentioned Prof. S. P. Thompson's, Mr. Mordey's, and Mr. Swinburne's, and these we will now consider in detail.

In Thompson's method of operating investigation (*a*) above, the E.M.F. of a single section on the armature can be sampled at different points in the revolution. The arrangement consists of two flat metal strips or brushes $b_1 b_2$ (Fig. 75) fixed to a piece of wood at a distance apart equal to the width between two consecutive commutator bars. A voltmeter is connected across $b_1 b_2$, which therefore measures the E.M.F. of a single section of the armature winding which is passing through the particular position of the field, corresponding to the position of the contacts. It is preferable that the compound brush $b_1 b_2$ should be mounted on a brush rocker capable of swivelling round the shaft over a degree divided scale, so that angular distances from some starting-point may be accurately obtained.

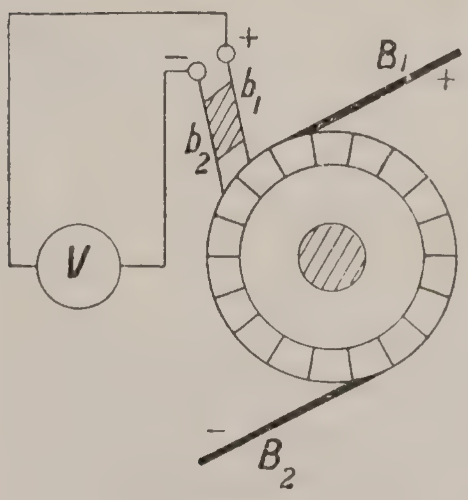


Fig. 75.

The method has the advantage that only a comparatively short range accurate reading voltmeter is needed, say, to about ten volts or so in the case of a 100-volt machine. The main brushes B_1B_2 must be arranged to allow b_1b_2 to pass them on the commutator.

The readings of the voltmeter will be different according to whether the machine is giving no current at all or its full-load current. If the machine is shunt wound it may in the former case be self-exciting, as the shunt current will be so small compared with the load current as to not affect the distribution round the commutator.

If now the readings on V are plotted on the ordinates of a curve with the corresponding angular positions right round the commutator on the abscissæ, the curve will not only show the variation of E.M.F. of the coil, but will show also the distribution of the magnetic field in the air gaps, the best position for the brushes and the "*angle of Lead*" which must be given to these when running on full load due to the shifting round of the resultant magnetic field RR (Fig. 73) to R_1R_1.

(75) Determination of the Distribution of Potential round the Commutator of a Dynamo. (Mordey's Method.)

Introduction.—The following is a convenient and simple method of finding the above-named distribution, and consists in measuring the potential between one of the main brushes and a single movable or *Pilot* brush capable of swivelling right round the commutator. It is then found that the potential increases or decreases from that main brush (according to whether it is the negative or positive one) round each half of the armature to the other brush, and that the variation is *regular* in a well-designed, but *irregular* in a badly-designed machine.

To represent the resulting variation or distribution graphically Prof. S. P. Thompson proposes drawing a circle OBC (Figs. 76 and 77) to represent the commutator and divide it into, say, 36 equal parts of 10° each, set off radially outwards from the circle,

lines ∝ potentials at the various angular positions of the pilot brush, thus getting the outer or potential curve *OAD*.

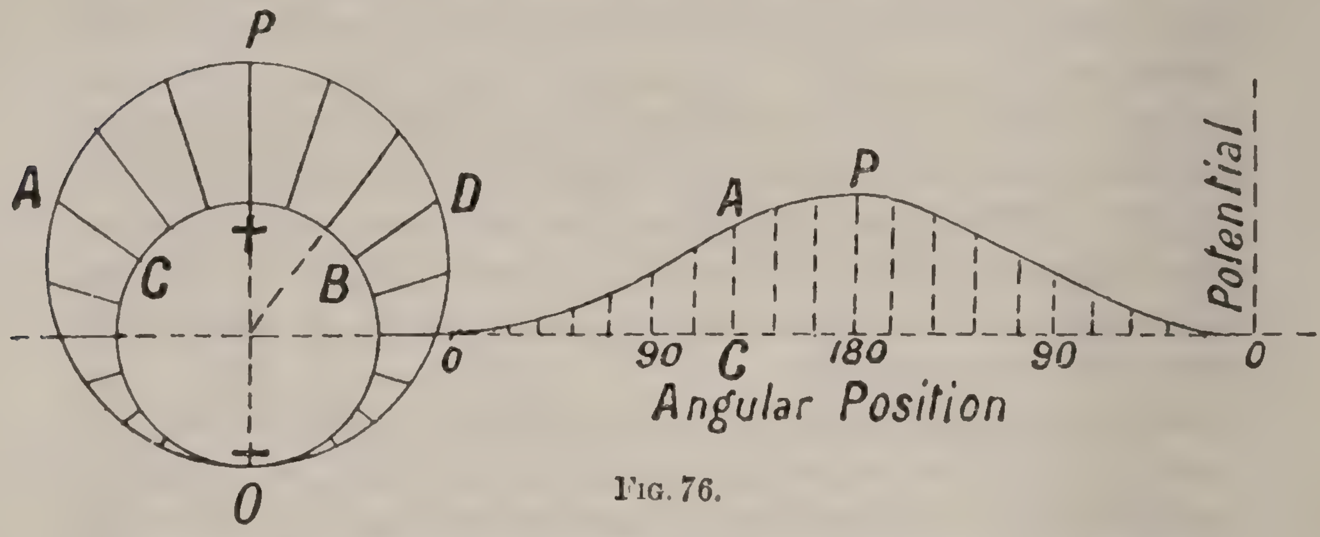


FIG. 76.

Next obtain the developed diagrams to the right of Figs. 76 and 77 by laying off a horizontal base to represent the length of the circumference of the circle *OBC*, then at the proper points along this angular line set up the radial lines from the left-hand Figure, due regard being paid to sign.

Fig. 76 is the result obtained with a well-arranged dynamo, and Fig. 77 with a badly-arranged one.

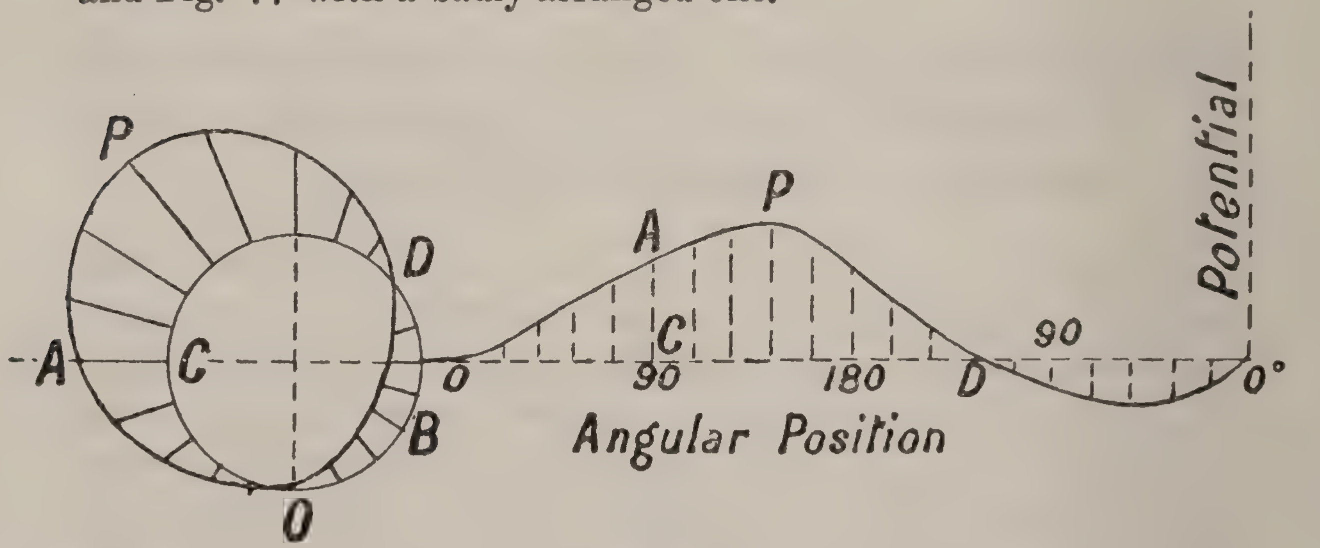


FIG. 77.

These curves show us several points, as, for example, the steepness of the curves in the right-hand diagrams enable an idea of the relative activity or idleness of the coils in these positions to be obtained, also the position of the brushes to give the best result and the distribution of field in the air gaps.

Fig. 77 may result from a machine in which the pole pieces are badly shaped, or the brushes badly placed.

Apparatus.—Dynamo to be tested, fitted with a third brush or pilot brush P capable of swivelling round the whole circle divided into degrees, and of making contact on the commutator at any position ; a rather long range accurate reading voltmeter V, and arrangements for taking a load from the machine when required.

Observations.—(1) Calling the two main brushes B_1 and B_2 and the pilot brush P, connect V between the negative main brush and P, the dynamo being shunt wound and excited off its main brushes B_1B_2.

(2) Run the machine on *open external circuit* at normal speed, and adjust P in line with the negative brush, which latter has been previously adjusted to give no sparking.

(3) Note the reading on V and the degree scale of P, and repeat every 10° right round the commutator *at constant speed.*

(4) Repeat 2 and 3 for a full-load current taken from B_1B_2, adjusting the speed (constant) to give the same voltage as before.

(5) Repeat obs. 1–4 with the same machine run as a motor.

(6) Tabulate your results in a convenient form, and plot a pair of curves for each test in the way indicated above.

Inferences.—State very clearly what you can deduce from your curves of distribution, and indicate in the developed diagram the positions of the poles, brushes, and resultant magnetic field of the machine.

(76) Determination of the Distribution of Potential round the Commutator of a Dynamo. (Mordey-Swinburne's Method.)

This is a neat modification of the preceding method, and consists in connecting a high resistance wire V_1V_2 across the main brushes B_1B_2, and finding by means of a sensitive detecting galvanometer G a position C along the resistance V_1V_2 such that G does not deflect. The point C is then at the same potential as P; hence since V_1V_2 is fixed, the distance V_1C or V_2C gives the relative potentials for various positions of P. The potentio-

meter V_1CV_2 can be easily calibrated by taking one single reading of the volts (V) across B_1B_2, whence the distances V_1C, for instance, in volts $= \frac{V_1C}{V_1V_2}$ of V where V_1C and V_1V_2 are in ohms, say, or some known units.

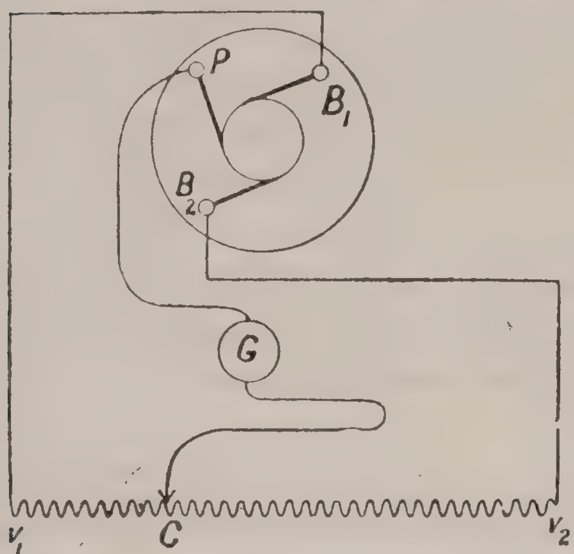


FIG. 78.

The speed must be constant throughout the test, so as to maintain V constant. Being a zero method it is very accurate, and has the advantage of not requiring a voltmeter which has to be equally accurate over its whole range, but only at one point.

(77) Analysis of the Total Internal Loss of Power in Direct Current Dynamos and Motors.

Introduction.—In test No. 82, p. 220, it is pointed out that the total internal loss of power in a direct current dynamo or motor is made up as follows—

(i) *Copper Loss* occurring in the armature and field coils, caused by heating due to the passage of the current.

This is at once easily calculable from the relations there given for finding the copper loss in either series, shunt or compound machines, when the resistances of the several coils, and the respective currents which each carries, is known. The loss in each circuit varies as the square of the current.

(ii) *Mechanical Friction* due to air churning or resistance, brush and bearing friction, each of which varies as the speed simply.

(iii) *Eddy Current* or *Foucault Current* loss occurring in the armature core, and also in the armature conductors, and varying as the square of the speed for the same excitation, since the eddy currents will be directly $\propto$ speed at constant excitation while the watts used in producing them will vary as the square of these currents, or if W_E = Watts wasted in eddy currents

and n = speed in revs. per min., then loss from this cause is $W_E \propto n^2 K_E$, where K_E is a coefficient depending on the eddy loss.

(iv) *Magnetic Hysteresis* in the core due to reversals of magnetization in it as it rotates and $\propto$ to its speed. If W_H = the loss from this cause and K_H its co-efficient, then $W_H \propto nK_H$; hence the total iron loss $W_I = W_H + W_E = nK_H + n^2 K_E$.

This equation has been made use of in several methods for separating these losses. Thus in Mr. Mordey's method, which is applicable to determining the losses in an *unwound* armature core as well as a wound one, the armature to be tested is driven, when in position between its own field poles, at different speeds (n), with its field (a) unexcited, (b) excited to a constant degree, (c) excited to various degrees, by an electromotor, and the power so required measured by a dynamometer or by knowing the efficiency of the motor accurately.

On plotting a curve between the speed (n) and the powers W required to drive at different speeds in a constant field, the constants K_H and K_E can be found from it.

Mr. Kapp's method is a slight modification of the preceding, and is only applicable to a ready-wound armature core. It consists in measuring the power W required to run the armature to be tested at different speeds in a constant field N, by running the armature itself as a motor "light," and noting the corresponding voltage V and current A taken at each speed (n).

If then T_a = total number of armature turns all round we have the fundamental relation $V = T_a N n\, 10^{-8}$.

But $W = AV = AT_a Nn\, 10^{-8} = nK_H + n^2 K_E$

$$\therefore A = \frac{K_H}{T_a N 10^{-8}} + n\frac{K_E}{T_a N 10^{-8}} = (a \text{ constant} + n \times \text{ a factor}).$$

On plotting therefore the curve between A and n to the axes OY and OS with (n) along OS, we shall obtain the straight line PQ. The ordinate OP is $\propto$ current required to overcome friction and hysteresis, while tan. $\theta \propto$ the eddy current effect. If OP is plotted to a scale of current, then $K_H = OP, NT_a\, 10^{-8}$, when $F = \dfrac{W - nK_H}{n^2}$ is also known.

We also have $\dfrac{OP}{QS} = \dfrac{\text{Hysteresis + Friction}}{\text{(Hysteresis + Friction) + Eddies}}.$

Thus the three separate factors or losses are each determined.

The following graphical method of separating the various losses is a simple and convenient one, and independent of any mathematical treatment. It is due to Mr. R. H. Housman, and is as follows—

Separately excite the field magnets to the normal amount and keep this constant. Note the current and speed of the armature when running light as a motor for different noted voltages applied to it. Plotting current, which $\propto$ torque with given field on the ordinates, and voltages which $\propto$ speed with given field on the abscissæ, or Joules per revolution on the ordinates and revs. per second on abscissæ, the straight line PQ (Fig. 79) is obtained cutting the current axis in P.

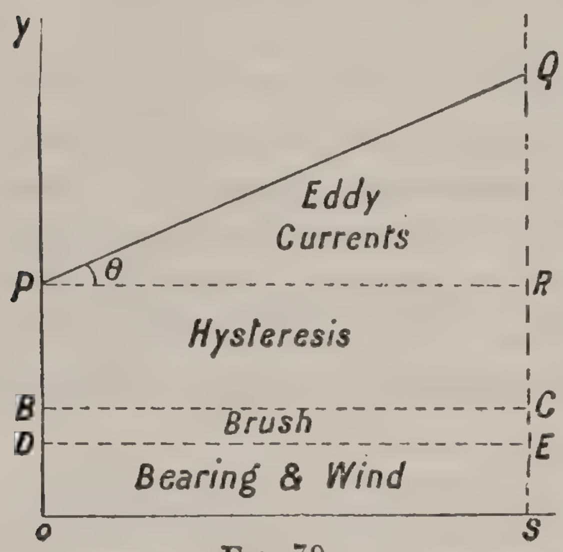


FIG. 79.

If Q is any point on PQ and QS is parallel to OP, then the total loss for that speed OS is given by $QS \times SO$. If PR is parallel to OS, then the area $PS \propto OS \propto$ power lost in hysteresis and friction together, and area $QR \times RP \propto OS^2 \propto$ power lost in eddy currents where $QP \propto RP \propto OS$. Repeating the above with a different excitation will give a second line $P'Q'$, usually parallel to PQ, showing that the eddy currents are constant for a given voltage.

It may be noticed that the total loss corresponding to any point such as Q on PQ = product of co-ordinates = $OS \times QS$, and not the area of the Fig. $POSQ$. In other words, the Fig. represents the nature of a dynamo Characteristic rather than the indicator diagram of a steam-engine.

To obtain the total mechanical friction losses, run the armature with brushes down, field disconnected and unexcited by a direct coupled motor, and note the increase of current required to drive over that needed for the motor alone. Plotting this current OB on the ordinates and drawing BC parallel to OS, the area $OC \propto$ total mechanical frictions, and $\therefore$ BR must be $\propto$ to the hysteresis loss alone. On noting this excess driving current with the brushes up, we get OD, and finally the area $OE \propto$ bearing and wind friction only. DC being $\propto$ the brush friction alone.

The total losses for a given voltage will be a minimum for a certain induction in the armature core, usually between 15,000 and 16,000 lines per square c.m. Since the hysteresis losses increase rapidly with increase of field, while the frictional losses increase with decrease of field due to the higher speed needed to obtain the same voltage.

For high inductions up to 18,000 or 20,000 the eddy currents cause the curve to bend upwards, and also the angle θ to be greater. This is probably due to the eddies generated by the stray leakage field through the shaft, etc. If the line PQ bends, it shows that the eddy-current losses are producing perceptible demagnetization on the field. Since both the eddy and hysteresis losses increase with armature current, these losses should really be measured with full-load armature current flowing by using the method of Fig. 86, which with careful adjustment of excitation will give considerable range of speed for constant armature current.

This question of the separation of the various losses is of great importance to the dynamo maker, enabling him to see in what way a machine is faulty, *i. e.* whether the eddy-current loss is excessive due to insufficient lamination, or the hysteresis too great due to too hard or inferior quality of iron. We will now consider a complete experimental analysis in detail.

Apparatus.—Exactly the same as that prescribed for test 95, and in addition an auxiliary motor should be available for coupling *direct* to the machine to be tested.

Observations.—(1) Carry out observations 1–3, test 95.

(2) Repeat 1 for an excitation 25% above and 50% below the normal.

(3) Disconnect all apparatus from the machine tested, and also the field from the armature. Connect the instruments up with the auxiliary motor, so as to measure the power taken to drive it. Demagnetize the field magnets of the motor to be tested by sending round the field coils a gradually diminishing (to 0) alternating current.

(4) Measure the voltage and current needed to run the auxiliary motor at some ten different recorded speeds between 0 and the maximum allowable.

(5) Direct couple the auxiliary motor to the armature tested, and with the brushes down, note the new power given to the auxiliary to drive the two machines at some ten different speeds, the field of the machine under test being entirely disconnected and unexcited.

(6) Raise the brushes and repeat 5, tabulating all your results as follows—

NAME . . . DATE . . .

Motor tested: No. . . .	Resistances: Armature	= . . .	Ohms @	°C.			Normal Voltage	= . . .
Maker . . .	„ Shunt	= . . .	„	„	„	„	Amps.	= . . .
Type . . .	„ Series	= . . .	„	„	„	„	Speed	= . . .
	Total copper losses	= . . .						

Speed in Revs. per		Tested Motor Self-driven (light).				Auxiliary Motor alone.			Motors coupled Brushes down.			Motors coupled Brushes up.			Friction losses.		
Min.	Sec.	Excitation.	Volts V_a.	Amps. A.	Watts (Total) AV_a.	Volts V_1.	Amps. C_1.	$C_1V_1 = W_1$.	V_2.	C_2.	$C_2V_2 = W_2$.	V_3.	C_3.	$V_3C_3 = W_3$.	Total $W_2 - W_1$.	Bearing and wind. $W_3 - W_1$.	Brush only $W_2 - W_3$.

(7) Plot all your results to the same pair of axes, having in each case the speed in revolutions per second on the abscissæ and the power in Watts required to be given to the shaft of the dynamo under test to produce those speeds under the various conditions mentioned in observations 1–6 on the ordinates.

(8) Calculate the various losses at normal speed as a percentage of the total loss in the whole machine at full load.

Inferences.—State very clearly all that can be inferred from your experimental results.

Note.—A variation of the preceding method for measuring the hysteresis and eddy current losses consists in measuring the watts absorbed by the armature in running the machine as a motor light at a series of excitations between 0 and the normal, the *speed being kept constant at normal value* by adjusting the volts on the armature by means of a main circuit rheostat in series with it.

Plotting a curve with armature watts as ordinates, and excitation as abscissæ, we find its lower portion to be nearly straight, and this part produced to cut the ordinates will give the watts which would be absorbed at zero excitation. Thus the differ-

ence between the watts at any given excitation, and at this zero value, will be the power lost in hysteresis, eddies, and mechanical frictions.

Again, if the curves are plotted between Joules per revolution as ordinates, and revs. per sec. as abscissæ, the friction line *BC* separating frictional and electro-magnetic losses has a fixed position in the diagram; whereas, with the axes denoting current and volts, a different friction line has to be drawn for each excitation, thus making it difficult to see what proportion of the whole loss is electrical and what frictional, when more than one set of curves corresponding with different excitation is drawn on the same curve-sheet.

(78) Measurement of the Coefficient of Magnetic Leakage "ν," and of the Relative Distribution of the Waste Field of Dynamos and Motors. (Ballistic Method.)

Introduction.—The present test has a most important bearing on the design of the magnetic circuit of a dynamo or motor, for since only a fraction of the total number of lines of magnetic force, generated by the field magnets, are usefully employed in cutting the armature conductors and so generating the requisite E.M.F., the results of the test enable the designer to allow for this discrepancy, providing he knows the coefficient of magnetic leakage "ν" for the particular form and type of machine in question.

In addition to this, the relative distribution of the waste field around the machine enables defects in the design of the magnetic circuit to be seen and corrected, for at the best the magnetic circuit of a dynamo or motor is very imperfect.

It should be remembered that leakage of magnetic lines of force will take place across any two points between which there is a difference of magnetic potential, the magnitude of which leakage will depend directly on this potential difference, and inversely on the magnetic resistance of the path.

The following is a convenient method of measuring or comparing the relative amounts of leakage in different parts of a dynamo,

and therefore the *static* leakage coefficient ν for the machine; the term *static* being here used to denote the value of ν obtained when the armature is at rest, for it is well known that an armature delivering current exerts a demagnetizing action on the field which directly promotes leakage. Assuming the normal excitation constant, the leakage will increase with the output, and it will largely depend on the degree of saturation of the iron and on the relative magnetic reluctances of the various parts. The method depends on the measurement of induced currents produced by moving either (1) an exploring coil so as to cut the field to be tested, or (2) the field in such a way as to cut the coil, the latter method being here adopted. Either the relative or absolute numerical values of the stray and useful flux in the various parts can be found, the relative values being obtained with reference to that part in which the flux is a maximum which can be taken as unity. Knowing these, the absolute values can be obtained by running the armature at a known speed and measuring the E.M.F. without allowing it to develop current and thereby distort the field. The useful armature flux can now be at once calculated, and from it, that in each of the various parts, or thus:—suppose we have a circuit consisting of a ballistic galvanometer, resistance box, earth inductor of N_1 turns, mean area A_1 square c.ms. in series with an exploring coil of N_2 turns, mean area A_2 square c.ms. wound round the magnetic field to be tested. If now the inductor, with its plane vertical or horizontal, is rotated rapidly through 180°, cutting the earth's field of strength F_1, then the total quantity of electricity set up in the transient current is $Q_1 = \dfrac{2N_1A_1F_1}{R_1} = K \sin. \tfrac{1}{2}\,\theta_1{}^\circ$ where K = ballistic *constant*, R_1 = total circuit resistance, $\theta_1{}^\circ$ = angular throw in degrees. If the exploring coil is now made to cut the field to be tested of strength F_2 by suddenly making, breaking, or reversing the exciting current, we get $Q_2 = \dfrac{N_2A_2F_2}{R_2} = K \sin. \tfrac{1}{2}\,\theta_2{}^\circ$ where θ_2 and R_2 have the same meaning as before. $\therefore$ Dividing we get $F_2 = F_1 \dfrac{2N_1A_1R_2}{N_2A_2R_1} \times \dfrac{d_2}{d_1}$ lines per square c.m. in the loop or search coil (in absolute measure) where d_1 and d_2 = scale deflections corresponding to θ_1 and θ_2.

As, however, it is the total field (A_2F_2) which we really desire to obtain, and denoting this by F_T

$$\text{we have } F_T = F_1 \frac{2N_1A_1R_2}{N_2R_1} \times \frac{d_2}{d_1} \text{ lines.}$$

Apparatus.—Earth inductor E; resistance box R; charge and short circuit key K; ballistic mirror galvanometer G (p. 569), having a small log decrement and periodic time about 8 or 10 seconds, so that this may be large compared with the time of flow of Q_1 and Q_2 which can therefore pass through the coil before it begins to move. A shunt wound dynamo to be tested; ammeter A; rheostat (r) (p. 599); quick break switch S; and source of current B.

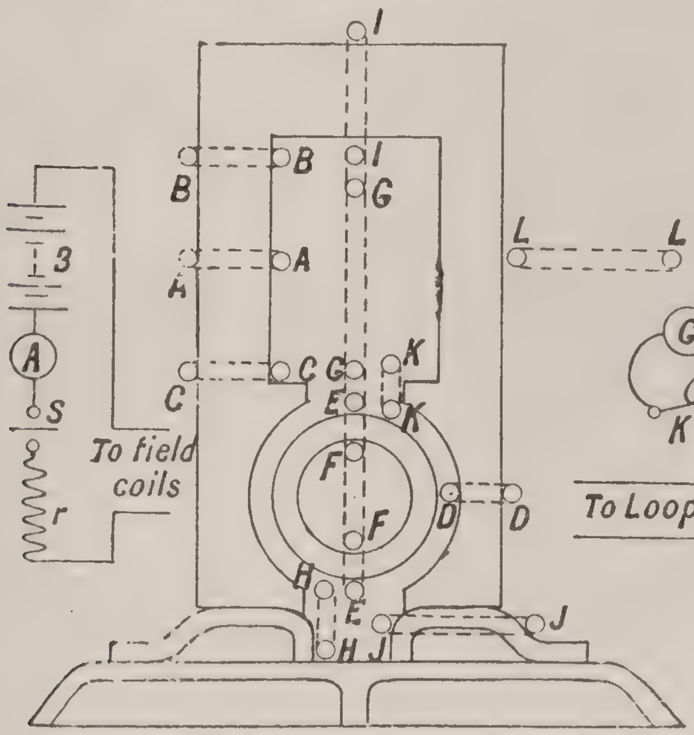


FIG. 80.

Observations.—(1) Adjust the needle of G to zero, and wind a single complete turn of wire on the dynamo at position A, connecting it up with the other apparatus as indicated in Fig. 80. The F.M. coils must be disconnected and excited separately from B.

(2) Close S and adjust (r), so as to get normal excitation through the F.M. coils.

(3) Close K, open S, and adjust R by trial so as to get a convenient throw on G, then note its value (D_1) on breaking, and (D_2) on making circuit at S, the excitation being that in 2. Repeat this twice and take the mean of each, calling it (d_2).

(4) Repeat 1–3 for each of the positions of the exploring loop indicated by the letters B, C, D, E, F, G, H, I, J, K, and L, respectively.

(5) Repeat 4 for excitations 50% higher and 50% lower than the normal, and in each case calculate ν from the formula,

$$\nu = \frac{\text{Total Field}}{\text{Useful Field}}$$

(6) Let down the brushes and run the machine at a known speed, measuring the E.M.F. E at each of the three excitations used, and tabulate as follows—

NAME . . DATE . . .

N_1 = . . . turns
A_1 = . . . sq. c.ms.
Galv. resistance g = . . . ohms.
Inductor resistance r_e = . . . ohms.
Total resistance R_1 = . . . ohms.

F_1 = . . . C.G.S. units.
d_1 = . . . Scale divs.
Total No. armature conductors C = . . .
speed = . . . revs. per min.
= . . . revs. per sec. (n).

Position of loop.	Turns on loop. N_2.	Area sq. c.ms. A_2.	Box resistance. R.	Total resistance. R_2.	Exciting current amps. A.	Throw on. Breaking. D_1.	Throw on. Making. D_2.	Mean throw at each position $d_2 = \frac{1}{2}(D_1 + D_2)$.	Armature E.M.F. E.	Useful Flux in Armature. $N = \frac{10^8 E}{C.n.}$	Flux at each position. $= N\frac{d_2}{d_{EE}}$	Flux F_T.	ν.

Inferences.—State clearly all the inferences which can be drawn from the results of the above experiments, and point out their bearing on the design of field magnets for dynamos and motors.

(79) Magnetic Characteristic of a Dynamo with varying Air Gaps.

Introduction.—It is of considerable importance, especially in the design of dynamos, to know the effect which the length of air gap, between the field magnet (F.M.) pole faces and armature core has on the excitation required to force a given number of magnetic lines of force through the core. For convenience the curve showing the relation between the amp.-turns (A.T.) or magneto-motive force (M.M.F.) which $= \frac{4\pi}{10} \times$ A.T. in the F.M.s and the total useful flux of lines (N) through the armature will be called the *Magnetic Characteristic* for the air gap used. The flux (N) can be found in two ways: (1) by using a ballistic galvanometer in series with a "search coil" temporarily wound on the armature and noting the throws produced on the galvanometer by making, breaking, or reversing known currents in the F.M. coils; (2) by running the armature mechanically and noting its E.M.F. speed, and number of conductors round periphery, N being then calculated from the fundamental formula $E = NnC \div 10^8$. This is the best and more practical method to employ, because the armature will now exert a slight demagnetizing action on the F.M.s tending to increase leakage and approximate more nearly to actual working conditions. The Exp. is divided into three distinct parts, viz. the determination of the relation between—

(α) The M.M.F. and flux (N) through armature with constant air gap.

(β) The air gap and flux (N) through armature with constant M.M.F.

(γ) The air gap and M.M.F. through armature with constant flux (N) in armature.

Apparatus.—The dynamo D capable of being driven mechanically; tachometer; voltmeter V; ammeter a; switch S; rheostat r (p. 599); supply of electricity.

The machine D to be tested must be specially constructed in order to be able to operate this test. As shown in Fig. 81, the pole pieces are each capable of being made to approach or recede from the armature by turning a massive screw bolt b fitted to

each, by means of a suitable key. The distance apart of the pole tips can be read off on a scale C fixed to the body of the machine.

Note.—The *pole tips* must *never* be *closer together than the two zero scale divisions* which will be termed their *normal* position in what follows, and must always be left at this distance after the test is over. *To increase this distance turn the screw clock-wise.* It will be noticed that the initial slopes of the curves in (α) are determined by the air gap, also that the air gap causes the curve to bend over.

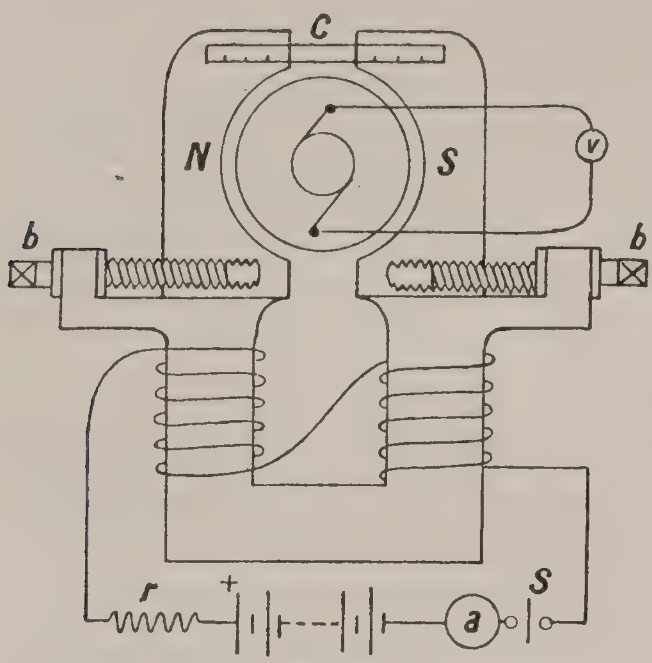


FIG. 81.

All lubricators must feed properly before the machinery is started.

Observations. — α — (1) Connect up as shown in Fig. 81, and adjust the pointers of all the instruments to zero.

(2) Set the pole tips at exactly the normal distance apart and adjust the speed so that with the maximum excitation allowable in the F.M. coils 25% above normal, the E.M.F. can be read off on v.

(3) With air gap and speed constant, adjust the excitation to about $\frac{1}{8}$ of the maximum allowable. Note this reading A and that on (v) viz. E.

(4) Repeat 3 for about eight ascending equal increments of current to about 25% above the normal excitation.

(5) Repeat 3 and 4 for the pole tips half-way and the farthest apart.

(6) Repeat 3–5 for the same current values descending.

(7) Plot curves in each case with M.M.F. as abscissæ and N as ordinates.

β—(1) Adjust the exciting current to the normal value and the speed so that the E.M.F. can be read off on v.

(2) With M.M.F. (*i.e.* A) and speed constant and the pole tips at exactly the normal distance apart, note the reading (E) on v.

(3) Repeat 2 for eight different distances increasing by $\frac{1}{8}$ at a time to the maximum possible.

(4) Repeat 2 and 3 for a return set of distances to the minimum (normal).

(5) Plot curves in each case with distances between iron of armature and pole face as abscissæ and N as ordinates.

γ—(1) Adjust the excitation to $\frac{1}{10}$ maximum and the speed so that a suitable low reading of, say, $\frac{1}{4}$ maximum voltage is obtained on v.

(2) With N (*i.e.* E) and speed constant and the pole tips at exactly the normal distance apart, note this distance (d) and the exciting current A.

(3) Repeat 2 for eight values of (d) rising by $\frac{1}{8}$ of the maximum at a time to the maximum, noting A, at each position, which is necessary to keep E constant.

(4) Repeat 2 and 3 for a return set of distances to the minimum (normal).

(5) Plot curves in each case with M.M.F. as abscissæ and (d) as ordinates.

NAME . . . DATE . . .

No. Armature conductors C = . . External diam. iron core = . . . inches.
Total F.M. turns (T) = . . . Internal „ „ „ = . . . „
Nett length „ „ = . . . „

Speed in Revs.		Distance between pole tips (d).	Distance between iron of face to core.	Exciting Current, A amps.		E.M.F. on (v) E.		M.M.F. $=\frac{4\pi}{10}$ A.T	Flux N. $=\frac{10^8 E}{C.n.}$
Per Min.	Per Sec. (n).			Increasing.	Decreasing.	Increasing.	Decreasing.		

Deductions.—State very clearly all the inferences which you can draw from your results and point out their bearing on dynamo design.

(80) Localization of Faults in Magnetizing Coils. (Induction-Ballistic Method.)

Introduction.—When a magnetizing coil of insulated wire is wound on a metallic bobbin, the latter is usually insulated on the inside by a thin strata of insulating material before winding on

the covered wire. Notwithstanding this, it may and does sometimes happen that the wire core becomes "shorted" to the metal-work of the bobbin, through the covering and insulation of the bobbin. This is particularly liable to be the case in shunt coils of dynamos which are wound on metal "formers," insulated with vulcanized fibre tissue before winding.

Such a fault, through poor contact of, in many cases, a very uncertain nature, gives trouble in the ordinary methods of testing for its position, by giving unsteady readings. Thus the ordinary resistance methods are extremely liable to be vitiated by variable contact resistance at the fault. The following method for localizing the position of the fault by means of *induced currents*, measured ballistically, is often a more convenient and reliable one for the purpose.

Apparatus.—Metallic bobbin or former F to be tested, wound with the magnetizing coil (m) which is "*shorted*" to frame at the point (f); high resistance ballistic galvanometer G; two-way key K (p. 587); battery of secondary cells B; switch S; ammeter A, and temporary primary magnetizing coil PP wound over the outside of the magnetizing coil mm proper, which is to be tested; rheostat R (p. 606); known high resistance box r.

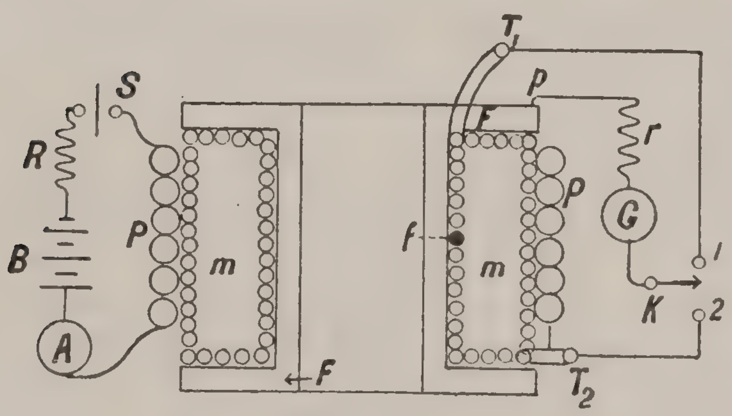


Fig. 82.

N.B.—It will be noticed that, as represented in Fig. 82, the fault (f) is on the first layer of turns next to the frame F, and we will suppose that the turn at (f) is making contact there with the metallic frame (F). Thus it will be seen that the point (f) divides the total number of turns on the whole bobbin into two parts between the leading out wires T_1T_2 of the coil, so that total turns = turns between T_1 and f + turns between T_2 and f.

Observations.—(1) Connect up as in Fig. 82, and adjust A and G to zero, the temporary coil PP having been previously wound on and a wire soldered to *any* point (p) on the metallic bobbin frame F.

(2) With R full in, close S and adjust the current on A to some convenient amount. Next also close K to stud 1 and adjust r to such a value as will give, say, $\frac{1}{2}$ or $\frac{3}{4}$ scale deflection d_1 on G when S is opened suddenly. Repeat two or three times with the same *constant* current, both *made* and broken in P.

(3) Close K to stud 2 and repeat 2 above with the same current, noting the new resistance out in r to give a suitable first throw on G.

(4) Repeat 2 or 3 for about four or five current strengths A so as to obtain finally different throws on G which will check one another, and calculate the position of the fault (f), or the number of turns to be unwound, to reach it, from the relation

$$\frac{N_1}{N_2}=\frac{\text{turns between } T_1 \text{ and } f}{\text{turns between } T'_2 \text{ and } f}=\frac{\text{mean 1st throw } d_1}{\text{mean 1st throw } d_2}\times\frac{r_1}{r_2} \text{ approx.}$$

where $r_1 r_2$ are the *total* resistances of $r+G$ when obtaining d_1 and d_2 respectively, and which are assumed to be very large compared with the contact resistance at f and also the resistance of the turns between f and both T_1 and T_2. If the resistance of the coil (m) is from 5 to 20 ohms then $(r+G)$ should if possible be at least 10,000 ohms.

NAME . . . DATE . . .
Coil tested : Total Turns $N=$ . . . Total Resistance R_0.
Galvanometer Resistance $G=$ . . . Ohms at . . . ° C.

Current for references only.	1st throws on G.		Box resistance.		Circuit Resist.		Ratio N_1/N_2	Turns to unwind N_2.
	mean d_1.	mean d_2.	r	r'	$r+G$ $=(r_1)$.	$r'+G$ $=(r_2)$.		

N.B.—It will be noticed from the formula in 4 that if r is adjusted so that $d_1=d_2$, then

$$N_1/N_2=\frac{r_1}{r_2} \text{ or } N_2=\frac{r_2}{r_1+r_2}N,$$

or again if r is kept constant throughout,

$$\text{then } N_1/N_2=d_1/d_2 \text{ or } N_2=\frac{d_2}{d_1+d_2}N.$$

If G is insensitive an iron core may be inserted in the coil to form a closed circuit if possible; this will increase the flux for a given current made or broken in PP, and therefore also the first throws d_1 d_2 on G.

This has the further advantage that N_1 and N_2 will now enclose the *same* number of lines of force, which is only approximately true if there is no iron core and the coil long.

It should be observed in passing that even a simpler method still than the one described above, for finding the position of the fault (f), would be to employ a slide wire or meter bridge or other convenient form of potential divider in the following manner. Connect the ends T_1 T_2, Fig. 82, of the faulty field coil to the extremities of a meter bridge wire and also to two or three Leclanché cells; connect the galvanometer G, which need not now be ballistic, but which must be sensitive, between the metallic former at p and the slider key of the bridge wire. Now move the key such that on tapping it G does not deflect. Then the lengths $T_1 f$ and $f T_2$ of the faulty coil are in the proportion of the corresponding lengths of the stretched wire either side of the K, and are therefore known if the gauge of winding and its resistance (which can be measured in the ordinary way) are known.

(81) Determination of the Rise of Temperature and Increase in Resistance of Magnet-windings.

Introduction.—Since every magnet coil has some resistance, which is usually considerable in shunt or pressure coils but small in series or current coils, it follows from Joule's law that heat must be generated in them when excited. The amount of heat developed per second by a current of (I) amperes flowing through, or a pressure of (V) volts across the terminals of, a coil of R ohms resistance is $\propto I^2R$ or $\frac{V^2}{R}$. Any coil must therefore have such an external surface for radiation of heat relatively to the amount of heat developed in it, that the "steady" temperature attained when the rates of production and dissipation of heat become equal is not high enough to deteriorate the insulation of the winding. The maximum limit to this "steady" final temperature is usually fixed at about 50° C., for it is found that the commoner insulating materials used generally begin to deteriorate with temperatures exceeding 60 to 70° C.

Admiralty specifications, however, prescribe that after a six-hours' run at full load, no accessible part of a machine may show a temperature of more than 70° F (= 38°·8 C) above the surrounding air. This would seem unnecessarily low, but from remarks to follow may not actually be so.

In the case of dynamos and motors the rise of temperature and its final steady value is required for the armature, series or shunt coils, commutator, bearings and frame. Further, it has been shown that the radiating facility of a surface in contact with iron is nearly twice as good as when it is exposed to air.

Except in special measurements and research, when perhaps thermo-couples and their equivalents may be used, the temperature rise of coils while energized is always obtained either (1) by thermometer, the bulb of which is placed on the coil and covered with a pad of cotton wool, or (2) by resistance measurement, obtained from the readings of an ammeter in series with, or a voltmeter across, the coil and the application of Ohm's law. This latter method is the one usually employed in a test room, is the most accurate of the two, and the quickest method of finding the "true mean rise" of temperature, especially with series coils. With shunt coils this resistance method can be effected by switching the supply off and then quickly measuring the resistance of the coil by the Wheatstone Bridge method. Usually the true mean rise of temperature by resistance tests is found to be at least 1·4 to 1·6 times greater than the apparent mean rise by thermometer due to the temperature of the layers of winding increasing from the outer one to that situated about three-fourths of the thickness of coil from it, and then decreasing again to the inner layer next to the iron core.

If R_c = the resistance of the coil cold, and R_h that when hot,

then $$R_h = R_c(1 + \alpha(t_h - t_c)) \text{ approximately,}$$

or $$\frac{R_h}{R_c} = \frac{1 \times \alpha t_h}{1 + \alpha'_c} \text{ more accurately,}$$

where t_c and t_h = the temperature in deg. cent. of the coil, cold and hot respectively, and α = the temperature coefficient of the material which for copper = 0·00428 ohm per ohm per 1° C.

$$= \tfrac{5}{9} \times 0{\cdot}00428 = 0{\cdot}00238 \text{ per °F.}$$

$\therefore$ the rise of temp. $= (t_h - t_c) = \dfrac{R_h - R_c}{a.R_c}$

$$= 233\frac{R_h - R_c}{R_c} \text{ deg. cent.} = 420\frac{R_h - R_c}{R_c} \text{ deg. Fahr.}$$

If now T = final temp. rise above surrounding air,
S = total heat radiating surface in □″ (exclusive of end flanges and internal surface, if any),
W = total watts wasted in the coil at full load = total I^2R,

then $T \propto W \propto \frac{1}{S}$ or $T = \dfrac{W}{S} \times K$,

where (K) = a heating constant depending on the depth of winding, amount of fanning by the armature, and whether the surrounding air is still or circulating, and may be taken as 75 for the usual shape and size of field coils of dynamos and motors, especially of multipolar types, excepting when iron-clad.

Hence $$T = 75\frac{W}{S} \text{ deg. cent.}$$

and since for shunt bobbins $W = VI_{Sh} = I^2{}_{Sh}R_{Sh}$.

$\therefore$ for a prescribed temp. rise (T) we have

$$\text{max. shunt current } I_{Sh} = \sqrt{\frac{T.S.}{75R_{Sh}}} \text{ amperes.}$$

Apparatus.—Magnet coil F (of, say, a dynamo) to be tested; ammeter (a) and voltmeter (v) each capable of dealing with the full-rated current and voltage for the coil; switch S_1; watch; small bulb thermometer and cotton wool; adjustable high resistance r for shunt coils, or low resistance for series coils;

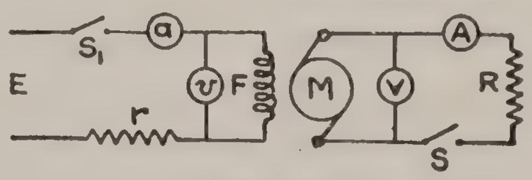


FIG. 83.

ammeter A, voltmeter V, switch S, and adjustable load resistance for the main circuit to the armature M. Separate means for driving M.

Observations.—(1) [*Armature M stationary*]. Connect up as shown on the left half of Fig. 83 and adjust a and v to zero, if necessary. Note the temperature of the air of the room by the

thermometer, secure the thermometer with its bulb *touching* the outside of the coil, and cover the bulb with a pad of cotton wool.

(2) With r full in, close S_1 and quickly adjust r so that v or a shows the normal value for the coil, and note the readings of both v and a, the thermometer and the time.

(3) By adjusting r maintain (a) constant in testing a series coil, or (v) constant in testing a pressure coil, either at the above normal value, and note the readings of v, a, the thermometer and time, say every 10 minutes for the first $1\frac{1}{2}$ hours, and then every 15 or 20 minutes, up to the condition when the variable quantity *becomes constant.* Then, again, take the temperature of the room and tabulate as follows—

NAME . . . DATE . . .
Coil Tested :—Type . . . Thickness . . . External Surface $S =$ . . . □″
Temp. of Room at Start = . . . °C. at End of test = . . . °C.

Time of Reading.	Minutes from start.	Amps (a).	Volts (v).	Watts $W = a.v.$	Resistance of Coil $R_h = \frac{v}{a}$.	Calculated Temp. Rise $T = 233\,\frac{R_h - R_c}{R_c}$	Thermometer Reading t.	Whether Motor at Rest or how Running.

(4) [*Armature M driven at Full Load and at Normal Speed*].—Repeat obs. 1–3 after the machine has cooled down to the temperature of the air.

(5) Plot curves to the same axes having time in "*minutes from start*" as abscissæ with values of R_h, T, and t as ordinates; calculate the "heating constant" (K) from the relation $K = \frac{TS}{W}$, and the maximum value of shunt current suitable for coil tested for the value of (K) found, and for a final temperature rise T of 50° C. above air.

Inferences.—Clearly state all that can be deduced from the results of the test, and point out their bearing on temperature testing.

(82) Efficiency of Direct Current Dynamos. (Swinburne's Electrical Method.)

Introduction.—This method, due to Mr. James Swinburne, has the advantage, firstly, in point of accuracy, of being solely an electrical one, and therefore far more accurate than a dynamo-

meter method in which the power required to drive is measured mechanically; secondly, of not requiring another similar machine for coupling to it, in addition to the one tested. The method, which is often termed the "Stray Power" method, is consequently very suitable for employment in workshop determinations, where usually no good transmission dynamometer is available for measuring the H.P. used in driving the generator under test, and is invariably used when Hopkinson's method cannot be applied as, *e.g.*, when no second similar machine is available. The principle of the present and all similar methods is based on the following, namely, that the total power put in = total power given out + total power lost internally or in symbols

$$W_I = W_O + W_L$$

where the suffixes I, O and L denote the input, output, and total losses in Watts (W) respectively.

Thus the commercial efficiency (η) of the dynamo is at once obtainable from the relation—

$$\eta = \frac{W_O}{W_I} = \frac{W_O}{W_O + W_L}$$

The output in Watts W_O developed by the dynamo is at once deducible from the product of the volts V and amperes C given out. The total loss W_L in Watts occurring internally in any dynamo is made up as follows—

(*a*) Copper losses L_c in armature and exciting coils due to heating by the passage of current, and which can easily be calculated when the currents and resistances are known.

(*b*) Friction losses L_F due to air churning, journal and brush frictions.

(*c*) Magnetic frictions or iron losses L_m due to Eddy or Foucault currents and magnetic hysteresis. Hence the total internal loss $W_L = L_c + L_F + L_m$, and to the quantity $(L_F + L_m)$ Mr. Swinburne has given the somewhat appropriate name of "*Stray Power.*"

The copper losses are calculable as follows—

Let C = the current given by the dynamo at its normal voltage V to some external circuit, and let R_a R_{Se} R_{Sh} be the resistances of the armature series coils and shunt coils respectively of any dynamo to be tested, of which R_{Sh} can be measured by a Wheatstone Bridge and R_a R_{Se} by the "Potential Difference" method (p. 84). We shall then have for a

Series dynamo $L_c = C^2 (R_a + R_{Se})$

Shunt dynamo $L_c = \frac{V^2}{R_{Sh}} + \left(C + \frac{V}{R_{Sh}}\right)^2 R_a$

Compound dynamo (long shunt)

$$L_c = \frac{V^2}{R_{Sh}} + \left(C + \frac{V}{R_{Sh}}\right)^2 (R_a + R_{Se})$$

Compound dynamo (short shunt)

$$L_c = C^2 R_{Se} + \frac{(V - CR_{Se})^2}{R_{Sh}} + \left(C + \frac{(V - CR_{Se})}{R_{Sh}}\right)^2 R_a.$$

The remaining losses, *i.e.* the stray power $(L_F + L_m)$, can readily be obtained by running the dynamo as a motor, the field magnets being separately excited so that the armature has the same magnetic induction as at full load, the E.M.F. supplied to it being at least equal to the *total E.M.F.* which the machine would develop when running on full load as a dynamo at normal speed. Thus the machine is running on no load other than its own friction, eddy currents, and hysteresis. If A = current flowing through the armature and V_a = the voltage across its terminals when the speed is up to normal, then we have

$$\text{Stray power} = (L_F + L_m) = AV_a - L_a$$

where L_a = copper loss in the armature for the current A in it.

Note.—Only a comparatively small current (A) at the proper E.M.F. mentioned above will be required to be furnished by the auxiliary source of current, and if R_a is very small, L_a can be neglected in comparison with AV_a in this last formula.

Apparatus.—Dynamo M to be tested, which for the purposes of discussion merely we will assume is shunt wound; voltmeter V; low reading long scale ammeter A; rheostats R (p. 606) and r (p. 599); tachometer; complete Wheatstone Bridge set ($W.B$); two-way voltmeter key K (p. 587); switch S_2; source of current E at a sufficiently high E.M.F.

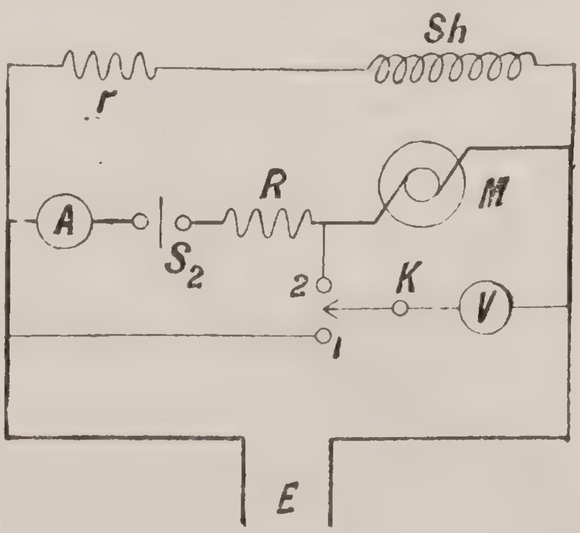


FIG. 84.

Observations.—(1) Connect up as in Fig. 84, and adjust the instruments V and A to zero if necessary. Switch on E, when the field should be then

excited to the normal amount, as can be seen by closing $K\,1$, and observing whether the normal voltage which the machine would give as a dynamo at the proposed speed is indicated on V.

(2) With R at its *maximum value* (not less than about 10 ohms) close S_2, adjusting R so as to give the armature the full requisite E.M.F. E. It will still run under the normal speed, since with so small a current the armature produces no demagnetizing action to quicken it up. Now adjust r so as to bring it up to the normal speed, and note by closing $K\,2$ the volts V_a across the armature terminals and the current A amps. flowing through it.

(3) Repeat 2 at the same excitation for some ten different speeds in all, both below and above normal (by varying R), entering the readings in the small tabular form—

Speed in r.p.m.	Stray Power readings			Total intake watts running light AV_a.
	Volts V_a.	Amps. A.	Watts $AV_a - A^2R_a$.	

(4) Open E, S_2 and K and measure by means of W.B. the resistance R_a of the armature and R_{Sh} of the shunt, remembering of course to disconnect one from the other while measuring their respective resistances.

(5) Calculate the power supplied and the commercial or nett efficiency of the dynamo for some ten different values of currents C (at, say, constant speed and voltage) taken from the machine, ranging from 0 to full load by about equal increments, and tabulate as follows—

NAME . . . DATE . .

Dynamo tested: No. . . . Normal Voltage = . . . Volts.
Type . . ,, Current = . . . Amps
Maker . . . ,, Speed = . . Revs.

Resistance: Armature = . . . Ohms. @ ° C.
,, Shunt = . . . ,, ,,
,, Series = . . . ,, ,,

Normal Speed Revs. per min.	Power developed assumed for calculation.			Stray Power Measurement in obs. 2.			Losses.		Calculated Power to Drive W_I $W_O + W_L$	Commercial Efficiency $=100\,\frac{W_O}{W_I}\%$.
	Volts V.	Amps. C.	Total Output $W_O = VC$.	Volts Va.	Amps. A.	$L_F + L_m = AVa - A^2Ra$ Watts.	Copper calculated Lc.	Total $W_L = Lc + L_F + Lm$.		

Note.—There will be only one value, that corresponding to normal speed, in each of the columns 1, 2, 5, 6, and 7 (counting from left to right) in the last table, but as many values in the remaining columns as there are values of amps. C assumed between 0 and full load.

(6) Plot the following curves having—

(*a*) Efficiency as ordinates and Watts developed as abscissæ.

(*b*) Stray power as ordinates and speed of armature as abscissæ.

(*c*) Watts developed as ordinates and Watts to drive as abscissæ.

Inferences.—State clearly all that can be inferred from your experimental results.

(83) Efficiency of Direct Current Generators. (Hopkinson's Electrical Method.)

Introduction.—The earlier methods of measuring the efficiency of direct current generators, in which the electrical output of the machine was obtained by the product of the ammeter and voltmeter readings, while the total mechanical input was obtained by means of some suitable form of transmission dynamometer, are more or less limited in their application from the fact that a reliable dynamometer is not always available. Even when it is, the method gives only an approximate result, for the error made in measuring the efficiency is proportional to the error made in measuring the input as given by the transmission dynamometer, and which is only too easy to make in an appliance such as this. It will therefore be evident that, given accurately calibrated instruments, any method of measuring the efficiency solely electrically will be capable of giving far more accurate results than could be obtained with any dynamometer.

The present method has this advantage, of being solely an electrical one, and requires two machines of as nearly the same output as possible, the accuracy of the test practically depending on how nearly alike in this respect the two machines are.

They must be capable of being placed in alignment with their shafts coupled mechanically together. The test can be made with either series, shunt, or compound machines, but the shunt is much the simplest.

Apparatus.—Accurate ammeters A and a_1 a_2; voltmeter V; rheostats R_1 R_2 for the field circuits (p. 599); change over voltmeter key K (Fig. 254); dynamo (a) to be tested coupled both mechanically and electrically to a similar machine (β) which runs as a motor. An auxiliary source of current (γ), such as a storage battery, or another dynamo giving an E.M.F. about equal to the normal of a and β, and able to supply the losses occurring in the machines a and β; switch S; rheostat Rh (p. 606, Fig. 274).

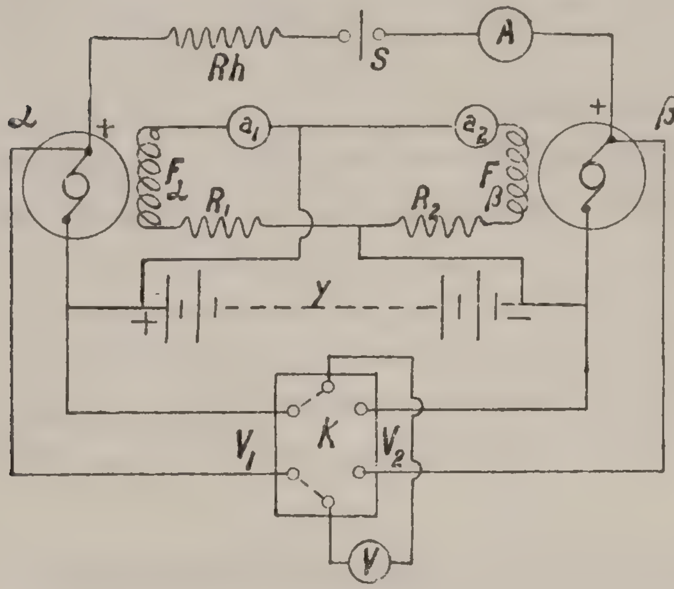


Fig. 85.

Observations.—(1) Connect up as in Fig. 85, and make sure that the E.M.F. of γ *assists* that of the dynamo a in driving the motor β in the right direction for self-exciting a.

(2) The respective fields F_a and F_β, in series with rheostats R_1 and R_2 respectively, are excited from the terminals of γ, as shown, to the normal amount roughly, except that of β, which is weakened to enable it to run as a motor.

(3) With Rh full in to start with, close S and adjust the auxiliary source of E.M.F. (γ) and the rheostat (Rh) so that the machines get up speed, and if possible obtain the normal full load current of a through the circuit.

(4) Slightly re-adjust R_1 and R_2 to bring $a\beta$ up to normal speed, then in quick succession measure the volts V_1 at the terminals of the dynamo a and the volts V_2, at the motor by means of the key K, at the same time noting the main current on A and the exciting currents a_1 and a_2.

(5) If possible obtain three or four different load currents through $a\beta$ from the normal downwards, and calculate the efficiency Σ from the relation

$$\Sigma = \sqrt{V_1 / V_2} \text{ approximately,}$$

and tabulate in a convenient manner.

Inferences.—Show how the above relation can be obtained, and state any assumptions made in obtaining it. What corrections would have to be applied to make it rigorously true? Obtain the true value of the efficiency Σ by applying the correction in question.

The test, though simple, requires a certain amount of experimental skill, especially in the case of series and compound machines. Moreover, the starting is somewhat troublesome.

By a slight modification in the connections, the test is a little easier to carry out, and this is shown in Fig. 86. Like the preceding arrangement it involves the use of an auxiliary generator or

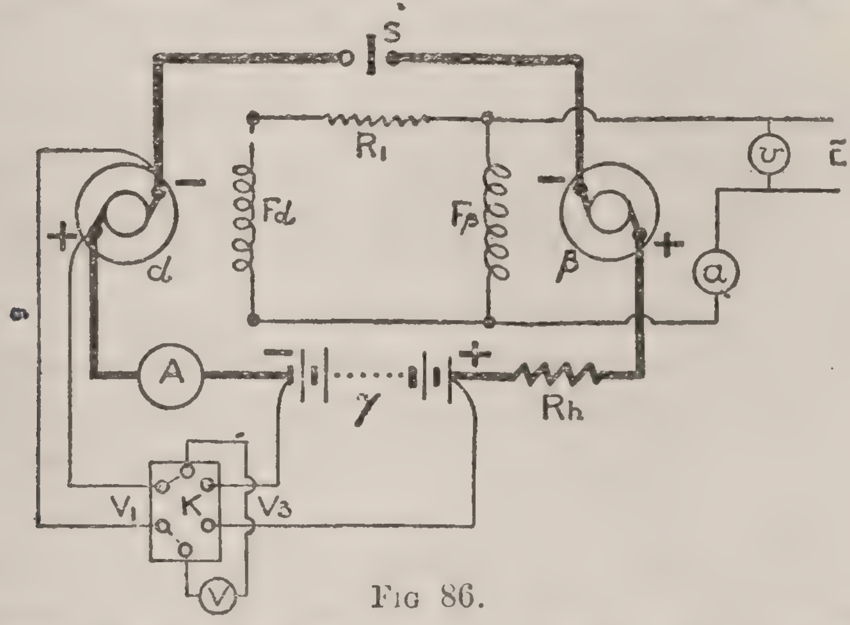


FIG 86.

set of secondary cells having the same current capacity as the machines under test, and a voltage of from 8 to 25% of that of the generator, according to their efficiencies. This, being, as before, in series with the generator and motor, takes the form of an added voltage to the system.

It is much better to excite the shunts from an independent supply instead of the auxiliary source.

In this arrangement the motor β must have the stronger field, and in order to start, the field $F\alpha$ of the generator α must either be broken or be made comparatively weak by means of the rheostat R_1.

Apparatus.—Similar to that for the preceding test; source of E.M.F. (E) necessary to fully excite the shunts $F\alpha$ and $F\beta$, the Auxiliary Source γ being as above mentioned.

Observations.—(1) Connect up as shown in Fig. 86, and adjust the ammeters A and a, and voltmeter V to zero, etc.

(2) With R_1 and Rh full in and the voltage (v) of the source E at the requisite value, close S, adjusting Rh to obtain full load current A through α and β, then simultaneously take the readings of a, v, A and the volts V_1 and V_3 across α and γ by means of the key K.

(3) Calculate the efficiency of either machine from the relation—

$$\Sigma = \sqrt{\frac{A.V_1}{A\ (V_1 + V_3) + a.v.}}$$

and tabulate your results in a convenient manner.

(84) Measurement of the "Nett" or "Commercial" Efficiency of Direct Current Dynamos. (Kapp's Electrical Method.)

Introduction.—The following, being an electrical method entirely, has the advantage that all the measurements are electrical, thereby enabling the efficiency to be determined with far greater accuracy than would be possible with any mechanical transmission dynamometer.

The method consists in coupling the generator to be tested both mechanically (with their armatures in alignment) and electrically to a similar type machine of as nearly equal power as possible, and which latter is made to run as a motor, driving the other, by the weakening of its field, with a rheostat. A small auxiliary generator, giving the normal voltage of the machine to be tested, is required, and must be so connected that it can be placed in quick succession across the terminals of the two coupled machines. The auxiliary source therefore supplies the necessary exciting currents together with the difference of the currents flowing in the two coupled machines. The test, though simple, requires a certain amount of experimental skill, especially in the case of series and compound machines.

Apparatus.—Dynamo (α) to be tested, assumed to be a shunt wound machine and having its field coils F_α across its terminals; another similar machine β to act as a motor and having a rheostat R_2 in its field F_β; "change over" switch C (Fig. 253);

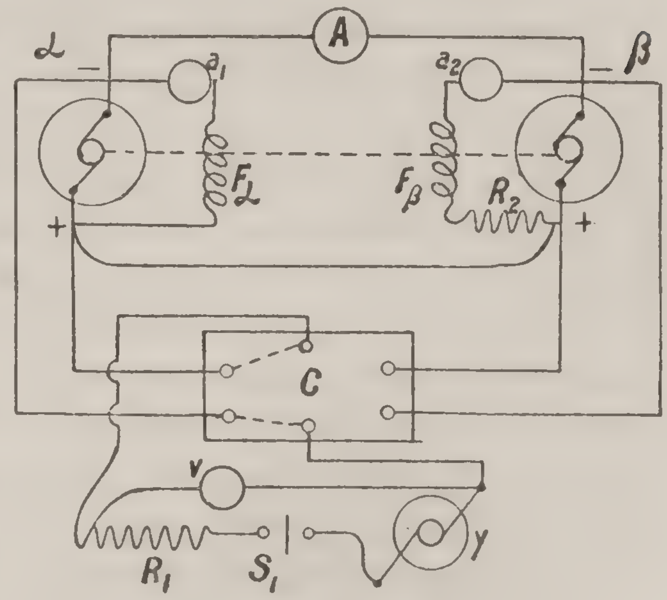


Fig. 87.

main rheostat R_1 (p. 606); ammeter A; voltmeter V; switch S_1 and auxiliary source of E.M.F. (γ), which may consist of the town mains (if the supply is continuous current), secondary battery, or small dynamo giving the normal E.M.F. of the generator α to be tested.

Observations.—(1) Connect up as shown in Fig. 87, and adjust the pointers of A and V to zero, if necessary. Arrange the machines α and β in alignment and couple their shafts together by a suitable coupling.

(2) Turn the "change-over" switch C to α, and with R_1 large close S_1 and gradually adjust R_1 and consequently the current until the machines start. Then when they are running at a constant speed, with V reading the normal voltage of α, note the ammeter reading A_α.

(3) Quickly "change over" C so as to place the auxiliary source γ across β and note the ammeter reading A_β for the *same* voltage V as before.

(4) Repeat 2 and 3 for some four or five different speeds, current, and voltages, and calculate the efficiency from the relation—

$$\text{Combined efficiency of the two machines} = \frac{A_\alpha}{A_\beta}$$

$$\text{Commercial efficiency of either machine} = \sqrt{\frac{A_\alpha}{A_\beta}}$$

Tabulate your results as follows—

NAME . . . DATE . . .

Generator tested: No. . . . Type . . . Maker . . . *Normal* Volts . . . Amps. . . . Speed . .
Machine Coupled: No. . . . ,, . . . ,, . . . Normal ,, . . . ,, . . . ,, . . .

Speed in Revs. per min.	Voltage V.	Currents in Amps.		Efficiency of	
		A_α.	A_β.	Combination A_α/A_β.	Generator tested or the other $100\sqrt{A_\alpha/A_\beta}$ %.

Inferences.—Show how the expression for the efficiency can be obtained, and dilate on the advantages and disadvantages of the method.

The preceding method can be slightly simplified by the following modifications. As in the above test, the following one involves the use of an auxiliary generator or set of secondary cells, having the same voltage as the machines under test and a current output of about 8 to 25% of that in the armature of α or β. Being in parallel with the machines to be tested, it takes the form of an added current to the system at the same voltage as the combination under test. The present tests are more convenient, generally speaking, and much simpler as regards starting than those of No. 83. Fig. 88 shows the connections, and the apparatus required is much the same as in the preceding method, except that the change-over switch C, Fig. 87, is dispensed with.

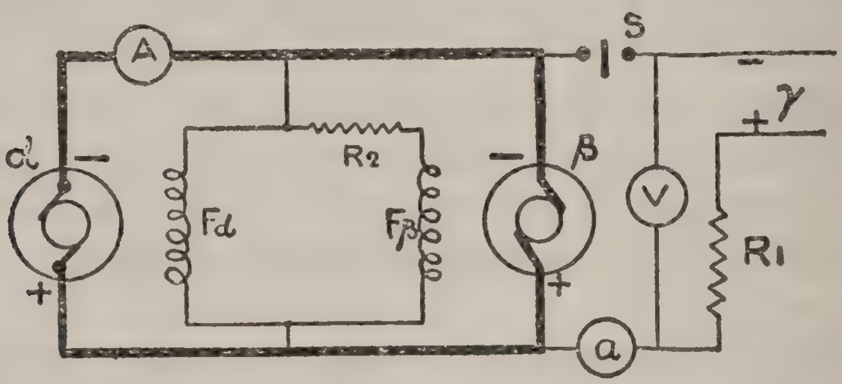


FIG. 88.

The fields F_α and F_β can be connected as shown in Fig. 88 instead of as in Fig. 87 if preferred, and it will then be noticed that 87 and 88 are electrically the same when the change-over switch C is kept as shown, and an ammeter (a) inserted in one of the leads connected to it. The source of supply γ, whether power mains or a third generator, must have a voltage at least equal to that of either α or β. Further, the losses in α and β are measured directly, and are small compared with the output of α and β; hence a small percentage error made in measuring them will be very small compared with the output of α and β, and will have but little effect on the resulting efficiencies. When two machines of the same size and type have to be tested, this method is almost always used in works for determining their efficiency and heating on a full-load time test.

Observations.—(1) Connect up as in Fig. 88 and set all the instruments to zero.

(2) To start up, put R_1 *full in* and cut out R_2 *to short circuit*, so that the fields F_α and F_β are as nearly as possible of equal and maximum strengths. Then close S and slowly cut out R_1, when the machines will start up as two similar motors in parallel on no load. The ammeter A will now read about half that of (a) because α will be taking about half the supply current.

(3) Now weaken the field F_β of the machine β by slowly increasing R_2, which will cause it to run faster and act as a motor, driving α as a generator. The reading of A will simultaneously fall, while that of (a) will remain nearly constant; and when A becomes zero, the voltage of α will have reached a value just balancing that of the supply γ, and (a) will indicate the current required to run α and β together at 0 load.

On still further increasing R_2 the current through (A) will be reversed, indicating that α is now commencing to supply, instead of receive, current.

Note.—For this reason A should be either of the moving soft iron needle type of instrument, or of the moving coil permanent magnet type connected in circuit through a reversing switch, otherwise a central zero moving coil type must be used.

(4) Take a series of load currents, as indicated on (A), differing by about equal amounts between 0 and the full-load value for either machine by still further increasing R_2—noting the readings of all the instruments and the speed at each load, V being constant at about normal voltage.

Note.—This circulating current A between the machines α and β will increase with the difference between their field strengths; and the limit is reached when the combination of a large current in the motor armature, and its weak field and high speed, causes excessive sparking.

Tabulate your results as follows—

NAME . . . DATE . . .

Machine α : No. . . . Type . . . Normal Output = . . . Volts = . . . Amps. = . . . Speed = . . .

„ β : No. . . . Type . . . Normal Output = . . . Volts = . . . Amps. = . . . Speed = . . .

Speed in r.p.m.	Generator (α).			Current from Mains (a) Amps.	Input into Motor β $(A+a)V$.	Efficiency of	
	Amps. (A).	Volts (V).	Output in Watts $W = A.V.$			Combination $E_1 = \frac{A}{A+a}$.	Either Machine $E = \sqrt{E_1}$.

(5) Plot curves having values of E and E_1 as ordinates, with W as abscissæ.

Inferences.—What errors, whether small or large, is the method liable to, and on what does the accuracy depend?

(85) Measurement of the Commercial Efficiency of a Generator by means of a Transmission Dynamometer.

Introduction.—This method of measuring the efficiency of an electrical generator, namely, by means of a transmission dynamometer, can be applied to a direct current generator equally as well as to an alternating current one. As, therefore, the application of the method to each of these two great classes of machines, to form two separate tests, was considered superfluous, preference was given to its application with an alternator, in that the output of a direct current generator is at once given by the product of the volts and amperes, while that of an alternator may present some difficulty to obtain accurately, the reasons for which are carefully explained. The actual measurement of the driving power by the dynamometer is obtained in precisely the same manner no matter what generator is being tested.

There are many different methods of finding the commercial efficiency of an alternator, depending in some cases on whether the armature rotates or is stationary, on the capacity of the machine, and on the facilities at hand for testing. In all cases the commercial efficiency = "mean" useful power developed ÷ total power absorbed by the alternator, the latter being = power

applied to the pulley to turn it + the power used in exciting. The mean or true power developed is easily obtained if a non-inductive resistance, such as a bank of glow lamps or water rheostat, is at hand which will carry the full load current of the machine, for then the true power = amperes × volts. This will not be true if the resistance is inductive owing to the "phase difference" between the current and voltage. For such a case the true power may be obtained by a non-inductive Wattmeter or the 3-voltmeter method (p. 379), etc. The power applied at the alternator pulley to drive it is very commonly obtained by indicating the engine, especially in large "sets." In the present case a transmission dynamometer is used to measure this power. It is of the spring type, and the means for recording the readings of it were devised by Prof. W. Stroud. The indications,

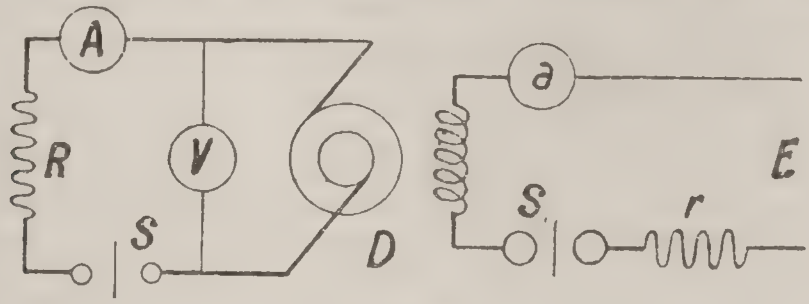


Fig. 89.

which are recorded electrically, represent the nett pull, or difference of tensions in the two sides of the belt in lbs. Then knowing the speed of the alternator and the diameter of its pulley, the H.P. can at once be deduced. For a full and detailed description of the dynamometer, see Appendix, p. 625.

Apparatus.—Alternator D to be tested; transmission dynamometer complete with its indicating galvanometer G (p. 625); tachometer, $a-c$ ammeter (A) and voltmeter (V); $D-C$ ammeter (a); switch S; and non-inductive resistance or bank of lamps R (p. 598); exciting circuit containing ammeter (a), rheostat r (p. 599), switch S_1 and exciting E.M.F. E.

Observations.—(1) Connect up as shown in Fig. 89, and see that all lubricators in use feed slowly. Adjust the secondary E.M.F. (p. 629) for use with the dynamometer, so that when placed directly across the terminals of G, a full scale deflection is produced. Then insert it in its proper place.

(2) With the alternator belt on the loose pulley on the counter-shaft, start the motor which drives this shaft, and note the mean deflection on G for different speeds. If this is appreciable it must be deducted from each of the readings which follow.

(3) Now throw the belt on to the fast pulley so as to start D, and without S being closed or the field excited, again note the mean deflection on G for different speeds.

(4) Adjust the speed of D to give normal frequency and the excitation to give normal voltage on V. Note the reading of G, with S still open.

(5) Close S and repeat 4 (keeping the speed constant) for about ten different load currents on A, rising by = increments to the maximum permissible by varying (R).

(6) Repeat 4 and 5 for frequencies of 40% and 75% of the normal respectively, and tabulate as follows—

NAME . . . DATE . . .

Alternator—No. . . . Maker . . . Normal output = . . . Watts. at . . . Revs. per min.
Resistance of exciting coils (r) = . . . ohms. Diameter of alternator pulley d = . . . ft.
" " armature (warm) r_a = . . . " Circumference " " πd = . . . ft.

Speed Revs. per min. N.	Frequency $\sim$ per sec.	Deflection on G.	Nett belt pull $(T-t)$ lbs.	Exciting Currents (a) amps.	Total H.P. absorbed $H_2 = \frac{(T-t)\,\pi d N}{33000} + \frac{a^2 r}{746}$	Output. Amps. A.	Output. Volts V.	Useful H.P. developed $H_1 = \frac{AV}{746}$.	Commercial efficiency $= 100 \frac{H_1}{H_2}$ %.

(7) Plot curves for each speed having A and useful H.P. developed as abscissæ, and V and efficiencies as ordinates respectively. Also between H.P. developed as ordinates, and H.P. required to drive as abscissæ.

Note.—The nett pull of the belt in lbs. must be obtained from the deflection of G with reference to the latest calibration curve of the dynamometer.

The Testing of Continuous and Alternating Current Electro-Motors.

General Introduction.—Since the production of the electromotor in its more practical form within recent years, the uses

to which it has been applied, for the electrical driving of workshops, haulage, electric traction, etc., etc., have assumed such proportions as to make the different forms and types of electromotor at the present day multitudinous. The systematic testing, therefore, of such machines becomes of considerable importance, in order that a comparison may be obtained and a judgment formed of the weak points of any particular type, together with its performance and qualities (whether good or bad) which it possesses.

No motor, least of all one intended for electric tram and railway work, should leave the makers' works or be installed in its proposed occupation without being first thoroughly tested for the following points—(*a*) *Resistance*, or conductivity of its electrical circuits; (*b*) *Insulation* resistance between *earth* or framework of the machine and the copper circuits both individually and collectively; (*c*) *Brake horse-power;* (*d*) *Efficiency;* (*e*) *Heating*, or rise of temperature of the various parts of the machine after a run at *full load* for a specified time. These tests we may now consider more in detail.

(*a*) Copper Resistance.—That of each of the copper circuits should be separately measured, by the Wheatstone Bridge in the ordinary way (p. 81) in the case of the shunt coils or other circuit of several ohms, and by the Potential Difference Method (p. 84) or voltmeter and ammeter method (p. 86) in the case of the armature and series coils or other low resistance.

(*b*) Insulation Resistance.—That of the various parts can be obtained by Tests Nos. 43 and 49 (pp. 113, 129) or other convenient method, at a pressure of something like three or four times the normal working pressure of the machine. The insulation resistance of the machine as a whole, when tested at the normal voltage, should not be less than 2000 ohms per volt, whence, of course, that of the individual coils or circuits will be much higher. Some makers merely test the separate parts under a pressure of 2500 to 5000 volts alternating, and if they stand this they are passed as satisfactory. This pressure can conveniently be obtained by means of a small testing transformer, stepping up from, say, 100 to 5000 volts, carefully set fuses being placed in circuit to prevent damage should the insulation break down.

(*c*) Brake Horse Power.—This may be measured in one of

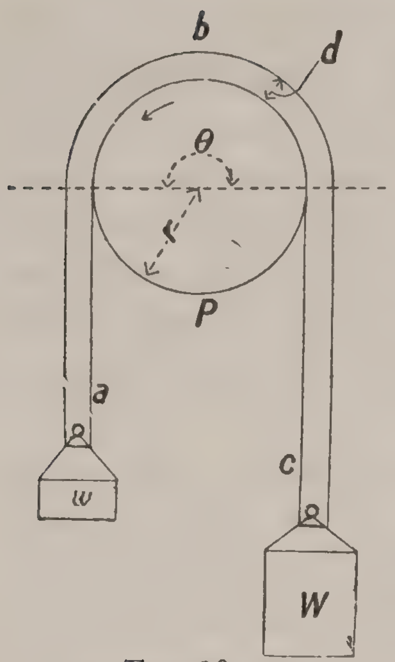


FIG. 90.

three ways, depending on the facilities at hand for testing; namely, by an absorption dynamometer, in other words, a modified form of Prony brake, by the "balance" or "cradle" method, or lastly by the electrical method. The last two methods will be described in conjunction with their application to tests which follow later on, but we will now consider the principle involved in the first-named method, reserving the description of some convenient forms of brakes until later. It will be sufficient if we consider the principle of the simplest form of brake, consisting of a rope or band *abc*, of diameter or thickness (d), lapped with any arc of contact θ (in circular measure), from a fraction of a turn to more than one turn, over the face of the motor pulley P, having a radius (r) and which rotates we will suppose counter-clockwise, as indicated in Fig. 90. To one end c is attached a large weight W, and to the other (a) a small one w. Now when the pulley P is at rest, W = tension on the right-hand or "tight" side of the rope, while w = the tension on the left-hand or "slacker" part of the rope. Then, as P rotates, the couple or torque T, due to the force of friction between the rope and surface of the pulley, tending to resist motion, and against which the motor does work, is—

$$T = (W - w)(r + \tfrac{1}{2}d) \text{ pound feet,}$$

where (W and w) are in lbs. and (r and d) in feet, ($W - w$) being the difference in tensions or nett load on the brake in lbs., and ($r + \frac{1}{2}d$) the *mean effective* radius in feet (of pulley and rope together) at which the nett load acts. If (n) = number of revolutions per minute made by P, then $2\pi n \div 60 = \omega$, the angular velocity of the pulley, and the work per second, or the rate at which work is done by the motor on the pulley $= \omega T$. Hence we have—

$$\text{H.P. developed} = (W - w)\ (r + \tfrac{1}{2}d)\ 2\pi n \div 33{,}000,$$

where 1 H.P. is equivalent to 33,000 foot-lbs. per minute.

All the power thus measured and appearing at the pulley is

wasted in heating this latter, and herein lies one of the chief difficulties in testing larger H.P.s, namely, the getting rid of the heat so generated by friction, for not only is the heat liable to burn the rope in two if the power of the motor is sufficient, but it also affects the co-efficient of friction (μ) between the rubbing surfaces, thereby causing the brake to jerk and preventing any steady readings being taken.

To obviate this trouble, either the pulley must be water-cooled (see p. 633), or readings must be taken immediately after adding a weight, and then the weight released from the rope. The trouble is further intensified by the motor running at such fast speeds, which is common to this type of driving power. By a slight modification of this form of brake, viz. substituting a spring balance for (w), the brake becomes automatically self-regulating for variations of μ, for then if μ suddenly increases, W rises, and (w), which now is the spring-balance reading, decreases, therefore $W-w$ increases and restores the brake to its first position. The coefficient of friction μ can be calculated thus—

Let $\theta =$ arc of contact (in circular measure) between cord and pulley, then $\frac{W}{w} = \epsilon^{\mu\theta}$

where $\epsilon =$ base of the Napierian logarithms $= 2{\cdot}71828$.

The friction surfaces (in contact) of the brake should be as large as possible, in order to readily dissipate the heat generated. Mr. Maw gives the following rule for finding the smallest dimensions of a brake pulley: if H.P. = horse-power to be measured by the brake, and $v =$ peripheral velocity of the pulley in feet per minute, and $(l) =$ width of rubbing surfaces in contact, measured axially, then $\frac{vl}{\text{H.P.}}$ must not be less than 700.

(d) Efficiency.—This can at once be obtained if the electrical H.P. absorbed by the motor for a given B.H.P. is known. If A amperes as read off on the ammeter is passing into the machine at a P.D. of V volts read on the voltmeter placed across the terminals of the machine, then the input or E.H.P. $= \frac{AV}{746}$ where 1 H.P. = 746 Watts.

Hence the commercial efficiency $\eta = \frac{\text{B.H.P.}}{\text{E.H.P.}} = \frac{\text{B.H.P.}}{\text{E.H.P.}}\ 100\%$.

(*e*) HEATING.—This may be limited by specification or the question of safety to the conductors, and also considerations of overloading. It is not advisable that the rise of temperature of any part of the machine should exceed 40° C. above that of the external atmosphere after a six hours' run on full load. The temperature can be obtained by placing the bulb of a thermometer on the part to be tested and covering it over by some cotton wool. This can only be done to the armature at the moment of stopping, and it will here be noticed that a sudden rise of surface temperature occurs in the armature at the moment of stopping, due, of course, to the ceasing of the ventilating action which goes on while it is rotating (see p. 216).

(86) Variation of Speed with Voltage across the Armature of a D.C. Electro-Motor (at Constant Excitation).

Introduction.—This is an important test, in that it will familiarize the student with the fundamental principles underlying the regulation and control of motors. It can be carried out on a series, shunt or compound-wound motor, so long as the corresponding change in the connections and means for maintaining *constant excitation* are made. As, however, the same result is obtained with each type of motor, we shall operate the test with the simplest type, viz. the shunt motor.

Note.—In a series motor the field regulating resistance, at least equal in value to the resistance of the series coils, must be shunted across them; whereas in shunt and compound motors it is connected in series with the shunt coils, and has a resistance and current-carrying capacity at least equal to those of the shunt coils.

Apparatus.—Shunt motor, of which M is the armature and F the field; main circuit variable rheostat R, ammeter A, and switch S, each capable of dealing with the full-load current of M; voltmeter V and supply mains $M_1 M_2$ of voltage E each for the rated voltage of M; field rheostat (r) and low-reading ammeter (a); tachometer.

Observations.—(1) Connect up as shown in Fig. 91 and adjust the pointers of V, a, and A to zero if necessary.

(2) With (r) all out and R *full in*, close S and gradually cut R out to short circuit as M gains speed, then adjust (r) to get normal speed (n). Note the readings of V, A, and a, which last-named must now be kept constant throughout the test by varying (r).—(See that the lubrication of M is working).

(3) With *the motor still running light* as in (2) above, vary R so as to obtain some eight different speeds (n) in about equal steps between 0 and the normal, and note the corresponding readings of V, A, and a (a being kept constant throughout).

(4) Repeat (3) *with the motor running at full load* (if arrangements permit), and for the same value of constant field current (a) as before.

Note.—The loading-up of motor can most conveniently be effected by means of an eddy-current brake or by taking any desired output from a coupled generator.

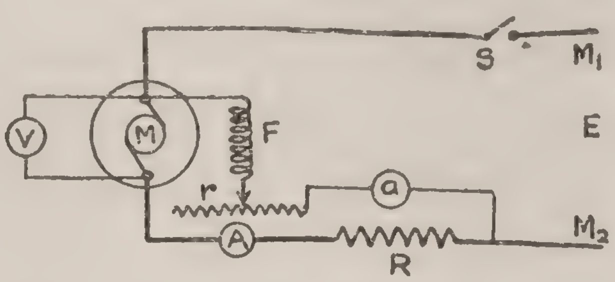


Fig. 91.

(5) Repeat (3) and (4) with, say, half the previous excitation maintained constant and tabulate all your readings as follows—

Name . . . Date . . .
Motor No. . . . Type . . . Armature Res. $r =$ . . . ohms . . .
Full Load:—B.H.P. = . . . Volts = . . . Amps. = . . . Exciting Amps. = . . .
R.p.m. = . . .

Motor running Light or Loaded.	Supply Volts (V) on Armature.	Back E.M.F. $v = V - A.r.$ of Armature.	Armature Amps. (A).	Constant Field Amps. used (a).	Armature Speed in R.p.m. (n).	Calculated Ratios.		
						$\frac{n}{V}$	$\frac{n}{v}$	$\frac{E}{A}$

(6) Plot to the same pair of axes, curves having speed (n) as ordinates with values of V, A, and $\left(\frac{\textit{supply voltage across } M_1M_2}{A}\right)$ as abscissæ.

Inferences.—State clearly what you can deduce from the table of results and curves, and show how they can be applied

to the design of a main circuit current rheostat for controlling the speed of the motor.

(87) Variation of Speed with Excitation in a Direct Current Electro-Motor (with Constant Supply Voltage on Armature).

Introduction.—The reader should peruse the introduction of the last test, the remarks in which apply to the present test also. In addition, it may be pointed out that when the motor is running light the back E.M.F. will remain nearly constant, since the power required to drive is usually very small, and is $\propto$ $(V-v)$, consequently the increase of speed will be almost inversely $\propto$ to decrease of field strength.

Apparatus.—That required for the present test is precisely as detailed for the last one, and need not be repeated again here.

Observations.—(1) Connect up exactly as shown in Fig. 91 of the last test, and adjust the pointers of V, a and A to zero if necessary.

(2) With (r) all out and *R full in*, close S and gradually cut R out as the motor gains speed, until V reads the normal voltage of the motor; then adjust (r) to get normal speed. Note the readings of A, a and V, which last-named must now be kept constant throughout the test by varying R (see that the lubricating arrangements of the motor M (Fig. 91) are working).

(3) With *the motor still running light*, as in (2) above, vary (r) so as to obtain some 8 speeds (n), differing by about equal steps between 0 and the normal value, and note the corresponding readings of A, a and V (V being kept constant throughout).

(4) Repeat (3) with the motor running at full load (if arrangements permit), and for the same constant value of V as before.

Note.—The most convenient way of loading up M (Fig. 91) is by means of an eddy current brake, or by taking the required output from a coupled generator.

(5) Repeat (3) and (4) with, say, half the previously normal value of supply voltage V across the armature, maintained constant, and tabulate as follows—

Name . . . Date . . .
Motor: No. . . . . Type . . . Armature Res. $(r) = . . .$ ohms.
Field Coil Res. $(r) = . . .$ ohms.
Full load: B.H.P. = . . . Volts. = . . . Amps. = . . . Field Amps. = . . . R.p.m. = . . .

Motor running light or loaded.	Armature			Field Flux $\propto 1/n$.	E/a.	Field Current (a).	
	Supply Volts V (const.)	Amps A.	Speed (n).			Ascending.	Descending.

(6) Plot to the same pair of axes, curves having speed (n) as ordinates with values of (A), (a) and (*supply voltage across* $M_1M_2 \div a$) as abscissæ, and between $1/n$ as ordinates with (a) as abscissæ.

Inferences.—State clearly what can be deduced from the table of results and curves, and show how these can be applied to the design of a field regulating rheostat for controlling the speed of the motor.

(88) Variation of Voltage, Current, and Speed, with position of the Brushes around the Commutator of a D.C. Machine at Constant Excitation.

Introduction.—Although the usual practice now is to design D.C. generators and motors with a fixed diameter of commutation and immovable brush-bars, special cases are met with in which provision is made for moving the brushes through considerable angular space round the commutator. As is well known, the terminal voltage of a generator and the speed of a motor is each capable of variation by moving the brushes, while a motor can even be stopped and reversed by so doing. In fact, where the variation of voltage or speed required is not large, it can be obtained by brush movement without expensive field regulators and without altering therefore the field strength—a feature which is sometimes valuable, while most machines will admit of quite an appreciable brush movement without much sparking, when running light, such is not the case when they are running on load, so that the scope of this test may be limited by the amount of sparking. Further, it will be found easier to

apply the test to a motor than to a generator, and we shall therefore operate the present test on a motor.

Apparatus.—Shunt motor, of which M is the armature and F the field; main circuit variable rheostat (R); ammeter A, and switch S, each capable of dealing with the full-load current of M; voltmeter V and supply mains $M_1 M_2$ of voltage E at least

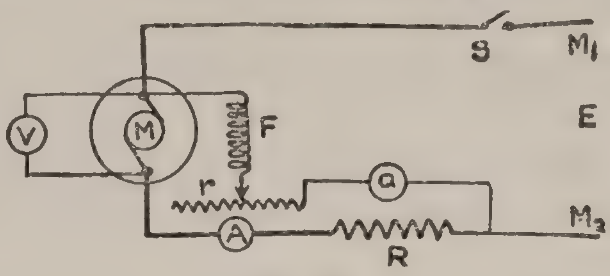


FIG. 92.

equal to the normal for M; field rheostat r and low reading ammeter (a); tachometer and, if possible, some scale for indicating the angular motion of the brushes round the commutator.

Observations.—(1) Connect up as in Fig. 92, adjusting (V) (A) and (a) to zero if necessary, and (R) *to a value not less than the Ratio* $\dfrac{\text{supply voltage}}{\text{full-load current}}$ *ohms*, so as to prevent the full-load motor-current being exceeded if the armature comes to rest or its speed increasing too rapidly as the brushes are moved.

Note.—This value of R will be given by blocking the armature and with normal excitation, noting the value necessary to give full-load current on A.

(2) Start the motor either by using the ordinary "Starter," or by (R) temporarily increased before closing S, and then gradually cutting out R until normal speed is reached—*the brushes being in the normal full-load running position*, and the excitation adjusted to normal full-load value.

Now note the values of V, A and (a), and the speed.

(3) Next keeping (a) constant at the above value, adjust R to the minimum value allowable and found in obs. (1), and note the readings of V, A and speed for normal position of brushes (as in obs. 2), and for a series of different positions throughout an angular distance $= \frac{1}{2}$ the polar pitch either side of their normal position.

(4) Repeat (3) at the nearest B.H.P. load to full load which it is practicable to run at, and tabulate as follows—

Name . . . Date . . .
Motor : No. . . . Type . . .
Full load : B.H.P. = . . Amps. = . . Volts = . . . R.p.m. = . . .
Armature Resistance r_a = ohms. Normal Excitation = amps.

Position of Brushes.	Speed in revs. per min. (n).	Armature		Back E.M.F. (e) $V - Ar_a$.	Field Strength $\propto e/n$.
		Volts V.	Amps. A.		

(5) Plot curves on the same axes having values of brush position (normal as origin) as abscissæ with values of e/n, V, A and speed respectively as ordinates.

Inferences.—State clearly what can be deduced from the results of your test.

(89) Efficiency and B.H.P. of Direct Current Series Wound Electro-Motors.

Introduction.—The series motor in general possesses some characteristic features which it may be well here to note in view of the prominent place this type of motor has, and still is, taking in electric traction and power work generally. Since it can be shown that the torque T of the motor is given by the relation—

$$T = \frac{CNA_a}{2\pi} = \frac{CN}{2\pi} \cdot \frac{E-e}{r_a} = \frac{CN}{2\pi} \cdot \frac{E-CNn}{r_a}$$

where C = number of armature conductors all round,
N = number of lines threading the armature or the useful flux,
A_a = number of amperes of current through armature,
E and e = impressed and back E.M.F.s of the mains and motor respectively,
n = speed in revs. per second,
and r_a = resistance of armature circuit.

It will be at once evident that the torque exerted is a maximum at starting, *i.e.* when $n = o$, and that it varies as the armature current A_a, since N also varies as A_a.

Again, when the motor is "*running light*" at its maximum speed $T = O$ nearly, for then the back E.M.F. generated almost = that of the mains E.

Thus a series motor tends to race directly the load is

suddenly removed, which is an undesirable feature for workshop driving.

The fact that T = maximum at starting, and that the motor will start on full load, is a most valuable property for traction work on tram and railway lines.

In the following Fig. and all after it, the motor is represented symbolically, (a) denoting the armature, commutator, and brushes, and FM the field magnet coils, which in this case, being series wound, are represented by a few curly lines.

Apparatus.—Electromotor (series wound) to be tested, fitted with an absorption dynamometer or brake (Fig. 295); ammeter A; voltmeter V; variable rheostat R (p. 606); switch S; battery or dynamo B, giving the requisite voltage needed for the motor, and speed indicator and set of half-pound and one pound weights for the brake, also a lubricant if necessary.

Observations.—(1) Connect up as indicated in Fig. 93, and adjust the pointers of A, V, and the tachometer to zero if necessary. See that all lubricating cups in use feed slowly and properly.

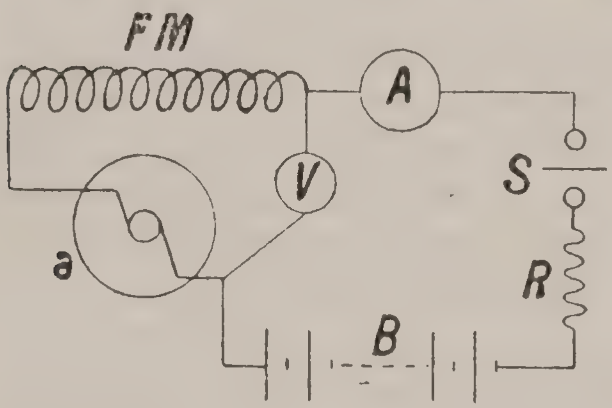


Fig. 93.

(2) See that R is at its full, then carefully remove the brake from the pulley and close S. Take a series of *gradually ascending and descending* observations (by varying R) for about ten different speeds, ranging by about equal intervals between the lowest readable on the tachometer and the maximum safe speed for the motor, noting this speed and the corresponding values of A and V at each.

(3) Replace the brake and repeat 2 for no weight in the pan. From 2 and 3 the loss in Watts in the brake can be found.

Note.—It will probably be necessary to exert a *very small* pressure by the finger on the brake in 3 to prevent it being carried round as the pulley rotates. No appreciable error need be introduced due to this. If a form of brake is used in which no loss of power can occur other than that incidental to its

use when actually measuring power, then omit obs. 3 and also the last seven columns in the next table, and substitute a column headed $A_1 V_1$ watts to run motor at no load.

Tabulate your results as follows—

Speed Revs. per min. (n).	Without Brake.						With Brake.						Loss in Brake in Watts $W_B = (A_2V_2 - A_1V_1)$.
	Volts.			Amperes.			Volts.			Amperes.			
	Ascending.	Descending.	Mean V_1.	Ascending.	Descending.	Mean A_1.	Ascending.	Descending.	Mean V_2.	Ascending.	Descending.	Mean A_2.	

(4) With the brake carefully replaced on the pulley and the smallest weight in the scale pan, close S and by varying R adjust the speed to the lowest convenient. Note this and the reading of V and A simultaneously, and the weight.

(5) Repeat 4 at the same constant speed for ten or twelve loads or weights in the pan, ranging from the smallest to that which will cause the current to rise to not more than 25% over normal.

(6) Repeat 4 and 5 for the maximum allowable speed and an intermediate one, each constant throughout, and tabulate your results as follows—

NAME . . . DATE . . .

Motor tested: No. . . . Type . . . Maker . . . Weight . . .

Resistance: Armature r_a = . . . ohms. Series coils r_s = . . . ohms.

Effective radius of Brake pulley and band r = . . . ft.

Normal B.H.P. = . . . Amps. = . . . Volts = . . . Speed = . . . revs. per min.

Speed Revs. per min. (n).	Weight in Pan W (lbs.)	Lesser Weight or Spring balance reading w (lbs.)	Nett Torque T = $(W-w)\,r$ (pound feet).	Volts V_3.	Amps. A_3.	Total H.P.		Commercial Efficiency $=\frac{H_1}{H_2}100\,\%$
						absorbed $\frac{A_3V_3}{746}=\text{H}_2$	developed $H_1=\frac{2\pi nT}{33000}+\frac{W_B}{746}$	

Note.—The true H.P. developed = H.P. calculated + H.P. lost in brake itself.

(7) With the brake removed from the pulley and R full in, close S and obtain the maximum speed allowable. Note this and also simultaneously the amperes (A_4) and volts (V_4).

(8) Replace the brake and add weights to the pan so as to obtain about ten different loads to the point when the largest load

stops the motor. Note the current A_4 and speed (n) at each, the *volts* V_4 having been *kept constant* by altering R to suit the load.

Note.—The current should not exceed about 40% above the normal. Tabulate your results as above.

(9) From observations 2 and 3 plot the following curves between

(*a*) Volts V_1 as ordinates and the corresponding speeds (n) as abscissæ.

(*b*) Brake loss W_B as ordinates and the corresponding speeds (n) as abscissæ.

From observations 4–6, plot for each speed curves between

(*c*) Efficiency and current as ordinates and corresponding B.H.P.s as abscissæ.

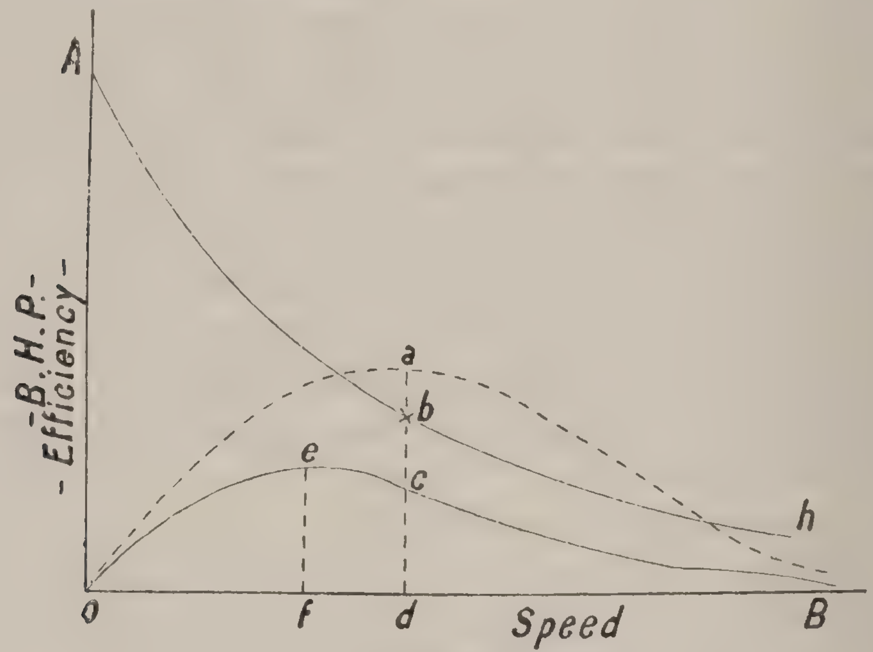


FIG. 94

From observations 7 and 8 plot the following curves between

(*d*) The speed in each case and E.H.P. input, B.H.P., and efficiency.

(*e*) The speed and current as ordinates and torque as abscissæ.

(10) Calculate the coefficient of friction (μ) between the brake band and pulley for various loads and for the arc of contact employed.

From the curves 9 (*d*) deduce the relation between the speeds that give maximum efficiency and maximum B.H.P. respectively.

Inferences.—State very clearly all the inferences deducible from your experimental observations. Explain fully the curves obtained in 9 (*d*) above.

Note.—The general form of the curves (*d*) 9 above are shown

in Fig. 94. The diagram due to Mr. Kapp is an exceedingly useful one for seeing the relative H.P.s and efficiency. OcB is the B.H.P. curve, OaB is the efficiency curve, and Abh is the E.H.P. (input) curve. The shape of this last varies with the type of series motor run off constant potential mains. The ordinates, such as (ad), of the efficiency curve $OaB = \frac{cd}{bd}$, at this and similar points to any arbitrary scale of ordinates.

(90) Efficiency and B.H.P. of 500 Volt Direct Current Series-Wound Tramway Motors.

Introduction.—The particularly heavy and trying work which a tramway or railway motor has to perform renders it all important to subject the machine to the most searching tests for defects or other faults at the outset. Such tests are twofold—

(1) A complete test of the motor at the works of the makers and again when fixed to the car.

(2) A test of its performance when driving the car on some approved route on the system. With regard to this case, the worst route of the whole system is chosen, *i.e.* one having the steepest gradients and sharpest curves.

The car is loaded with an artificial load, such as sand bags, for instance, equal to the full load of passengers which it is intended to carry. It is then run as continuously as possible along that route with five-seconds stops every five minutes for say two hours.

This test is considered satisfactory, if, at the end of that time, all has gone on satisfactorily and the temperature of the armature and commutator of the motor has not risen above the prescribed limit.

Next, with regard to the "Works Tests." Besides the efficiency test at various loads, the motor should be run at the average speed it will run at in practice, say that corresponding to eight or nine miles per hour of the car, for four to six hours at the maximum load which the motor is intended for.

Except in the case of electric railways, where the car or engine axle is direct driven, single reduction gear between motor and car axles is almost universally employed of between 4·75 and 4·86 to 1.

The sizes of tramcar wheels are usually either 30 inches or 33 inches in diameter. The remarks mentioned in the *Introduction of Test* 89 should be carefully read and remembered, when the performance of the motor on test will at once be obvious.

Caution.—*The operators of the controlling rheostat switch-gear, etc., must stand on the india-rubber mat provided, and must on no account touch any live metal work on the circuit of the* 500 *volt generator and tramway motor.*

Great care must be taken to insure that the rheostat in the main circuit of the motor is FULL IN *before closing the main switch, and also that it is at once re-inserted* before *pulling out that switch on stopping.*

The apparatus and connections of the preceding test are those now to be obtained, and the observations, as there given, to be carried out. In addition to curves 9, *a—e,* plot two on the same curve sheet, having speed as abscissæ, and both torque and amperes as ordinates.

(91) Relation between the Starting Torque and Current in a D.C. Electro-Motor.

Introduction.—It is very instructive to compare the results obtained in applying the principle of the present test to series, shunt, and compound wound motors, but it may also be applied to alternating current motors. The torque or turning effort by the armature on its shaft, measured in terms of a pull, acting at a given radius or leverage from the shaft centre, and tending to turn it, is expressed in pound-feet, usually, in this country. In a motor it results from the inter-action of the field magnets and the field of the armature as set up by the currents flowing through it, and is $\propto$ to the product of these two field strengths.

If C = the number of effective conductors on the armature;

N = the effective magnetic flux cut by them or flowing in the core;

A = the armature current;

then it can be shown that the torque (T) is given by the relation

$$T = \frac{CN}{8{\cdot}52 \times 10^8} \times A. \quad \text{lb. ft.}$$

an expression independent of the speed of the motor.

$$\therefore T \propto \text{armature current} \times \text{field flux} \propto AN.$$

In a series motor, the field flux N will vary as A varies, up to the point of magnetic saturation of the field magnets, when it will be practically constant. Any further increase in A will give the relation

$$T \propto A.$$

This also holds for a shunt motor, in which the field excitation is practically constant and near saturation.

In a compound or differential motor, however, the shunt and series windings oppose each other magnetically, and hence on starting it may happen that they nearly balance, thus giving practically zero starting torque.

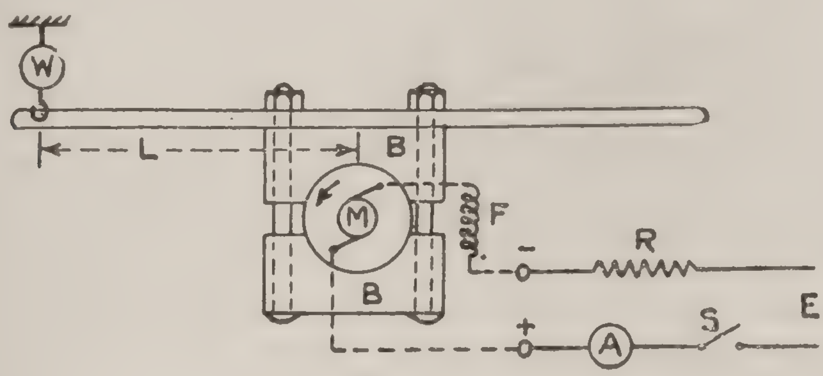


FIG. 95.

In capstan, windlass, and all traction work, maximum torque is required on starting; and since the series motor fulfils this condition and is exclusively used for the purpose in D.C. work, we will consider this type of motor to be the one used in the present test.

Apparatus.—Series motor M to be tested on a suitable D.C. supply E; brake-blocks B with yard-arm L and spring balance W; ammeter A; switch S; and variable current rheostat R.

Observations.—(1) Connect up as in Fig. 95, setting A and W to zero if necessary. Clamp B B to the pulley of the motor so as to prevent slipping (rotation), and with the yard-arm horizontal. Attach the spring balance W at a measured distance L from the pulley centre.

(2) With *R full in*, close S, and adjust R so as to give some ten currents through M and A differing by about equal amounts

between O and the full-load current of M, noting the readings of W and A at each.

Note.—The connections of field F and armature of M must be such that the latter *tends* to turn in the direction shown. Also the yard-arm may have an equal overhang as shown, or be otherwise balanced when horizontal.

Further, as there will be a good deal of static friction at the motor bearings, the armature should be rotated by hand a few revolutions before attaching B B, so as to well oil the journals; and even then the *mean* of several readings of W, taken for each value of A by disturbing the position at which (B) rests, by the hand.

(3) Take an ascending and descending series of value of A, and tabulate as follows—

Motor: No. . . . Type . . . B.H.P. = . . . Volts. = . . . Amps. = . . .

Amps. (A).	Pull. W (lbs.)	Torque $T = W.L.$ lb. ft.	Values of A/T.

(4) Plot a curve having values of A as ordinates, and T as abscissæ.

Inferences.—Point out all that can be deduced from the table of results and shape of the curve.

(92) Efficiency and B.H.P. of Direct Current Shunt-Wound Electro-Motors.

Introduction.—If a shunt motor, supplied at constant potential, has a very low armature resistance, high shunt coil resistance and field magnets giving a field relatively very much more powerful than that due to the armature, the variation of "lead" of the brushes will be slight and the motor will be almost self-regulating in speed for wide variations of load, *i. e.* it would run at constant speed independent of the torque. The falling off of the speed in shunt motors as the torque increases will be the less as the field magnetism is the more powerful. The brushes, in two pole machines, should press at opposite ends of a diameter, and to ensure sparkless running must have a "backward lead." In all

cases the efficiency = total power given out ÷ total power put in, both being reckoned in similar units. The input is easily deduced from ammeter and voltmeter readings, but the output is more difficult to obtain accurately. In the present test it is obtained by means of an "absorption dynamometer," which we will assume to be the modified form of Prony Brake introduced by Raffard. Such a brake wastes in heat all the power given out by the motor through friction, but at the same time forms a measure of this power. The arrangement is such that the brake automatically adjusts itself to variations of the coefficient of friction between the rubbing surfaces due to heat. In brake tests of this nature just sufficient lubrication (such as soap and water) and no more ensures smooth working without sudden jerks due to seizing, and this, together with experience in manipulation, is the secret of the success of such tests.

If (r) = mean effective radius of *pulley and band together* in ft., n = number of revs. per min., W = weight in lbs. in scale pan, and w = weight in lbs. at the slack side of the pulley; then the angular velocity of the pulley $\omega = 2\pi n \div 60$, and the couple or torque resisting motion $T = (W - w)r$; then the work done per sec. $= \omega T$, and the B.H.P. $= (W - w)\ 2\pi rn \div 33000$.

Apparatus.—Shunt motor (M) to be tested; voltmeter V; ammeters A and a; rheostats R_1 (p. 606) and R_2 (p. 599); switches S_1 and S_2; source of current B; speed indicator. A set of $\frac{1}{2}$ lb. and 1 lb. weights are provided with the brake, together with a pump and tank by means of which a slight dripping of lubricant may be allowed to fall into the central rotating pulley and band.

Note.—For further remarks on the testing of motors see the "General Introduction" on the subject, p. 232, *et seq.*

Observations.—(1) Connect up as indicated in Fig. 96 and adjust the pointers of all the instruments to zero where necessary. See that all lubricating cups in use feed slowly and properly.

(2) Uncouple the absorption dynamometer from the motor shaft. Set $R_1 R_2$ at their maximum values, and close S_2, adjusting the exciting current (a) to the normal value by means of R_2.

(3) Close S_1 and take a series of *gradually increasing* and *decreasing* observations (by varying R_1) for about ten different

speeds ranging by about equal intervals between the lowest readable on the tachometer and the maximum allowable for the motor in question, at *constant normal excitation*, noting the speed and corresponding values of A and V at each.

(4) Repeat 2 and 3 for exciting currents (a) 50% below and 20% above normal respectively.

(5) Re-couple the brake and motor together and repeat 2–4 for no weight in the pan. From 2–5 the loss in Watts in the brake can be found.

N.B.—It will probably be necessary to exert a very small pressure by the finger on the brake in 5 to prevent it being

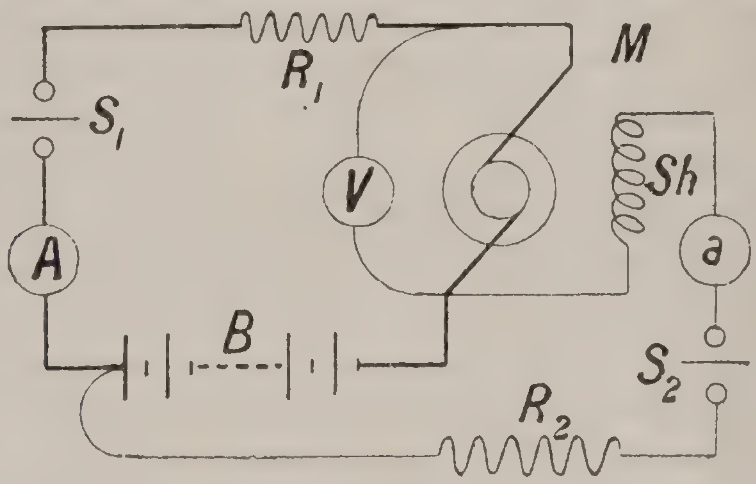


FIG. 96.

carried round as the pulley rotates. No appreciable error need be introduced due to this. Tabulate[1] your results as folllows—

Speed Revs. per min. (n)	Exciting current Amps. (a).	Without Brake.						With Brake.						Loss in Brake in Watts $W_B = (A_2V_2 - A_1V_1)$
		Volts.			Amps.			Volts.			Amps.			
		Ascending.	Descending.	Mean V_1.	Ascending.	Descending.	Mean A_1.	Ascending.	Descending.	Mean V_2.	Ascending.	Descending.	Mean A_2.	

[1] If a form of brake is used in which no loss of power can occur other than that incidental to its use when measuring power, then omit obs. 5 and also the last 7 columns in the above table and substitute a column headed A_1V_1 Watts to run motor at 0 load.

(6) Adjust both the exciting current (a) and the speed (n) to the normal for the motor being tested and keep both *constant*, then take a series of readings of A_3 and V_3 for some ten different loads varying from the smallest weight in the pan to the one which will give an armature current not exceeding 25% over normal.

(7) Repeat 6 for a 50% smaller excitation at the *same speed.*

(8) Repeat 6 and 7 for a 50% smaller speed.

(9) For a constant voltage across the armature maintained by means of R_1, load the brake with different weights, and note these and the corresponding speeds and currents through the armature at constant normal excitation, and tabulate as follows—

NAME . . . DATE . . .

Motor tested: No. . . . Type . . . Maker . . Weight . .
Resistance—Shunt coils r_s = . . . ohms. Armature r_a = . . . ohms.
Effective radius of brake pulley and band r = . . . ft.
Current in Shunt coils a_s = . . . amps.

Speed in Revs. per min. n.	Weight in pan W (lbs.).	Lesser weight w (lbs.).	Nett Torque $T = r(W-w)$ pound ft.	Total H.P. developed $H_1 = \frac{2\pi n T}{33000} + \frac{W_B}{746}$	Armature.		Total H.P. absorbed $H_2 = \frac{A_3V_3 + a_s^2 r_s}{746}$	% efficiency $= 100 \times \frac{H_1}{H_2}$
					Amps. A_3.	Volts V_3.		

Note.—The true H.P. developed = H.P. calculated + H.P. lost in brake itself.

(10) From experiments 2–5 plot curves between, volts V_1 as ordinates, and speed (n) as abscissæ, also with brake loss as ordinates and speed as abscissæ.

From experiments 6–8 plot curves for each speed, having efficiences as ordinates and total H.P. developed as abscissæ, also between the latter and speed as ordinates.

From experiment 9 plot the mechanical characteristic curve, having speed and current as ordinates and torque as abscissæ.

Inferences.—What can you deduce from the results of your experiments, especially from observation 9?

(93) Efficiency and B.H.P. of Direct Current Compound Wound Electro-Motors.

Introduction.—The Compound Wound motor is an automatically self-regulating one for maintaining constant speed independently of the magnitude of the load.

Without considering the theory of this regulation, which is outside the province of the present work, and for which the reader should refer to standard theoretical works, it may be

remarked that the desired result is obtained by employing the series and shunt coils to magnetize the field magnets differentially, *i.e.* while the shunt magnetizes, the series coils *de-magnetize.* This differential compounding results in the production of a nett field at any particular load sufficiently greater than what would be given by an equivalent pure shunt motor to cause the back E.M.F. to rise sufficiently to maintain the speed constant. The efficiency of such motors cannot manifestly be so high as one of the same size which is not wound in this way, since an extra amount of power is used up in producing the demagnetizing force which actually destroys part of the field.

Apparatus.—Precisely similar to that required for the shunt motor test (p. 248).

Observations.—These are the same as for the above-mentioned shunt motor test, and will not consequently be repeated here.

The experimenter should refer to and carry the present test out in the same way, but in coupling up at the onset, care must be taken to connect so that the series coils oppose the shunt and tend to demagnetize the magnets. Exactly similar tabulation of results and plotting of curves must be carried out with the inferences deducible.

N.B.—The applied E.M.F. to the motor should be maintained constant.

(94) Efficiency and B.H.P. of *Small* Direct or Alternating Current Electro-Motors. (Cradle-Balance Method.)

Introduction.—In testing small motors, such as from $\frac{1}{40}$ to $\frac{1}{4}$ of a H.P., difficulties present themselves in measuring the power developed by them or the work which they will do, owing to the relatively large amounts of extraneous friction introduced in applying the usual brake tests. In fact, in the case of the smaller power motor, this source of friction would entirely vitiate the results and make them worthless. The following method practically gets over this difficulty entirely, and may be carried out in one of two ways—

(*a*) The motor to be tested is suspended freely with its armature spindle in centres, or on friction wheels, the field magnet

system with its bed-plate, etc., being carefully *balanced* by counterpoise weights so as to bring the centre of gravity of the system in a line with the spindle. On the motor being supplied with electrical energy, and made to rotate and do work against the friction introduced at the face of its pulley by a stretched cord passing once round, the armature reacts on the field magnets tending to rotate them in the opposite direction with a certain force.

If then this action is resisted by a weight or force W attached to the field magnet system at a leverage L, then the moment of this force resisting the tendency, *i.e.* the torque, $= WL$.

Thus the arrangement is practically an electro-magnetic dynamometer in which the magnetic friction between armature and field magnets takes the place of mechanical friction in the ordinary dynamometer.

The preceding arrangement of the method has the disadvantage that the weight of the heaviest portion of the machine is resting on the shaft, and consequently there will be a bearing friction assisting the magnetic pull of the armature on the field.

A better arrangement in this respect is one devised by Prof. C. F. Bracket, which is merely a slight modification of the preceding one. It consists in fixing the motor in a "*cradle*" supported freely by knife edges resting on steel or agate planes, or on friction rollers, carried in a suitable fixed frame. The whole suspended system is very carefully balanced by means of counterpoise weights so that the centre of gravity lies in the axis of the motor shaft, this latter having been set in a line joining the knife edges of the cradle.

A horizontal balanced lever controls the cradle, the end of it either supporting weights or being attached to a spring balance. Thus it will be seen that now the weight of the armature only is on the bearings, and it is being used under ordinary conditions. The balanced lever might be graduated and a sliding weight used to run along it to balance the torque, but the arrangement of the spring balance shown in Fig. 285 is the simplest and easiest to manipulate. This method has a further advantage that the friction at the journals of the *motor* does not affect or vitiate the measurement, but in the case of the application to a dynamo it should be remembered that it does. It should be borne in mind

that a fruitful source of error may arise due to loss of power in driving the speed indicator. When small motors are being tested, care should be taken to choose an indicator that is very easily driven, and to drive it by means of a spiral or helical spring of, say, thin hard-drawn brass. Any eccentricity between the two shafts does not then matter so much as it would if they were direct coupled.

Apparatus.—That required for this test is precisely similar to what is detailed under one or other of the preceding methods of testing series, shunt, or compound wound direct current motors or alternating current single phase motor, according to which of these types of motors the one being tested belongs. In addition the cradle absorption dynamometer is needed, for a complete description of which see p. 621.

Observations.—As, with the exception of the somewhat different type of brake herein to be manipulated, the whole test will be precisely similar to one of the foregoing motor tests, depending on which kind of electromotor is to be tested, the *rationale* of this present test will not be repeated here. The experimenter should refer to the proper corresponding test and carry out the present one in an exactly similar manner, tabulating and plotting the results in just the same way.

Though the following expression will be found in connection with the description of the cradle dynamometer on p. 621, we may repeat that if W = weight or force applied at the end of the cradle lever in order to keep the same at zero when the motor is doing work, and if L = distance between its point of application and the fulcrum of the cradle, then the torque exerted by the motor $= WL = T$, and the work it does per sec. $= \omega T = 2\pi n T$.

Where n = speed in revs. per sec.—

$$\therefore \text{H.P. developed} = \frac{2\pi n T}{550}.$$

Consequently if different tensions are applied on the cord wrapped round the motor pulley, causing it to do various amounts of work, thereby taking in different currents (A) amps. at different voltages V, the efficiency of the motor at each load is—

$$\text{Efficiency} = \frac{\text{H.P. developed}}{\text{H.P. absorbed}} = \frac{2\pi n T \times 746}{550 \times AV.}$$

(95) Efficiency and B.H.P. of Direct Current Electro-Motors by Swinburne's Electrical Method.

Introduction.—In the usual brake tests it is difficult and often impossible to obtain very accurate results, owing to variation of the co-efficient of friction between the rubbing surfaces and the resulting jerky behaviour of the brake. The advantage of any method, therefore, of measuring the input and output of a motor by solely electrical means will at once be apparent, as it is possible to obtain much more accurate results with such a method.

The present method, which is purely an electrical one, is due to Mr. James Swinburne, and is sometimes termed the "*Stray Power*" method. The principle of it and all similar methods is based on the fact that

Total Power given out = Total Power put in − Power lost internally, or in symbols, $W_O = W_I - W_L$; where the suffixes O, I, and L denote the *output*, *input*, and total *losses* in Watts (W) respectively.

We thus at once obtain the commercial efficiency of the motor to be $\frac{W_O}{W_I} = \frac{W_I - W_L}{W_I}$.

The input in Watts W_I given to the motor is at once obtained by the product of the volts and amperes of the supply. The total loss W_L in Watts we will consider now more in detail, and which in any machine is made up as follows: (a) the copper losses L_C in armature and exciting coils due to heating by the passage of current, and which can easily be calculated when the currents and resistances are known; (b) the friction losses L_F due to air churning, journal and brush friction; (c) magnetic frictions or iron losses L_m due to eddy or Foucault currents and magnetic hysteresis.

Hence total internal loss $W_L = L_C + L_F + L_m$, and to the sum $(L_F + L_m)$ Mr. Swinburne has given the somewhat appropriate name of "*Stray Power.*" The copper losses are calculable as follows—

Let C = total current flowing into the motor from the supply, and let R_a, R_{se}, and R_{sh} be the resistances of the armature, series

coils, and shunt coils respectively of any motor of which R_{sh} can be measured by a Wheatstone Bridge, and R_a, R_{se} by the "Potential Difference" method (p. 84) or ammeter and voltmeter method, p. 86. Then we shall have for a

Series motor $L_C = C^2(R_a + R_{se})$,

Shunt motor $L_C = \frac{V^2}{R_{sh}} + \left(C - \frac{V}{R_{sh}}\right)^2 R_a$, where V = normal working voltage,

Compound motor (long shunt)

$$L_C = \frac{V^2}{R_{sh}} + \left(C - \frac{V}{R_{sh}}\right)^2 (R_a + R_{se}),$$

Compound motor (short shunt)

$$L_C = C^2 R_{se} + \frac{(V - CR_{se})^2}{R_{sh}} + \left(C - \frac{(V - CR_{se})}{R_{sh}}\right)^2 R_a.$$

The remaining losses, *i.e.* the stray power $(L_F + L_m)$, can readily be obtained by running the motor at no load, *i.e.* with no other load than its own friction, eddy currents and hysteresis, at normal excitation of the field. Then we have

$$(L_F + L_m) = AV_a - A^2r_a = \text{Stray Power},$$

where A now = current flowing into the motor armature at voltage V_a across the armature, and A^2r_a is the copper loss in the armature occurring for this current and voltage.

Note.—Only quite a small current *at the normal voltage* of the motor is required to be furnished by an auxiliary source of E.M.F., and if R_a is very small, A^2r_a can be neglected in comparison with AV_a in this last formula.

Apparatus.—Motor M to be tested, which for purposes of discussion merely we will assume is shunt wound; voltmeter V; low reading long scale ammeter A; rheostat R (p. 606); tachometer; complete Wheatstone Bridge set (W.B.); two-way voltmeter key K (p. 587); switch S_2; source of E.M.F. (E) at least equal to that for which the motor was built; rheostat r (p. 599) in the field coil circuit.

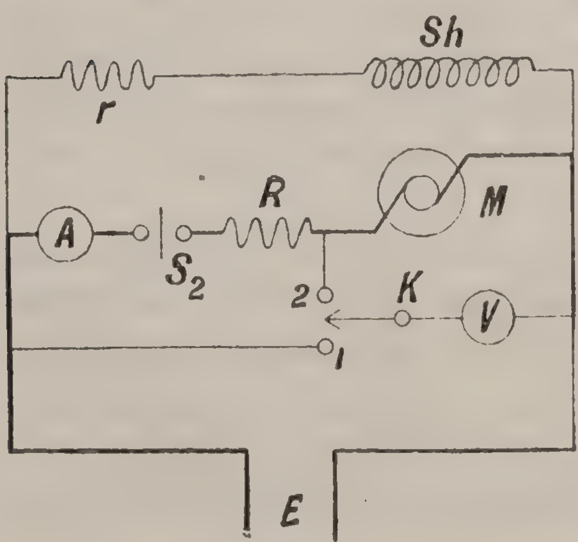


Fig. 97.

Observations.—(1) Connect up as in Fig. 97, and adjust the instruments V and A to

zero if necessary. Insert E, when the field should then be excited to the normal amount, which can be seen by closing $K1$ and noting whether the normal voltage is read off on V.

(2) With R at its *maximum value* (not less than about 10 ohms), close S_2, adjusting R and if necessary the excitation by the rheostat r, so that the machine runs at its normal speed. Now note, by closing $K2$, the volts V_a across the armature terminals and the current A amps. flowing through it.

(3) Repeat 2 at the same excitation for some ten different speeds in all, both below and above normal, and tabulate as shown.

(4) Open E, S_2 and K, and measure by suitable means the resistance R_a of the armature and R_{sh} of the shunt, remembering of course to disconnect one from the other at the time.

(5) Calculate the B.H.P. and commercial efficiency of the motor at *normal voltage* V for some ten different assumed values of current C supplied to the machine ranging from 0 to full load by about equal increments, and tabulate as shown in the larger table.[1]

Speed in R.p.m.	Stray Power Readings (from obs. 2 and 3).			Total Intake Watts Running Light AV_a.
	Volts V_a.	Amps. A.	Watts $AV_a - A^2R_a$.	

NAME . . . DATE . . .

Motor tested: No. . . . Normal Voltage = . . . Volts. Resistance: Armature = . . . Ohms.
Type . . . „ Current = . . . Amps. Shunt = . . . „
Maker . . . „ Speed = . . . Revs. per min. Series = . . . „

Normal Speed Revs. per min.	Power supply assumed for calculation.			Stray Power Measurement in Obs. 2.			Losses.		Calculated Output $W_O = W_I - W_L$.	Commercial Efficiency $= 100 \frac{W_O}{W_I}$ %
	Volts V.	Amps. C.	Total Input $W_I = VC$.	Volts V_a.	Amps A.	$L_F + L_m = AV_a - A^2R_a$ Watts.	Copper (calculated) L_C.	Total $WL = (L_C + L_F + L_m)$.		

(6) Plot the following curves having

(*a*) Efficiency as ordinates and output W_O as abscissæ.

(*b*) Stray power as „ and speed as „

(*c*) Output W_O as „ and input W_I as „

Inferences.—State very clearly all that you can infer from the above experimental results.

[1] See note to larger table, p. 222–3.

S

(96) Efficiency of Direct Current Electro-Motors by Poole's Electrical Method.

Introduction.—This method,[1] due to Mr. Cecil P. Poole, is an electrical one entirely, and enables the efficiency of an electromotor to be obtained without using an absorption brake. The rated B.H.P. of the machine is assumed for the purposes of calculation, and the whole essence of the test consists in obtaining the armature current at which this rated output is obtained.

Let A be this full load armature current.

V = the normal voltage which the motor should have.

W = the normal rated B.H.P. of motor, reckoned in Watts.

v and a = the measured quantities as detailed below.

Then the armature core + friction losses $w = (V - v)\,a$ Watts,

and the armature resistance $r = \frac{v}{a}$ ohms.

$$\text{Hence } A = \frac{V - \sqrt{V^2 - 4(W + w)\,r}}{2\,r} = \frac{V}{r}\left(0{\cdot}5 - \sqrt{\tfrac{1}{4} - \frac{(W - w)r}{V^2}}\right)$$

This value for A is based on the assumption that the core losses, armature friction, windage, and eddy currents in the pole pieces all remain constant from 0 to full load. While this is not strictly the case, the error introduced is practically negligible.

Apparatus.—Suitable source of supply of slightly higher voltage than the normal required for the motor to be tested; rheostat p. 599; low-reading long scale ammeter (a); voltmeter V to read the normal voltage; low-reading long-scale voltmeter (v); and if the motor is shunt or compound wound, an ammeter to measure the normal shunt current a_{sh}; switch.

N.B.—If the resistance of the shunt r_{sh} be known, then the last-named ammeter may be dispensed with for $a_{sh} = \frac{V}{r_{sh}}$ amperes.

Observations.—(1) Connect up the above apparatus so that the rheostat and ammeter (a) are in series with the armature alone, and the switch, so that it cuts off the supply entirely from motor and all apparatus, the voltmeter V being across the armature terminals, the shunt coils of the motor being across the mains.

[1] The rationale of the method first appeared in the *American Electrician*, to which the author is indebted for it.

(2) With the rheostat *full in*, close the main switch when the shunt will at once be fully excited. Now gradually cut out resistance in the armature circuit, thereby running up the speed, until V reads the normal voltage across the armature, thus running light. Now note the small armature current (a) amperes flowing.

(3) Switch off the shunt circuit and block the armature to prevent it moving. With the rheostat *full in*, close the main switch and again pass the same current (a), as in observation (2) above, through the armature while stationary, noting the corresponding fall of potential (v) volts across the armature terminals by means of the low-reading voltmeter, and switch off.

(4) Repeat observation (2 and 3) twice or three times and take the mean of the respective values of (a) and (v), calculating the efficiency Σ of the motor on full load from the relation—

$$\Sigma = \frac{100\ W}{V\ (A + a_{sh})}\ \%$$

and tabulate your results as follows—

NAME . . . DATE . . .
Motor Tested : No. . . . Maker . . . Rated B.H.P = . . .
Normal Voltage = . . . Speed = . . . Current = . . .
Resistances : Armature = . . . Shunt = . . . Series = . . .

Armature Current running free. (a) amps.	Volts across Armature Still (v).	Losses $(V-v)\ a = w$	Armature Resistance $r = \frac{v}{a}$	Field Current a_{sh}	Full load Armature Current A	Efficiency of Motor Σ

Inferences.—State clearly any advantages or disadvantages which you consider the method possesses.

General Considerations Relative to the Testing of Asynchronous Alternating Current Induction Motors.

While it is not proposed to discuss either the construction or the theory of action of such machines, certain considerations relative to the testing of both single and polyphase induction motors may with advantage be noted. In all cases they are self-starting by reason of the *rotating magnetic field* set up by the supply current, whether single or polyphase, flowing in the windings of the stator or fixed portion of the motor. It is, however, only in single-phase types that after reaching full speed the rotating field (produced only during the starting-up period) is changed by switching to a simple alternating, or reversing, or pulsating field.

The speed attained at the end of the starting period with no pulley load is called "full" or "synchronous" speed, but in all induction motors the speed of the rotor decreases as the load increases.

If (f) = the periodicity of the supply in cycles per sec.,
(n) = the speed of the rotor in revs. per sec.,
(p) = the number of pairs of poles in the stator,
then synchronous or full speed is the speed of the rotating field $= \frac{f}{p}$ revs. per sec., while the difference between the speeds of the field and rotor, called the "slip" $= \frac{f}{p} - n = \frac{f - np}{p}$ revs. per sec., and $\left(\left(\frac{f - np}{p}\right) \div \frac{f}{p}\right) \times 100 = \left(\frac{f - np}{f}\right) 100$ = the slip in percentage of full speed, which varies from about two or three in large motors to as much as twelve in very small ones. We therefore see that the slip equals the periodicity of the rotor currents.

Measurement of Slip.—The last-named fact is made use of in the following method of measuring slip, but is applicable only in the case of induction motors with slip-ring rotors. Connect preferably a moving-coil permanent magnet D.C. ammeter in one

of the leads between rotor and starter, then since such an instrument indicates for currents in one direction only, the number of impulses given to, or kicks (K) of, the pointer per min. in the same direction will directly equal the number of complete cycles per min. of the induced slow period rotor currents—in other words the slip. If (f) = periodicity of the supply to the stator, then the percentage slip $= \frac{K}{60 \times f} 100.$

Thus, if $K = 120$ kicks per min., the slip $= \frac{120}{60 \times 50} \times 100$ $= 4\%$ with a 50 ~ per sec. supply.

If a dead-beat moving soft iron needle A.C. ammeter is used, the number of kicks per min. would be doubled, for the same value of f and slip, and would, even if the ammeter was sufficiently dead beat to indicate with such rapidity, be impossible to count. With even a very dead-beat moving coil D.C. ammeter, a 5 or 6% slip is about the maximum measurable by this method. A slight variation of the above method consists in counting the oscillations of a light pivoted compass needle placed above or below one of the leads between rotor and starter, the lead having a direction N. and S. so that the needle lies parallel to it when no current is flowing. The slip is then obtainable as before.

The above are direct methods of measuring slip, but if a long-range accurate tachometer is available, the slip can be obtained usually with sufficient accuracy by reading the rotor speed (n_1) running light, and (n_2) at any load when the slip is given by $\frac{n_1 - n_2}{n_1} \times 100\%$.

Determination of Slip by Calculation.—If an induction motor has a three-phase wound rotor and both the rotor current (A_2) and resistance (R_2) per phase in each case are known, the slip (S) in cycles per sec., or in percentage of the supply frequency (f_1), or in revs. per min. or per sec., can be calculated for the corresponding load as follows—

If f_2 = frequency of the rotor currents,

W_2 = mechanical output from the rotor of the motor (in watts),

p = number of pairs of stator poles,

W_1 = power (in watts) transmitted electro-magnetically by the rotating field in the stator to the rotor,

w = power (in watts) lost in the rotor $= 3A_2{}^2R_2$ approximately.

Then torque $\times \dfrac{2\pi f_2}{p} = W_2$

and torque $\times \dfrac{2\pi f_1}{p} = W_2 + w (= 3E_1A_1 \cos\phi$ for a 3-phase stator supply)

$$\therefore \frac{f_1}{f_2} = \frac{W_1}{W_2} = \frac{W_2 + w}{W_2}$$

and by a well-known rule in proportion we therefore have

$$\frac{f_1}{f_1 - f_2} = \frac{W_1}{W_1 - W_2} = \frac{W_1}{w} = \frac{W_2 + w}{w}$$

where $W_1 - W_2 = w$

$$\therefore \text{ the slip} = \frac{f_1 - f_2}{f_1} = \frac{w}{W_2 + w} = \frac{3A_2{}^2R_2}{W_2 + 3A_2{}^2R_2}.$$

Determination of Frequency, Slip, and Speed (Stroboscopic Method).

Introduction.—Although the measurement of such quantities as those mentioned above—by this method—is by no means common, it probably only needs the advantages and accuracy of the method to be realized in order to bring it into much more general use.

Measurement of Frequency and Slip.—The phenomenon and principles of stroboscopy can be applied in the measurement of either the frequency of an alternating current supply from an alternator, or the slip of an induction motor, as follows: A black disc having white radial lines is fixed concentrically on the shaft of an A.C. motor run from the supply, and is illuminated by an A.C. arc lamp fed from the same supply. Now the illumination from the lamp will vary periodically and flicker with the supply frequency, and when the speed of the stroboscopic disc corresponds with this supply frequency, *i.e.* when the angular velocities of the two are equal, the white lines will always be illuminated in the same place and appear to be at rest. If the

speed of the disc is greater than that corresponding to the frequency of supply, the white lines will appear to slowly rotate in the *same* direction as the disc; whereas if the speed has a smaller value, the lines will appear to rotate in the *opposite* direction. The last condition will obtain with an induction motor, and if the number of white lines equals the number of pairs of stator poles, they will rotate in the opposite direction to that of the disc with the same number of revolutions per min. as those lost by the motor, *i.e.* as the slips.

For example: the rotating field in a 2-pole stator on a 50 ~ supply will make one revolution in the periodic time of the current, or will rotate with a speed of $50 \times 60 = 3000$ revs. per min. If the slip between rotor and field is 5% ($= 5 \times 30 = 150$ revs. per min.), a single white line on the black disc will appear to rotate backwards at a speed of 150 revs. per min., and will also make one complete revolution in the periodic time of the current.

With a 4-pole motor and the same slip and supply frequency, the speed of the rotating field equals 1500 revs. per min., slip equals 75 revs. per min., and each of the two white lines will appear to rotate at 75 revs. per min., which can be counted against time, and the slip thereby at once obtained. Sectors, alternately white and black, can be painted on the disc or even the pulley, and used instead of the black disc with radial white lines if so desired.

Measurement of the Resistance of Single and Polyphase Windings.—This is usually effected by the ammeter-voltmeter method with direct current (see p. 86) applied to a single phase-winding in the case of a single-phase generator or motor, and to each of the phase-windings separately of 2-phase machines.

Thus, if (r) equals resistance of each phase-winding, we see that the total copper loss in a single phase machine equals $A^2 r$, and in a 2-phase machine equals $A_1^2 r_1 + A_2^2 r_2$; or if in the latter case the resistances of the two windings are equal as they should be, and usually are, we have $r_1 = r_2 = r$, and if $A_1 = A_2$ then the total copper loss $W_c = A^2 \times 2r$, where A is the current in either phase, and $(2r)$ the so-called *equivalent* resistance of the machine.

In 3-phase windings, the resistance between any two terminals

is, with star connection, that of 2 phase-windings *in series* (as seen from Fig. 143 *a*), and therefore $= 2r$; while with mesh connection (Fig. 143 *b*) we see that between any two terminals there are two circuits *in parallel*, composed of 1 phase-winding in parallel with the other 2 phase-windings in series, or (r) in parallel with ($2r$), *i.e.* a terminal resistance of

$$\frac{1}{\frac{1}{r}+\frac{1}{2r}} = \frac{1}{\frac{2+1}{2r}} = \frac{2}{3}r.$$

Now, if without troubling to trace the connections in order to see whether they are star or mesh, the resistance between the three pairs of phase-terminals are measured and added together, the sum $\div 2$ will equal the *equivalent* resistance of the whole stator or rotor windings, and the total copper loss in the stator or rotor $=$ (line-current)$^2 \times$ equivalent resistance.

(97) No-Load "Open Circuit" Test of an Induction Motor on a varying Voltage, constant Normal Frequency Supply. (Rotor running Light at No Load.)

Introduction.—Under these conditions the motor will run at its maximum possible speed, namely that corresponding almost, but not quite, to true synchronism, and therefore with an almost zero slip—the small difference being necessary for overcoming the small losses due to windage, mechanical and magnetic frictions, and copper loss due to the no-load running current. The test can be operated, of course, on single-, two-, or three-phase motors, but we shall assume the use of a three-phase motor here on account of the connections being slightly more complex.

Such a motor may have either a *squirrel-cage* (short circuited) rotor or a *wound* rotor with slip rings. If the former, it may be started up from full voltage mains either by a *star-delta* switch or through an auto-transformer or sectioned choker, depending on its size. If it has a wound rotor, the starting rheostat connected to this is put to "full in" and the stator then switched directly to the supply. The starter is then gradually cut out to short circuit as the speed increases. If

now, with the motor running at full speed, normal voltage, and frequency, the voltage is gradually decreased, the speed will remain practically constant until the lower voltages are reached, when it will fall off rapidly.

Both the stator and rotor current will also decrease gradually, the former owing to a decrease in magnetizing and core-loss current producing the stator flux and depending on the voltage, the latter in an inverse proportion to the strength of the rotating field and voltage.

As the voltage falls the idle or magnetizing component of the current will also decrease, while the energy component overcoming frictions will remain much the same in value, hence

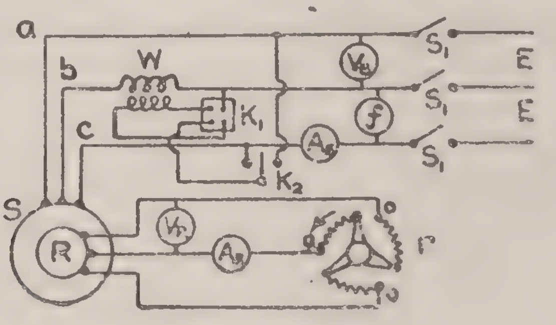


FIG. 98.

the ratio of idle to energy current will decrease and the power factor will in consequence rise.

If $V_S =$ normal voltage per phase on the stator

and $V_R =$ the corresponding maximum voltage per phase of the rotor indicated when this is turned through a polar pitch with slip rings open-circuited. Then $\frac{V_S}{V_R}$ is called the ratio of transformation, which is not equal to the ratio of the stator and rotor currents owing to the magnetizing current of the stator. The induced E.M.F. in the rotor circuits varies directly as the slip, and has a frequency $f_R =$ the slip. Thus the reactance of the rotor circuits ($= 2\pi L_R f_R$) bears a constant ratio to the slip.

As the rotor current is not usually measured, the ratio of transformation enables it, and also the most suitable starting resistance, to be calculated, knowing the stator current at any load.

Apparatus.—Source E of three-phase alternating current, preferably a motor-driven alternator, the speed and field of

which are each variable between wide limits; three-phase switch S_1; voltmeters V_S V_R; ammeters A_S A_R; frequency meter f; wattmeter W; reversing key K_1 (p. 585); and two-way key K_2 (p. 587) for fine-wire circuit of W; three-phase induction motor, of which (S) is the stator and (R) the rotor and (r) the starter.

Note.—By the use of only one wattmeter, with its pressure-coil connected through K_2 to the remaining two mains in rapid succession, we assume the motor to be electro-magnetically balanced, or equally loaded, in the three phases (see p. 389). On no load, such is usually the case, for the current A_S is small, and therefore any inequality in the ampere turns, resistance, or reactance of the windings is nearly negligible in effect compared with what it would be on load. With phase-windings *unequally* loaded or balanced, two wattmeters must be used to obtain the true power absorbed (see p. 392).

The ammeter A_R in the rotor circuit should be very dead beat and of the moving needle type, and should be of low resistance so as not to throw out the balance of the rotor currents. If the rotor R is of the squirrel-cage type, V_R A_R and r cannot be used.

Observations.—(1) Connect up as in Fig. 98, levelling and adjusting to zero, such instruments as need it. On starting any machine always see that its oiling arrangements are working properly before doing anything else.

(2) With R running light at no load, adjust the voltage V_S and frequency f of the supply E to the normal values for the motor, and note the readings of all the instruments under this supply condition, and also (with the same *frequency* kept *constant*) for a series of values of V_S (by field regulation) decreasing by about equal amounts to the point where the speed *begins to decrease*, and from this point by *smaller and more gradual* decrements of V_S until the speed decreases too rapidly to read.

Note.—After the speed begins to fall, sudden changes in V_S must be avoided, while the simultaneous readings of all the instruments must be taken rapidly.

W must be read, first with its volt coil across (ab) and then with it across (bc) at each voltage V_S, by using the key K_2. Further, in some cases (not all), this change of connection will be accompanied by a reversal of direction of the deflection of W depending on the magnitude of the power factor. The re-

versing-key K_1 must in such cases be turned through 90°, so as to bring the deflection on to the scale again; but the reading must now be considered $-^{ve}$ and subtracted from the other to give the total watts (see p. 392), otherwise when the readings across *ab* and *bc* are both on the scale and therefore both $+^{ve}$, their sum gives the total watts.

(3) With the frequency (f) constant at normal value for the motor, raise the voltage V_S from 0 by small and very gradual steps until the speed begins to increase too rapidly to enable readings to be taken. The simultaneous reading of all instruments at each voltage must be done rapidly. Tabulate all your results as follows—

Induction Motor: No. . . . Maker . . . Type . . .
Full-load output: B.H.P. = . . . @ . . . r.p.m. Volts = . . . Amps . . . Frequency . . .
Equivalent Res. (hot) of Stator Windings = . . . ohms: of Rotor Windings = . . . ohms.

Speed by		Supply						Apparent Watts $\sqrt{3}\,A_S V_S$.	Power Factor $\cos\phi = W/\sqrt{3}\,A_S V_S$.	Rotor		Transformation Ratio V_S/V_R.	Value of A_S/A_R
					Watts.								
Tachometer.	Kicks of A_R.	Frequency (constant) f.	Volts V_S.	Amps A_S.	W_{ab}.	$\pm W_{bc}$.	Total $W = W_{ab} \pm W_{bc}$.			Volts V_R.	Amps A_R.		

(4) Plot the following curves (from obs. 2 and 3) having values of stator volts V_S as abscissæ with (1) speed, (2) intake watts W, (3) stator amps. A_S, (4) cos ϕ, (5) ratio $\frac{V_S}{V_R}$ as ordinates in each case.

Inferences.—From a careful study of the shapes and dispositions of the curves relatively to the axes and of the tabular results, state all that can be deduced.

(98) No-Load "Short-circuit" Test of an Induction Motor on a varying Voltage, constant Normal Frequency Supply. (Rotor kept stationary and short-circuited.)

Introduction.—Under these conditions the slip will be 100% since the speed is zero, while the power absorbed will almost

wholly consist of copper loss $\propto$ to the square of the current. The only remaining source of loss is that due to hysteresis and eddy currents in the iron which will be small, owing to the low induction density reached with even the maximum voltage it is possible to use in this test. Further, it will be noticed, from a reference to test No. 137, that the motor approximates to a static transformer with a stator primary and rotor secondary under the conditions for maximum magnetic leakage which the stator windings maintain in the air gap between stator and rotor.

Under stationary conditions the ratio of stator to rotor current is practically a constant and approximates to the ratio of transformation of the motor. Under running conditions the ratio of currents departs from constancy, due to the no-load current taken by the motor at normal voltage, and no longer approximates to the ratio of transformation.

Apparatus.—That indicated for the preceding test (p. 265), but without K_1K_2, an additional wattmeter now being used with its current coil in main a or c (Fig. 98), one end of each of the volt circuits of the two wattmeters, to be denoted by w_1 and w_2, being connected to the third main as indicated in Fig. 143*c*. The reason for now using two wattmeters is that the heavier stator currents to be used in this test, which will depend mostly on the resistances of the windings, will show up any slight want of symmetry, and may (or may not) result in the unequal current loading of the three phases—a condition necessitating the use of two wattmeters (p. 392). A_S A_R w_1 and w_2 must also now be capable of taking the heavier currents used in the present test.

Observations.—(1) Connect up as in Fig. 98, levelling and adjusting to zero such instruments as need it. See that the lubricating arrangements of the supply set are working properly.

(2) With the *rotor R short-circuited* and *prevented from rotating* and the supply-frequency (f) constant at normal value, take the readings of all the instruments at each of a series of supply voltages V_S, increasing from zero to a value which will produce a stator current A_S of, say, 50 % in excess of that of full load, and tabulate as for the last test (p. 267).

(3) Plot curves, having values of V_S as abscissæ, with values of W, A_S, A_R and cos ϕ respectively, as ordinates. Also curves between A_S as ordinates, with A_R as abscissæ, for this test, and from the readings of the last test No. 97 on the same curve-sheet for comparison.

Inferences.—From a careful study of the curves state clearly all that can be deduced from the test.

(99) Efficiency and B.H.P. of Single Phase Alternating Current Electro-Motors.

Introduction.—The somewhat rapid development of the distribution of electrical energy by single phase alternating currents in recent years has brought with it the introduction of single phase alternating current motors, of which, up to comparatively recently, there has been no practical commercial instance. Now, however, there are several forms, but none of them are able to compete with the direct current motor in the matter of efficiency and powers of starting under load with the amount of electrical power absorbed in doing so. There are two classes of alternating single-phase motors, known as the *Synchronous* and *Asynchronous* types. The former cannot start themselves but have to be run up, by a separate source of power, into synchronism with the periodicity of the supply current; then, on being switched into circuit, they run perfectly synchronously with the generators, *i.e.* at constant speed, for wide variations of load from 0 to considerably over full load, and are of course separately excited. The latter class are self-exciting and self-starting (on very small loads) by using suitable means, but are non-synchronous, and the difference between the speeds of rotation of the magnetic field and the rotating armature is called the "Magnetic Slip" or "*Slip*" simply. This generally only amounts to a small percentage at full load.

The self-starting property is obtained by producing a rotatory magnetic field at starting, caused by diphasing the current in two separate circuits by means of the suitable use of either self-induction or capacity, one circuit being cut out when the motor gets up speed.

The fixed portion of the motor (*i.e.* field magnets) through the

winding of which the supply current flows is usually termed the "*Stator.*" The rotating portion (*i.e.* the armature) is termed the "*Rotor*," and usually consists of short-circuited conductors carried on a well-laminated drum. There is no electric connection in most cases to the rotor, or between rotor and stater. It will also often be found that the best efficiency is not at normal full load, which is analogous to the series wound direct current motor in this respect. Speaking broadly, it may be said that single-phase motors should be self-starting, and this on a current certainly not exceeding that taken at full load. The *power factor* should be high.

A motor built for a given periodicity will not give as a rule its full power when supplied with a current of a much higher periodicity, while it will take too much current with a lower periodicity.

The efficiency of any motor = the total power given out ÷ the total "mean power" absorbed, both being reckoned in equivalent units. In the present and similar tests the true mean input cannot be obtained by the product of the amperes and volts, as in the case of direct current motors, owing to the "phase difference" between the current and pressure, but must be obtained by means of a non-inductive Wattmeter. The output, or B.H.P., is obtained by an absorption dynamometer, which is a modified form of Prony brake. Such brakes waste, in heat, all the power developed by the motor from friction, but at the same time give a measure of this power. No lubricant is usually needed, but a little black lead may be applied to the pulley if the brake is jerky.

Apparatus.—Alternating current motor M to be tested; brake complete with weights; non-inductive Wattmeter W; alternating current ammeter A and voltmeter V; switch S; rheostat R (p. 597); tachometer; source of alternating current E, preferably one that can be varied.

Tests.—(1) Connect up as indicated, and adjust the pointers of all the instruments to zero, levelling such as need it. See that all lubricators in use feed slowly, and that the resistance switch (S) is open.

(2) Adjust the speed and excitation of the alternator so as to give the normal voltage and frequency required for M, and remove the brake.

(3) Make R a maximum; *i.e.* in the present case put resistance

switch S to *start*, and when the motor has got up speed throw S over from start to *full*, then, when the speed has become steady, note it and the readings of A, V, and W.

(4) Place the brake in position, and with no weight in the pan, again note the motor speed and readings of A, V, and W.

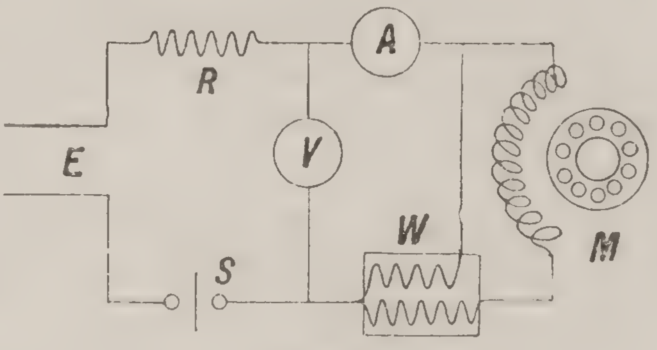


FIG. 99.

(5) Repeat 4 for about ten loads, rising by about equal increments of weight to the maximum, the voltage and frequency being kept constant.

(6) Repeat 4 and 5 for a higher and lower frequency than the normal.

(7) Determine the power required to just start M by removing the brake, turning S to *start*, and adjusting the speed of the alternator to give normal frequency (to be kept constant).

(8) Carefully and gradually raise the voltage (at constant speed) by means of the excitation until M just starts, then instantly note the readings of A, V, and W. Repeat this three or four times and take the mean.

(9) Repeat 7 and 8 for about five different frequencies, rising by about = increments to about 20% above normal.

(10) Determine the relation between the speed of M and frequency of supply by removing the brake, and when the motor has got up speed, turning the Switch (S) to "full." Then for constant normal voltage note the speed of M and readings of A, V, and W for about ten different frequencies, rising up to about 20% above normal.

(11) Determine the effect of variation of voltage at constant normal periodicity with the motor running light by altering R, and noting the speed and readings of all the instruments. Tabulate all your results as shown.

NAME . . . DATE . . .

Motor: No. . . . Type . . . Made by . . . Effective Diam. of Pulley and band $d =$ ft.
Normal B.H.P. = . . . at . . . volts and . . . revs. per min., and frequency = . . . $\sim$ per sec.
Alterations per revolution of Dynamo $K =$ . . .

Speed in revs. per min. of		Frequency $\sim \frac{K \times n}{60}$ per sec.	Volts V.	Amps. A.	Power Absorbed.			Weight in Pan W lbs.	Lesser weight or balance reading w lbs.	Nett Weight in Brake $(W-w)$ lbs.	B.H.P. developed $H_2 = \frac{\pi d N (W-w)}{33000}$	Power Factor $\cos\theta = \frac{W}{AV}$.	Angle θ.	Efficiency $\frac{H_2}{H_1} \times 100$ in %	Slip.
Alternator n.	Motor N.				Apparent Watts AV.	True Watts W.	True H.P. $H_1 = \frac{W}{746}$								

Note.—The nett weight on brake = (weight of scale pan and weights) − reading of spring balance.

(12) In experiments 1–6 plot curves having values of (a) power factor, (b) efficiency, (c) true power absorbed respectively as ordinates, and B.H.P. developed as abscissæ, for each frequency.

(13) In experiments 7–10 plot curves having (d) true power required to start, (e) speed of motor, as ordinates and frequency as abscissæ, in each case.

Inferences.—What can you deduce from your experimental results? Taking the cost of electrical energy for power purposes at 2d. per B.T.U., find the cost per B.H.P. per hour, and also when M is running on no load at normal voltage and frequency.

(100) Efficiency-Load Test of a Polyphase Induction Motor. (Absorption - Brake Method.)

Introduction.—The efficiency of any electro-motor

$$= \frac{\text{B.H.P. developed}}{\text{E.H.P. absorbed}} = \frac{\text{B.H.P.}}{\text{B.H.P.} + \text{internal losses}}.$$

The internal losses in an induction motor comprise (1) copper losses in stator and rotor windings, (2) iron losses (hysteresis and eddy currents) in stator and rotor cores, (3) mechanical friction due to windage, journals, and brushes, if it possesses a slip-ring rotor.

If A_S and r_S = the current and resistance, respectively, per phase of stator winding and A_R and r_R = the current and

resistance, respectively, per phase of rotor winding; then, if this latter is of a three-phase type, the stator copper losses are—

$A_S^2 r_S$ for single-phase; $2A_S^2 r_S$ for two-phase; and $3A_S^2 r_S$ for three-phase induction motors, while the rotor copper loss is $3A_R^2{}^2 r_R$.

For a given iron core, we have seen (p. 354) that the expression for the iron losses contains two variables only, namely, the *frequency* and the *induction density* $\propto$ to flux, and dependent solely on the supply voltage and number of stator turns. Hence the iron loss is independent of load. Further, since the frequency of the rotor currents and consequently of the flux in the rotor core equals the slip, which is only some 5 % of the speed of the stator field, it follows that the iron loss in the rotor core will be small compared with that in the stator core and the other losses, and will increase slightly with speed. The friction losses being $\propto$ to speed, will be sensibly constant at all loads in an induction motor, since the speed of such a motor has a variation of some 5 % only. It will therefore be at once realized that the copper losses (increasing as the square of the current) are mostly responsible for the rapid increase of the total internal loss as the load increases.

The supply current to the stator of an induction motor is composed of two components—

(*a*) One which may be termed the no-load or magnetizing component, producing the rotating magnetic field, and which is not only in quadrature with the supply voltage but nearly constant at all loads.

(Owing to the air-gap between stator and rotor-cores, the ampere turns of excitation, and hence the magnetizing component necessary to produce a given flux, is much higher, and the power factor much lower, than if the magnetic circuit was a closed one, and therefore an induction motor takes a considerable no-load current which may be from a quarter to one-third of full-load current. The smaller the air-gap the smaller this current, the greater the power factor and output of the motor for a given size. For this reason the air-gap of such motors is reduced to a mere clearance for rotation.)

(*b*) The other, which may be called the load-component, out

of phase with the voltage, but producing a field in the stator equal and opposite to that produced by the rotor currents in the stator, and hence balancing the demagnetizing effect of the rotor-induced currents on the stator field.

This load component increases directly with the B.H.P. output of the motor and $= A_R \times \dfrac{\text{turns on rotor}}{\text{turns on stator}}$.

Thus it will be seen that for the rotating field to have a constant strength, the stator current taken at no load will just suffice to produce this requisite field-strength and provide for the iron and friction losses. As the load increases, the increase in the rotor ampere terms is balanced by an equal and opposite increase in stator ampere turns, and we have the following relation, viz. that the

$$\text{Total stator amp. turns} = \overline{\text{total rotor amp. turns} + \text{no-load amp. turns}},$$

or

$$\text{Total stator current} = \overline{(\text{rotor current} \div \text{ratio of transformation}) + \text{no-load current}},$$

the line above denoting that the sum is vectorial and not algebraical.

The total power given out, *i.e.* the B.H.P., can be measured either by means of an absorption dynamometer brake in the manner already clearly defined in the previous tests, or by making the motor to be tested drive a direct current dynamo, the *commercial efficiency* of which is accurately known at various loads. This method should be adopted whenever possible, as it has the advantage, when carried out properly, of being more accurate than the ordinary brake methods. The method consists in suitably driving the dynamo from the motor to be tested either by means of a *thin* supple (pliable) belt or by the direct coupling of their shafts (placed accurately in alignment), through a flexible coupling or helical spring sufficiently strong for the purpose, thus avoiding the difficulty of getting their shafts *exactly* in true alignment. The belt arrangement also obviates the same difficulty, but it must be very pliable, otherwise errors will be introduced due to the extra power absorbed by the slipping and bending of this belt round the pulleys.

Thus measuring the electrical power developed by the dynamo which is at once given by the current $\times$ voltage, and knowing its commercial efficiency e, the B.H.P. of the motor can easily

be calculated and $= \frac{\text{D.C. output}}{746} \div e$. Further, if $n =$ the speed in revs. per min., the torque T of the motor is given by the relation $T = \frac{\text{B.H.P.} \times 33000}{2\pi n}$ lb. ft.

In all cases the efficiency of any motor = total power given out at its pulley ÷ total power absorbed, both being reckoned in equivalent units. A multiphase alternating current motor is *self-exciting*, *self-starting*, but asynchronous as regards speed and the periodicity of the supply. The starting torque can be made equal to that of the best direct current motor without an excessive percentage over load in the current taken.

They can be wound to run direct on 5000 volt circuits and

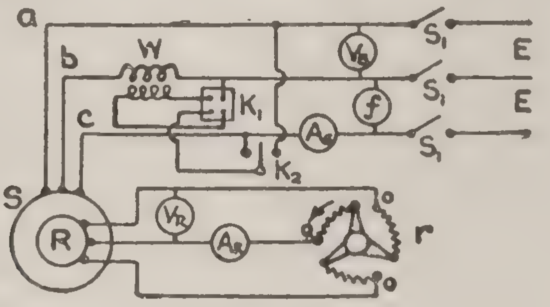


FIG. 100.

over without much fear of the insulation breaking down, and their great advantage, except in the larger sizes, lies in the fact that there are no rubbing contacts of any kind to get out of order, and consequently there is no sparking. In the present case we will assume that the motor to be tested is of the three-phase type, as perhaps the measurements of input are not so obvious as in the two-phase system.

Apparatus.—Source of three-phase alternating current E; three-phase motor SR to be tested either coupled mechanically to a direct current dynamo D of known commercial efficiency, or fitted with a Prony brake, p. 634; Wattmeter W; alternating current ammeters A_S A_R and voltmeters V_S V_R; triple pole switch S_1 S_2 S_3; tachometer; and if a coupled dynamo load is used, direct current ammeter A and voltmeter V; rheostat R (p. 606); switch S.

Note.—For a detailed description of power measurements in multiphase circuits, see pp. 388–400.

Observations.—(1) Connect up as in Fig. 100, and adjust all the instruments to zero, levelling such as require it. See that

all lubricating arrangements in use feed properly on starting the motor in the usual way.

(2) With the motor quite free, take readings on all the instruments concerned when M thus runs "light" at its normal frequency and voltage, noting the speed.

(3) If a dynamo load is used stop SR, couple the shafts of D and SR together and start the combination up again, with R at its maximum when S is closed; or if a Prony brake is used take a series of about ten different loads from D or on the brake, varying from the smallest to the largest permissible corresponding to the maximum current allowed for SR. Note simultaneously the readings of all the instruments at each load and also the speed, the supply voltage and frequency being constant throughout.

(4) Repeat 3 for speeds 20% above and 20% below normal respectively, if possible, by varying the speed of the generator.

(5) Adjust the direct current load to a convenient amount, then, keeping V_S *constant*, alter the speed of the three-phase generator by successive steps, and note the corresponding effect on W_1 W_2 and the speed of the motor.

(6) Keeping the *speed* of the three-phase *generator constant*, alter V_S by successive steps and note the effect on W_1 W_2 and the speed of the motor, using the same load. Tabulate all your results as follows—

NAME . . . DATE . . .

Multiphase Motor: No. . . . Type . . . Maker . . .

Normal: Volts = . . . Amps. = . . . Speed = . .

Res. Warm per Phase Winding: Stator = . . . ohms.

Rotor (r_w) = . . . ohms.

Dynamo D: No . . . Type . . . Maker . .

Normal: Volts = . . . Amps. = . . . Speed = . .

Diameter of Brake Pulley d = . . . ft. if used.

Speed Revs. per min.								Watts.		For Use with Dynamo Load.						For Use with a Prony Brake.					
of Motor N.	of Supply Alternator.	Frequency of Supply ⁀ per sec.	Slip of Motor.	Amps. A_S.	Amps. A_R.	Volts V_S.	Volts V_R.	W_1.	W_2.	Volts V.	Amps. A.	Watts AV.	Commercial Efficiency at Loads AV.	B.H.P. of Motor $H_1 = \frac{AV}{746\,\eta}$.	Torque in lb. ft. $= 33000\,H_1 \div \pi N$.	Pull on Tight Side W lbs.	Pull on Slack Side w lbs.	Nett Pull $(W - w)$ lbs.	B.H.P. of Motor $H_1 = \frac{\pi d N (W - w)}{33000}$	Total H.P. absorbed $H_2 = \frac{W_1 + W_2}{746}$.	Efficiency of Motor $\Sigma = 100\frac{H_1}{H_2}$.

(7) Plot the following curves from observations 3 and 4 for each speed having efficiency, power factor, slip, speed, current, and intake Watts as ordinates and. B.H.P. as abscissæ, also curves having Torque as abscissæ with A_S, A_R and slip as ordinates.

And from observations 5 and 6, curves between voltage V_S and supply frequency as ordinates with speed of the motor as abscissæ in each case.

Inferences.—State very clearly all the inferences which can be drawn from your experimental results, and point out their bearing on electrical driving by multiphase current motors.

(101) Determination of the performance of an Induction Motor at all loads without loading it at all. (Heyland's Method.)

Introduction.—It sometimes happens to be inconvenient and even impossible to brake, or otherwise absorb the B.H.P. of large induction motors, and in other cases to supply the large amount of electrical power required by them at full load from a generating plant which may be already running nearly at full load. The difficulty is met fortunately in such cases by the following method entailing the construction of the well-known "*Heyland Diagram*" from very simple "*no load and short circuit*" readings on the motor. The intake current and power, the output, the power factor, and the slip, etc., and hence efficiency, can then be deduced for all B.H.P.s and a complete set of curves drawn showing the performance of the motor. The temperature rise at any load cannot however be obtained with this method, and the best and most economical way of determining it is to let the motor drive a generator which is capable of absorbing the required B.H.P. and simultaneously return the output of this generator to the motor supply. Thus in running a 6 hours' temperature test on, say, a 500 B.H.P. motor having an efficiency of 95%, the power wasted would only

be some 10% or about 50 E.H.P. as against over 500 H.P. if the output of the generator had been taken up in rheostats.

Apparatus.—Three-phase induction motor M with phases equally balanced (presumably) complete with starter. A tachometer; a voltmeter; an ammeter reading up to at least full load intake amperes, and a source of supply SS at the normal voltage and frequency for which the motor is built and capable of reduction to about $\frac{1}{4}$ full normal voltage at full normal frequency, as before, together with—

For motors with *mesh-connected stator*—2 similar wattmeters having a capacity of about $\frac{1}{6}$ of the full-load output of the motor.

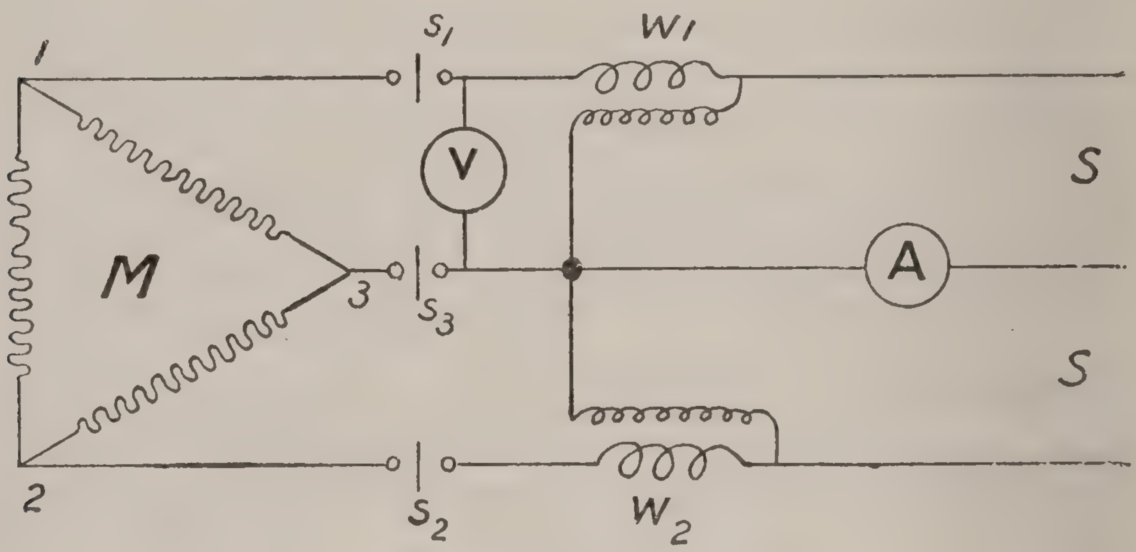


Fig. 101.

For motors with well-balanced *star-connected stator*—1 wattmeter having a capacity of about $\frac{1}{18}$ of the full-load output of the motor.

The ohmic resistance of a complete stator phase (and of a rotor phase for reference later, if needed) will be required, and can be obtained from a separate measurement with a high-reading ammeter and low-reading voltmeter, by the fall of potential method (*vide* p. 84).

Observations.—(1) Connect up as in Fig. 101 if the motor stator is mesh connected or if the stator is star connected but not well balanced, and adjust the pointers of those instruments which require it, to zero. The terminals of the stator of the motor M are 1, 2 and 3, whether it is star or mesh connected.

(2) Close the switch S_1, S_2, S_3 and start up M, finally short-

circuiting the starter. Then with the supply at normal voltage and frequency note the speed of M on the tachometer and the readings of all the instruments, the motor running quite light and at "no load."

N.B.—If the power factor of the system is low, one of the wattmeters will read negatively. Reverse the connections to its shunt coil and take the reading which must be considered $-^{ve}$ and subtracted from the reading of the other wattmeter to get the total true power.

(3) Open S_1, S_2, S_3 and when the motor comes to rest, clamp the shaft in any convenient way to prevent it rotating, and place the starter at short-circuit. Now apply any convenient lower voltage, say, $\frac{1}{3}$ to $\frac{1}{4}$ of the normal value at full normal frequency, and again read all the instruments as in Test 2 above and switch off and open the starter.

Note.—A lower voltage has to be used in this test for the larger-sized motors, because the normal voltage would cause dangerously large currents to flow which would probably damage the stator winding in even their brief application. The true static current will now be the observed current × by the ratio of the two voltages, while the corresponding static watts will be those observed × by the square of this ratio (see table). The reason for this is that the static watts are nearly all copper loss and hence $\propto$ to (current)^2.

(4) Measure in a convenient manner (pp. 84 and 263) the resistance R_t between any two terminals of the stator and rotor, preferably while warm. Then the resistance of each complete stator phase (star connections) $r_s = \frac{R_t}{2}$; and for (mesh connections) $r_s = \frac{3}{2}R_t$. Also in the case of a slip ring rotor obtain the ratio of transformation given by the ratio of any stator voltage to the corresponding rotor voltage with rings open-circuited and rotor in the position giving max. volts.

Record your results as follows—

Motor: No. . . . Maker . . .
Rated Full Load: B.H.P. = . . . Line Volts (V) = . . . Line Amps. = . . . Speed = . . .
Slip at ,, ,, = . . . Normal frequency = . . .
Stator Phases connected in . . .
Resistance:—complete Stator Phase, r_s = . . . Rotor Phase, r_r = . . .

Motor running quite freely at 0 *Load* and *Normal Line Voltage and frequency.*										
		Amperes in		Wattmeter Readings.				Power Factor		
Speed of Rotor by Tachr.	Volts across Stator Phase V_o.	Line A_L.	Each Stator Phase A_o.	w_1.	w_2.	$w_1 - w_2 = w$.	Total Intake True Watts $Kw = w_o$.	From $\frac{w_1}{w_2}$ and curve (see p. 510).	Cos. $\theta_2 = \frac{\sqrt{3}w_o}{3A_L V_o}$.	Angle of Lag θ_2.

Stator of Motor supplied with lower voltage than normal, but at normal ∼. Rotor ,, ,, at Standstill and Shorted										
	Amps. in Line			Wattmeter Readings.			Total Intake True Watts at			
Applied Line Volts V_S.	At Volts V_S A^1_L.	At Volts V $\left(\frac{V}{V_S}A^1_L\right) = A_M$.	Stator Amps. at Volts $V = A_S$.	w_1.	w_2.	$w_1 - w_2 = w$.	Volts V_s $Kw = w^1{}_S$	Normal Volts $\left(\frac{V}{V_S}\right)^2 w^1{}_S = w_S$.	Power Factor Cos. $\theta_1 = \frac{\sqrt{3}w_S}{3VA_M}$.	Angle of Lag θ_1.

Note.—With mesh-connected stators: Volts per phase = line volts and amps. per phase $= \frac{1}{\sqrt{3}}$ line amps.; with star-connected stators: Volts per phase $= \frac{1}{\sqrt{3}}$ line volts and amps. per phase = line amps.

It will be seen that the Power Factor for the "no load" and for the "short-circuit" tests is found from the relation—

$$\text{Cos. } \theta = \frac{\text{True Watts absorbed per phase}}{\text{Amps. per phase} \times \text{volts per phase}},$$

but it is perhaps more convenient to calculate from the experimental readings by means of the fraction given in the above table.

From the data contained in the above tables, the Heyland Diagram can be constructed, giving the performance of the motor.

Construction of Heyland Diagram.—This will be understood more easily by working it out from tests recently made by the author on a 360 B.H.P. 500 volt three-phase induction motor, running at a speed of 300 revs. per min. with a normal frequency = 50 ~ per sec. The stator windings were star connected, the rotor windings being also star connected, and led out to three slip rings which were connected to a starting resistance. The method of procedure with this motor was as follows—

With an ammeter in one line and a wattmeter connected between the neutral point and one terminal of the motor so as to measure the true watts absorbed by one phase (see Fig. 151, p. 396), the following measurements were made—

Motor running light at normal speed, frequency, and voltage with rotor short circuited.—Wattmeter reading = 3600 watts per phase $= \left(\frac{w_o}{3}\right)$.

Line current $(A_L) = 142$ amperes.

Resistance of each phase of stator (cold) $r_s = 0{\cdot}0122$ ohm., or by calculation about $0{\cdot}013$ ohm. (hot) on the assumption of a maximum temperature rise of 70° F. above that of the air.

Resistance of each phase of rotor (cold) $r_r = 0{\cdot}0054$ ohm.

Ratio of transformation in rotor 500 : 325.

Bearing in mind always that the diagram is constructed with reference to one phase of the motor, and not the motor as a whole. Further that in testing motors with mesh-connected stators it would be the total power absorbed that would be measured by the two line wattmeter method (Fig. 101, p. 278) instead of that per phase.

Hence the power factor of any phase, whether in star- or mesh-connected stators, can be calculated best from the general relation

$$\cos. \theta_2 = \frac{\sqrt{3}w_o}{3A_L V},$$

where w_o = total power in watts absorbed by motor running light with a line current A_L and line voltage V. The numeral 3 reduces w_o to watts per phase, and $\sqrt{3}$ reduces the line voltage V or line amperes A_L to the corresponding quantities per phase in the case of star- or mesh-connected stators respectively. Thus in the present case

$$\cos.\,\theta_2 = \frac{\sqrt{3}w_o}{3A_L V} = \frac{\sqrt{3}\,.\,10800}{3\,.\,142\,.\,500} = 0{\cdot}0878$$

or $\theta_2 = 85°$.

Motor at standstill with rotor short circuited and stator supplied at normal frequency.—With these conditions we require the line current that would flow, and also the true watts absorbed, at normal line voltage. As, however, a line voltage as great as the normal value would, in most motors, produce an abnormal current that would be inconvenient to measure and liable to damage the windings, a smaller voltage sufficient to give a convenient line current is applied. Thus in the present case

the line current $A_L' = 404$ amps.

,, pressure $V_S = 127$ volts.

Wattmeter reading $\left(\frac{w_s'}{3}\right) = 8000$ watts per phase.

From which we calculate the following static standstill values—

Line current

$$A_M = \frac{\text{normal volts}}{\text{applied volts}} \times 404 = \frac{500}{127} \times 404 = 1591 \text{ amps.}$$

Total watts absorbed

$$w_s = \left(\frac{500}{127}\right)^2 \times (3 \times 8000) = 372{,}000 \text{ watts,}$$

whence $$\cos.\,\theta_1 = \frac{\sqrt{3}w_s}{3A_S V} = \frac{\sqrt{3}\,.\,372000}{3\,.\,1591\,.\,500} = 0{\cdot}2701$$

or $\theta_1 = 74{\cdot}3°$.

Current Circle.—Referring to Fig. 102, draw two lines OE giving the phase of the supply volts and OA perpendicular to one another, and from the point O, as origin, set off a straight line OC (= current which would be taken by the stator if directly across normal voltage), making an angle θ_1 of 74·3° with OE and another line OD (= no-load current) making an angle θ_2 of 85° with OE. Now choose a convenient linear scale of current, *e.g.* in the present case, 100 amps. = 1 cm., and thus make

$$OC = 1591 \text{ amps.} = \frac{1591}{100} = 15{\cdot}91 \text{ cms. long,}$$

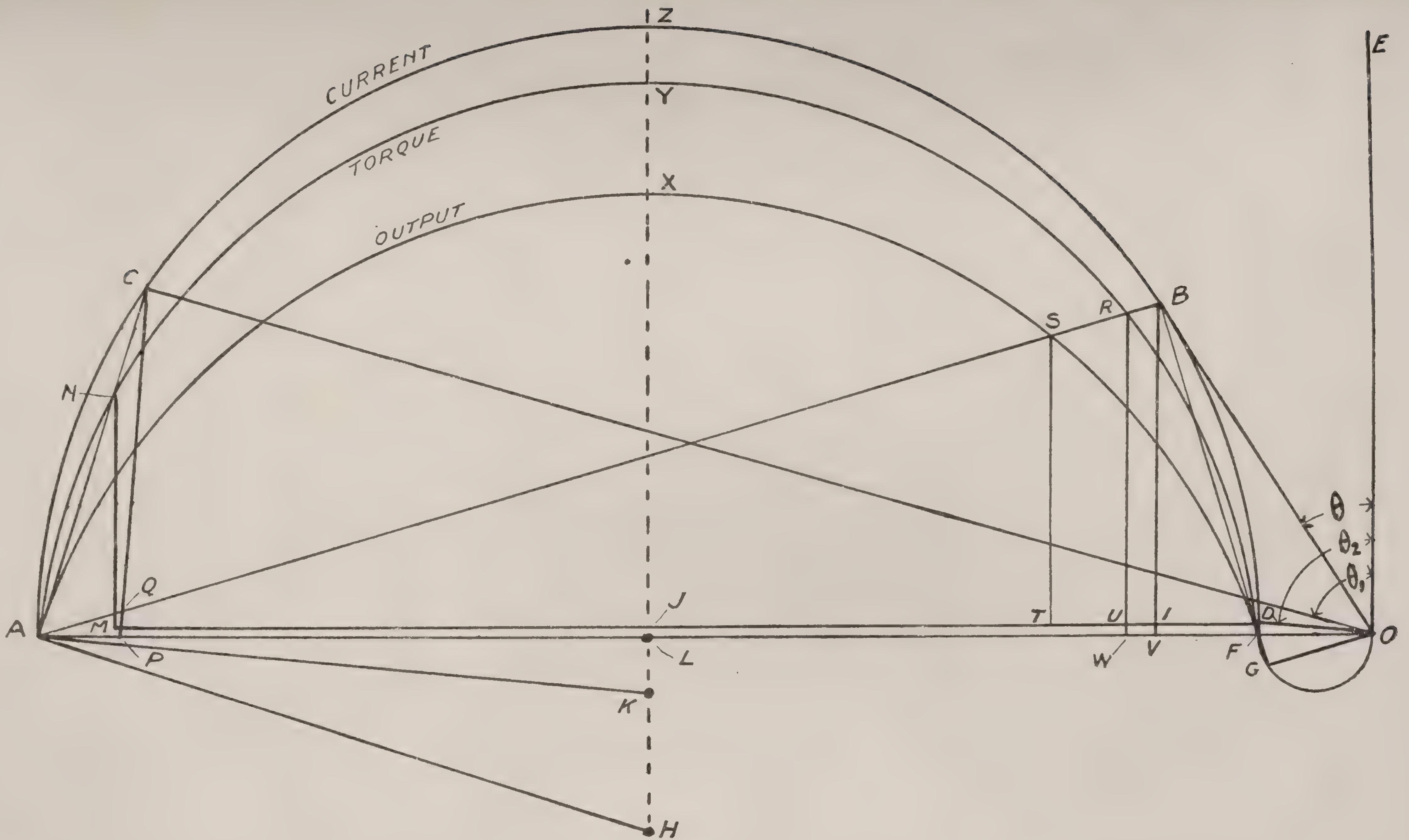


FIG. 102.

and make $OD = 142$ „ $= \frac{142}{100} = 1{\cdot}42$ cms. long.

Then with a centre L (on OA) draw a semi-circle $ACZD$ through the points C and D, and cutting OA in A and F, thus determining the point A. From D draw $DF =$ energy component of no-load current perpendicular to OA and cutting it in F, then DFO is the no-load triangle of currents $OF =$ the no-load magnetizing current $\cos\theta_1 =$ the no-load power factor. Rotor current $= FB \times$ ratio of transformation stator intake $\propto BV$ and AZF is the current circle for the motor.

Output Circle.—Join CA, and from A draw AH perpendicular to AC, cutting the perpendicular HZ, to OA through L, in the point H. With centre H draw the output semi-circle AXF through the points A and F. Since the angle $CAH = 90°$, the line CA is a tangent to the output circle, the ordinate of which at A is zero, thus satisfying the condition of no output corresponding to this point and the point C, output of motor $= ST$ which has a max. value $= XJ$.

Torque circle.—The torque of an induction motor is proportional to the rotor flux (R_F) × the rotor current (R_A) and in fig. 102 the right-angled triangle ACO is a triangle of fluxes in which $AC \propto R_F$; $AO \propto$ stator flux (S_F) which is also $\propto$ applied voltage per phase and $OC \propto$ leakage flux L_F. Consequently the actual rotor flux will be $\propto AC -$ copper drop in stator resistance, and we proceed by first finding the voltage represented by AC when the volts per phase of stator are represented by AO. Thus—

$$\frac{\text{volts per stator phase}}{\text{volts represented by } AC} = \frac{\text{length of } AO}{\text{length of } AC} = \frac{16{\cdot}7 \text{ cms.}}{4{\cdot}55 \text{ cms.}},$$

whence—volts represented by

$$AC = \frac{AC}{AO} \times \text{volts per phase} = \frac{4{\cdot}55}{16{\cdot}7}\left(\frac{500}{\sqrt{3}}\right) = 78{\cdot}66 \text{ volts.}$$

Now the copper drop for stator phase (for the short circuit current $A_S = 1591$ amps.)

$$= A_S r_s = 1591 \times 0{\cdot}013 = 20{\cdot}68 \text{ volts.}$$

Therefore from the point C mark off along CA the length $CN = 20{\cdot}68$ volts, which is proportional to

$$\frac{A_s r_s}{78 \cdot 66} \cdot AC = \frac{20 \cdot 68 \times 4 \cdot 55}{78 \cdot 66} = 1 \cdot 196 \text{ cms.}$$

Now find a centre K in ZH, such that a semi-circle $ANYRF$ described from it passes through the points ANF. This is the required torque circle. Since the rotor flux is represented by AN, the total torque is $\propto AN \times CF$ which is $\propto$ to the area of the right-angled $\triangle ANF$ (the line NF not being shown in Fig. 102). But the base AF is constant. Hence the total torque is $\propto$ to the altitude of the triangle. For example the total torque is $\propto RW$ for the stator current OB. Again, in the no load triangle of currents ODF, the no load wattless magnetising current $= OF$ lagging 90° in phase behind the E.M.F. OE, and having a load current component in phase with $OE = FD$. The resultant no load stator current as read off on the ammeter $= OD$. Hence, since useful torque = total torque − torque spent in overcoming internal frictions for the same stator current OB, we have useful torque $\propto RW - UW \propto RU$. Starting torque would be NM at the starting current OC and a power factor $\cos \theta_1$.

Slip of the Motor.—Since we know that the slip is directly $\propto$ to the rotor current NF, and inversely $\propto$ to the rotor flux AN, we see that it will have its maximum value at C and minimum value at D, for $\frac{NF}{NA}$ (which is $\propto$ to slip) has its maximum value unity at C when the motor is at standstill.

From C therefore draw CP perpendicular to AK cutting AO in P. Then the slip $\propto \frac{NF}{NA} \propto \frac{QP}{AP} \propto QP$, since AP is constant and the triangles ARF and APQ are similar.

Now maximum slip corresponding to the point C and the motor at standstill is $\propto CP$, which scales 4·35 cms. and represents 100 % slip.

The slip at the output load ST when taking a stator current OB is $\frac{\text{length } PQ}{\text{length } PC} \times 100\ \% = \frac{0 \cdot 3}{4 \cdot 35} \times 100 = 6 \cdot 9\ \%$.

Stator copper loss $= (BV - RW)$ watts and rotor copper loss $= (RU - ST)$ watts each to a scale of watts suitable for input BV and output ST, BR and $RS =$ ohm drop of volts in stator and rotor respectively to same scale as OA gives stator volts per phase.

Application of Diagram.—The diagram can now be employed for determining the performance of the motor at any load corresponding to any point such as B taken anywhere on the current curve AZF. Suppose that we choose the point B as being the point of contact of the tangent OB with the current circle AZF. Join AB, cutting CP in Q, the circle AXF in S, and AYF in R, and draw the perpendiculars ST, RUW, and BIV.

Power Factor.—For any given point on AZF the power factor is given by the cosine of the angle between the join of this point with O and the line OE. At B it has the maximum value possible, since OB is a tangent to the circle, namely

$$\cos. \theta = \cos. 32{\cdot}8^\circ = {\cdot}8406.$$

Stator Current per phase $= OB$, which is also the line current in the present case since the stator is star connected.

This scales 4·95 cms. corresponding to 495 amperes (since 100 amps. = 1 cm.).

Total Apparent Watts absorbed $= \sqrt{3}\,.\,V\,.$ line amps. $= \sqrt{3}.500\,.\,495 = 428{,}600$ watts.

Total True Watts absorbed $= \sqrt{3}\,.\,V\,.$(component of OB in phase with and parallel to OE).
$= \sqrt{3}\,.\,V\,.\,BV$ with star connections $= \sqrt{3}\,.\,500\,.\,416 = 360{,}200$ watts.

The same result is given by $3(BV) \times$ volts per phase.

Stator copper loss for this load $= 3$ ($r_s \times$ component FB^2 of stator current) $= 3\,.\,0{\cdot}013\,.\,(435)^2 = 7381$ watts.

This loss is proportional to BR which $= 0{\cdot}43$ cm., and the **Rotor copper loss** is $\propto$ to RS which $= 1{\cdot}0$ c.m., and this loss therefore $= \frac{1}{\cdot 43} \times 7381 = 17{,}165$ watts.

The total loss in the motor at the load corresponding to the intake stator current $OB = 10{,}800 + 7381 + 17{,}165 = 35{,}346$ watts.

The output of the motor therefore $= 360{,}200 - 35{,}346 = 324{,}854$ watts $= 436$ B.H.P.

The efficiency of the motor therefore $= \frac{324{,}854}{360{,}200} = 90{\cdot}18\ \%$ when giving 436 B.H.P. or 21 % overload with a power factor already found of 84 %.

The ordinate ST thus represents 436 B.H.P. or the scale of the ordinates of the output circle is $\frac{436}{ST} = \frac{436}{3{\cdot}65 \text{ cms.}} = 119{\cdot}5$ B.H.P. per 1 cm., consequently the rated full load of the motor, namely 360 B.H.P., will be given by an ordinate such as ST', but only 3·014 cms. long.

The efficiency, power factor, and slip, etc., can now be worked out for this new full load point which gives another point, such as B on AZF nearer to F and a line such as AB, at a smaller angle to OA. Hence the performance at any load can be determined.

Further, if T = the torque in pound feet,
and ω = the angular velocity of the rotor $= 2\pi n$,
where n = the revs. per min. of the rotor at a given B.H.P.,
then $$\text{B.H.P.} = \frac{\omega T}{33{,}000} = \frac{2\pi n T}{33{,}000} \text{ or } T = \frac{33{,}000 \text{ B.H.P.}}{2\pi n}.$$

Now since at B the B.H.P. = 436 and the slip = 6·9 % $\therefore n = \frac{300 \times 93{\cdot}1}{100} = 279$ r.p.m. $\therefore T = \frac{33{,}000 \times 436}{2\pi . 279} = 8210$ lb. ft.

Hence the scale of the ordinates, such as RU, of the torque circle is known from all other ordinates from the above, and $= \frac{8210}{RU} = \frac{8210}{3{\cdot}9} = 210{\cdot}5$ lb. ft. per 1 cm.

The maximum torque which the motor can exert before pulling up is represented by JY, and the maximum B.H.P. by JX.

The starting torque corresponding to the current CO is NM.

The Heyland diagram becomes more accurate the smaller the no load losses as compared with the copper losses, *i. e.* the larger the motor tested. The performance of the motor is slightly better than given by the diagram, while for motors smaller than for 4 or 5 H.P. the line MD should be drawn downwards from D at an arbitrary angle of about 25° to OA for greater accuracy,[1] in correcting the small error (slightly affecting the accuracy of the diagram) due to the greater proportion of no-load to load losses in small motors.

[1] For higher accuracy see Theory of Induction Motors by Diagram, G. Ossanna: Zeitschr. Elektrotech. Wein, 17, pp. 223–248 (1899), and Circle Diagram, by J. L. la Cour (same journal), 21, pp. 613–645 (Nov. 1903).

Note.—If OG be drawn perpendicular to BF produced, then AB and OG will always be parallel at all loads and perpendicular to BG.

The sides OB, BG, and GO of the triangle OBG, represent the stator current, rotor current, and magnetising current respectively. Now, in any electro-magnetic circuit the *total flux* = the *useful flux* + the *leakage flux*, while $\frac{\text{total flux}}{\text{useful flux}}$ is called the *leakage factor* ν, which is always greater than unity, and $\frac{\text{leakage flux}}{\text{total flux}}$ is called the *coefficient of magnetic dispersion* σ which should be always much less than unity.

In the Heyland diagram, Fig. 102, the leakage factor

$$\nu = \frac{OA}{FA} = \frac{OF + FA}{FA} = \frac{OF}{FA} + 1$$

and the dispersion coefficient

$$\sigma = \frac{OF}{OA} = \frac{OA - FA}{OA} = 1 - \frac{FA}{OA} = 1 - \frac{1}{\nu}.$$

(102) Complete Test for Efficiency, Slip, Power Factor, and Temperature Rise, of Three-Phase Induction Motors. (Sumpner and Weekes Method.)

Introduction.—This method,[1] due to Dr. W. E. Sumpner and R. W. Weekes, is an application of the principle of the ordinary Hopkinson test of a pair of d.c. dynamos (p. 228) to the test of a pair of three-phase induction motors. The arrangement, which entirely avoids the use of a brake, is extremely convenient for obtaining the temperature rise due to a long run at any particular load, and is shown in Fig. 103. M and G are the two machines to be tested, of which M is made the motor and G the generator. Their stator terminals T are connected to the main supply SS, which is at the normal voltage and frequency required by G and M. A belt B drives the rotor pulleys, which must be of different diameters in order to enable the generator G to run at a higher speed, and the motor M at a lower speed than that of synchronism.

[1] Communicated by the authors to "Section G" of the British Association, August 22, 1904, at Cambridge.

If D_M and D_G are the diameters of these pulleys: then assuming the rotors are to remain short circuited during the test, the ratio $\frac{D_M}{D_G}$ must be such as to cause the right slip for the load required. For example,—if the slip of each machine M and G $= 2 \cdot 5$ % at full load, and if the probable efficiency of each $= 85$ %, then 15 % of the load is lost in each, and the overload of the motor $M = 30$ % roughly, corresponding with $2 \cdot 5 + \frac{30}{100} \times 2 \cdot 5$ or $3 \cdot 25$ % roughly. The other machine G working as a generator developes full load and has a negative slip of $2 \cdot 5$ %. D_M and D_G must therefore differ by $2 \cdot 5 + 3 \cdot 25$ or $5 \cdot 75$ %, assuming no mechanical slip of the belt B on the pulleys. If, however, $1 \cdot 25$ % be allowed for this, the pulleys must differ by $5 \cdot 75 + 1 \cdot 25$ or 7 % in diameter, and the machines under test will, when switched into circuit, take a perfectly definite load which can be maintained for any length of time.

An interesting feature of the test lies in the fact that G, the machine used as the generator, gives current of about the same power factor as that of the current supplied to the motor M, while the current from the supply mains SS is the difference of the power components of the machine currents, together with the sum of the inductive components of these currents. Consequently, the power factor of the main current from SS is very small, and may be only $\frac{1}{3}$rd of that of the machine current, the main current from SS may be equal to, or greater than, that through the machines, while the actual power taken from SS may be less than $\frac{1}{3}$ of that circulating round the machines G and M.

Alteration of load with two given pulleys can be obtained by alteration of resistance in the rotor circuits between starter and slip rings, or by using a low resistance starter which can stand the full load rotor currents. The C^2R losses in these rotor resistances, if appreciable, must be deducted from the power taken from SS in order to get the total loss in the two machines G and M and in the belt drive. The machines can be run at various loads, with resistance variation such as above, if the pulleys are chosen with sufficient difference to obtain the maximum slip required.

The magnitude of the mechanical slip at the pulleys is determined by the ratio of their circumferential speeds, a quantity difficult of determination with any accuracy in ordinary belt drives, but most easily found in the present method. Thus—

U

let S_F = frequency of the supply current,
G_F = „ „ generator rotor current,
M_F = „ „ motor rotor current;

then ratio of the rotor speeds $R = (S_F + G_F) \div (S_F - M_F)$ and ratio of the circumferential speeds of the pulleys $= R\frac{D}{D_M}$

G and M_F can be very accurately determined, and are each small compared with S_F, so that although S_F cannot be so accurately found, the value of R is not much affected by small errors in S_F.

The belt losses are easily determined, as shown in the table, and are caused by (*a*) extra bearing friction and in bending and driving the belt, (*b*) heating of the pulleys due to belt slip.

Apparatus.—The two similar three-phase induction motors to be tested: suitable pulleys and belt; four alternating current ammeters a_1, a_2, A_1, A_2; an alternating current voltmeter V; four wattmeters w_1, w_2, W_1, W_2; source of three-phase supply SS of normal voltage and frequency for which the machines under test have been built; three-throw switch s_1, s_2, s_3.

N.B.—Switches must be used with the stator connections if the rotors are of the "short-circuited" type, but are not wanted if the rotors are supplied with slip rings and starting resistance. Only two wattmeters will be needed if one is connected between neutral point and a terminal in the case of each machine, since in this case total power = 3 × power of one coil, which is sufficiently accurate for commercial work. Two wattmeters to each machine, as shown in Fig. 103, is, however, the best and most accurate arrangement, and has the additional advantage that the ratio of the readings of a pair of wattmeters gives the power factor independent of the usual method of getting the power factor from true watts ÷ apparent watts. With two wattmeters the reading of one will be $-^{ve}$ owing to the low power factor of the supply current.

Observations.—(1) Connect up as shown in Fig. 103, and adjust those instruments to zero which require it.

(2) With the supply SS at the normal voltage and frequency needed for M and G, and the belt B off, close s_1, s_2, s_3, and start up M and G, which must run in the *same* direction. If they do not, stop them, change the connections at T, and start up again. Note the readings of all the instruments, and denote those of w_1 and w_2 by w_{01} and w_{02} respectively.

(3) Stop M and G, place the belt on, and start up again, with only one of the machines in circuit, and acting as a motor, but driving the other by belt with its stator excited and rotor open circuited. Note the readings of all the instruments, and denote those of w_1, w_2 by w_{B1} and w_{B2} respectively.

Thus the belt loss $W_B = (w_{B1} + w_{B2}) - (w_{01} + w_{02})$.

(4) In Tests 2 and 3 above, and in all future load tests, the slip of each machine can most conveniently and accurately be determined by measuring the frequency of the rotor currents by suitably shunting any ordinary low reading d.c. voltmeter to the

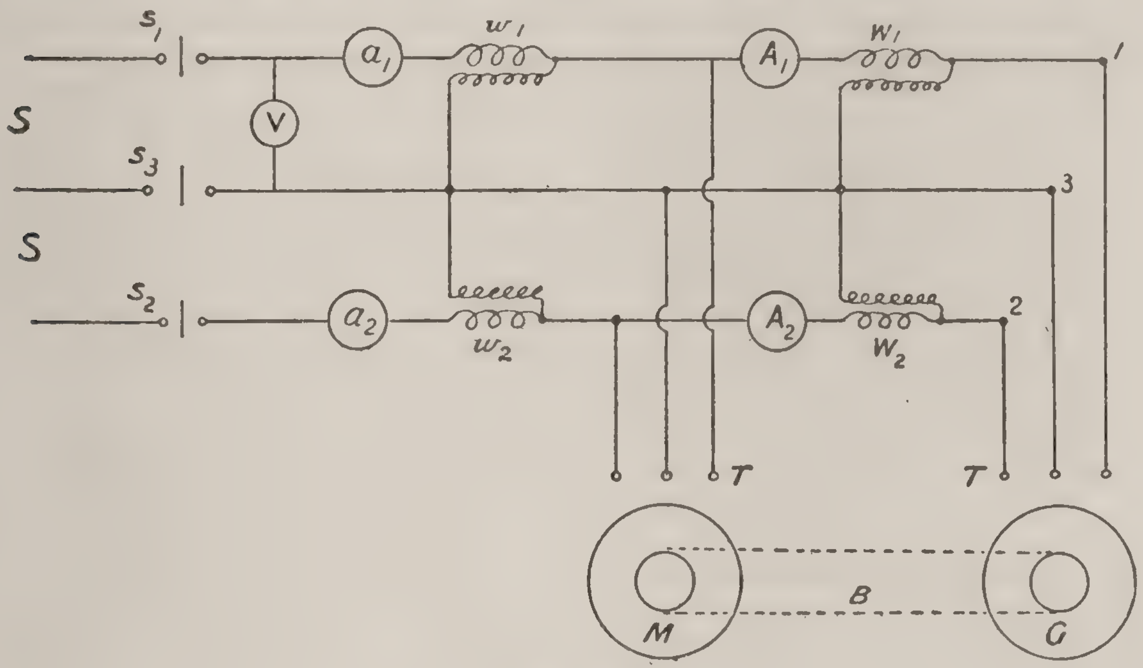


FIG. 103.

slip rings of the rotor, and noting the number of periods made by the pointer in, say, 20 seconds, these being slow enough to be easily counted. If M_F = number of periods per second, in, say, the case of the motor rotor currents and S_F = frequency of the supply current in periods per second, then the slip of the motor = $\frac{M_F}{S_F} 100\,\%$.

(5) Take readings at different loads, obtained by altering the resistance in the rotor circuit and tabulate as shown on page 293.

The values of the losses given in columns 31 and 32 are accurate enough for commercial purposes if the two machines are of the same make and of about the same rated full-load output and if subjected to the same voltage. A greater degree of accuracy may be obtained, however, by subdividing these

losses. In addition to the diameters of the pulleys indicating which machine is the motor and which the generator, the latter is the one which must always run at the higher speed. If the belt is removed and the machines run light, the wattmeters W_1 and W_2 will read negatively if G is the generator.

Column 44 is obtained from the curve Fig. 104, which is drawn in the following manner.

The ratio of the two readings of wattmeters W_1 and W_3 connected as shown in Fig. 103, varies from + 1 to − 1. Hence the power factor cos. ϕ can be calculated from the readings of W_1 and W_3 by substituting different values of ϕ (the angle of lag) between current and voltage in the formula shown.

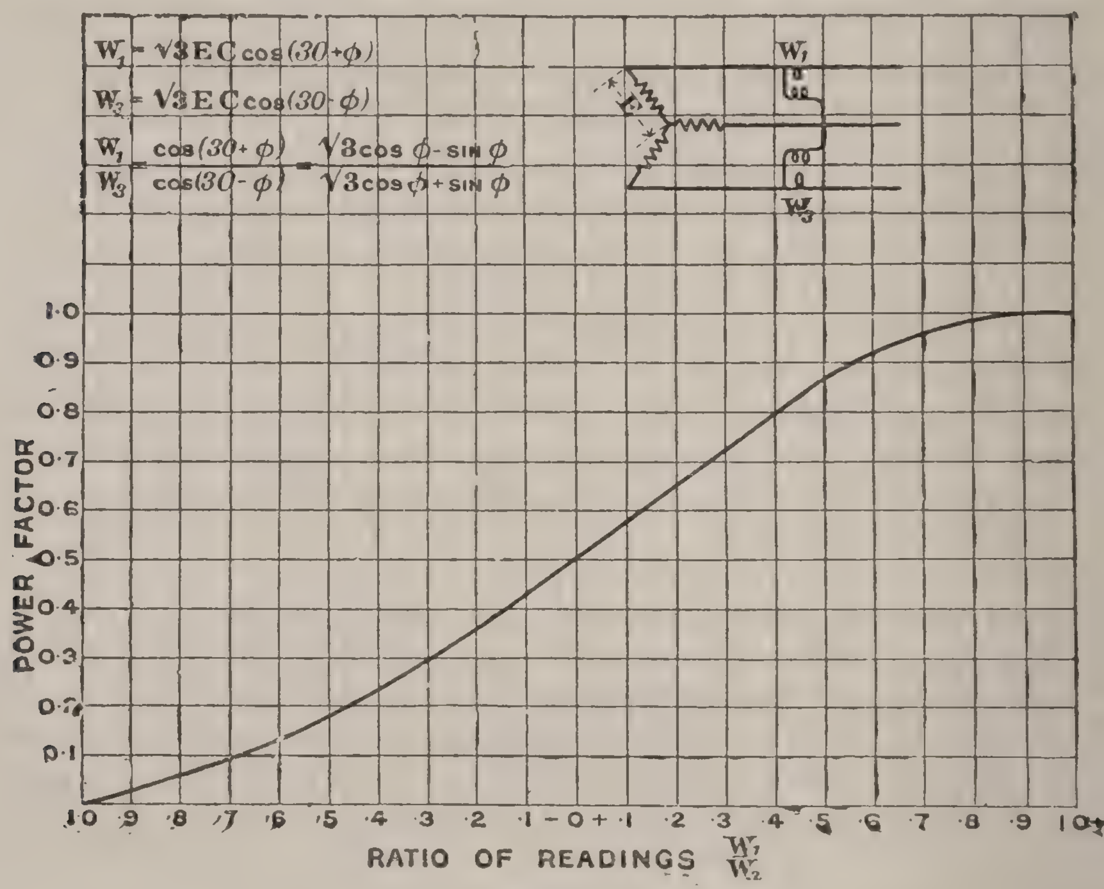


Fig. 104.

The value of $\frac{W_1}{W_3}$ so found plotted against the value of cos. ϕ gives the curve, Fig. 104. As an example of the application of this curve, let $W_1 = 6800$ watts, $W_3 = 18000$, then $\frac{W_1}{W_3} = 0{\cdot}37$ and cos. $\phi = 0{\cdot}77$. Again, let $W_1 = (-)\ 2000$ watts, $W_3 = 4000$, then $\frac{W_1}{W_3} = (-)\ 0{\cdot}50$, and cos. $\phi = 0{\cdot}18$.

Machine No. . . . Maker . . . Full Load,—Volts . . . Amps. . . . Speed . . . used as Generator G.
Machine No. . . . Maker . . . Full Load,—Volts . . . Amps. . . . Speed . . . used as Motor M.
No. . . . Resistance of each : Stator Coil . . . Rotor Coil . . . Diam. of Motor Pulley D_M = . . .
No. . . . Resistance of each : Stator Coil . . . Rotor Coil . . . Diam. of Genr. Pulley D_G = . . .
In Test No. 2 w_{01} = . . . w_{02} = . . . In Test No. 3 w_{B1} = . . . w_{B2} = . . .

Hour of Reading.	Time in Hours from Start.	Volts V.	Amperes.		Wattmeter Reading.		True Watts.		Power taken from Mains SS $w_1 + w_2$.	Amps. A_1.
			a_1.	a_2.	d_{w1}.	d_{w2}.	w_1.	w_2.		
1	2	3	4	5	6	7	8	9	10	11

Amps. A_2.	Wattmeter Reading.		True Watts.		Output of Generator G $W_1 + W_2 = W$.	No. of Periods of Rotor Currents in 20 Secs.		Frequency of Rotor Currents.		Frequency of Supply Mains SS S_F.
	dW_1.	dW_2.	W_1.	W_2.		M_P.	G_P.	$\frac{M_P}{20} = M_F$.	$\frac{G_P}{20} = G_F$.	
12	13	14	15	16	17	18	19	20	21	22

Mechanical Slip of Belt $\frac{S_F+G_F}{S_F-M_F} \times \frac{D_G}{D_M}$.	Electro Magnetic Slip of		Belt Loss $(w_{B1} + w_{B2}) - (w_{01} + w_{02}) = W_B$.	Temperature in Degrees F. or C. of the			Nett Power given to M and G $w_1 + w_2 - W_B = w$.	Total Losses in		Motor Input $W + w_1 + w_2 = M_I$.
	Motor $\frac{M_F}{S_F} \times 100 = M_S\%$.	Genr. $\frac{G_F}{S_F} \times 100 = G_S\%$.		Air.	Motor Stator.	Generator Stator.		Motor $\frac{M_{Sw}}{M_S + G_S} = M_L$.	Generator $\frac{G_{Sw}}{M_S + G_S} = G_L$.	
23	24	25	26	27	28	29	30	31	32	33

Motor Output $M_I - M_L = M_O$.	Motor Efficiency % $\frac{M_O}{M_I} 100$	Motor Output in Fraction of its Rated Full Load.	Generator.		Power Factor of Motor.				P. F. of Generator.	
			Input $W + G_L = G_I$.	Efficiency $\frac{W}{G_I} 100$	Obtained with Y Connected Wattmeters. $W_1 + w_1$.	Obtained with Y Connected Wattmeters. $W_2 + w_2$.	$\pm \frac{W_1 + w_1}{W_2 + w_2}$	Cos. ϕ from Curve.	$\pm \frac{W_1}{W_2}$.	Cos. ϕ from Curve.
34	35	36	37	38	39	40	41	42	43	44

(103) Relation between Efficiency, Slip, Torque, Load, etc., in an Induction Motor with Variable Rotor Circuit Resistance.

Introduction.—The present test is obviously just an extension of, and similar in almost every way to, test No. 100, which is therefore repeated here but with different amounts of the starting resistance (r) in circuit instead of it being all cut out to short-circuit as in that test. Consequently there will be one set of curves, such as was obtained in test No. 100, for each different value of rotor circuit resistance used in the present investigation.

Further, if, say, five different rotor circuit resistances were used, giving five complete sets of curves as in test No. 100, then any of the variables plotted, say, against load, for the different constant rotor resistances can be transferred and replotted against rotor resistance, *e. g.* there would be five efficiency-load curves, then if a straight line was drawn through, say, full-load point, parallel to the axis of efficiency, and cutting the five efficiency curves; the five different efficiencies obtained by the five intersection points can be plotted against the five values of rotor resistance of the efficiency-load curves to give a curve of five points between efficiency and rotor resistance only.

If E_S = E.M.F. induced in each stator circuit due to the rotating field,

$N_S N_R$ = number of turns per phase in each stator and rotor winding respectively,

$\omega_S \omega_R$ = angular velocities of rotating stator field and rotor respectively,

r_R = resistance of each rotor circuit,

L_R = self-induction of each rotor circuit,

p = number of pairs of poles in the rotating stator field,

f = frequency of the stator supply voltage and currents,

K = the slip.

Then we have $K = \frac{\omega_1 - \omega_2}{\omega_1}$ and the frequency of the induced E.M.F. and currents in the rotor circuits will $= \frac{\omega_1 - \omega_2}{\omega_1} \times f =$

Kf = slip in cycles per sec., and it can be shown that the running torque T is given by the relation

$$T = \frac{N_R^2 E_S^2 r_R K}{N_S^2 \omega_1 (K^2(2\pi f L_R)^2 + r_R^2)} = \frac{N_R^2 E_S^2 r_R K}{N_S^2 \frac{2\pi f}{p}(K^2(2\pi f L_R)^2 + r_R^2)}$$

where f/p = speed of the stator rotating field in revs. per sec.,

and $2\pi f/p = \omega_1$ = its angular velocity,

$K 2\pi f L_R$ = reactance per rotor circuit when in motion and which is $\propto$ to slip,

and $K^2(2\pi f L_R)^2 + r_R^2$ = (impedance)^2 per rotor circuit when in motion.

Further, if A_R = current per phase in the rotor, and F = leakage flux, then the coefficient of self-induction per phase of the rotor $= \frac{(N_S)^2 F}{10^9 A_R}$ henries, which can also be calculated from the shape of slots and winding in them.

The above expression for T shows us that the running torque T of the induction motor is $\propto$ to the square of the stator voltage, *i.e.* to the square of the stator flux, and increases as both the supply frequency and reactance per phase of the rotor decreases, becoming a maximum (bd) when

$$K = \frac{r_R}{2\pi f L_R} \text{ and } T_{\text{max.}} = \frac{N_R^2 E_S^2}{N_S^2 \frac{2\pi f}{p} 2(2\pi f L_R)} = \frac{N_R^2 E_S^2 p}{8 N_S^2 \pi^2 f^2 L_R}$$

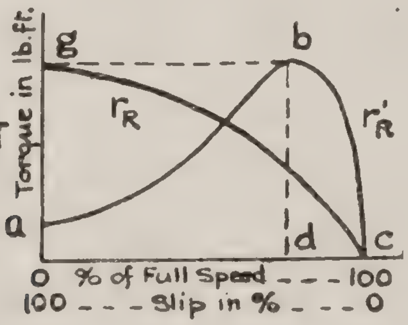


FIG. 105.

Thus, since the last expression for the maximum value of the running torque does not contain r_R, we see that it is constant and independent of the rotor circuit resistance, but for different values of rotor resistance will attain the same maximum value bd at a different slip, as indicated in Fig. 105. The motor starts

at 0 with 100% slip and reaches the same maximum running torque $bd = og$ for slips of cd% and $co = 100$% (*i. e.* full speed) respectively, with rotor circuit resistances r^1_R and r_R; since the stator current depends on the constant no-load current and rotor current, and the latter will always be the same for a given torque, it follows that each value of torque will have a definite stator current which is independent of the rotor resistance.

Apparatus.—Precisely that for the preceding efficiency-load test No. 100, the starting resistance in the rotor circuit being of sufficient current-carrying capacity to enable it to carry, without overheating, the full-load rotor currents.

Observations.—Connect up as in Fig. 100, and carry out the tests precisely as directed in that test, for as many different values of starter resistance as possible.

Tabulate as shown on p. 276, adding two extra columns for values of starter resistance (r) and total rotor circuit resistance $r_R = (r_w + r)$ respectively.

Plot the following curves having torque T in lb.-ft., and rotor current A_R as ordinates with percentage of full speed (or slip), and stator current A_S respectively as abcissæ for each value of rotor circuit resistance (r_R).

Also curves having efficiency, power factor, slip and true stator watts absorbed as ordinates with B.H.P. as abcissæ for two widely different values of r_R.

Inferences.—From a careful study of the numerical results and curves state clearly what can be deduced.

(104) Relation between the Starting Torque, Current, Voltage, and the Rotor Circuit Resistance of an Induction Motor. (Rotor at Standstill.)

Introduction.—Under these conditions the slip will be 100%, since the speed is zero. The power absorbed will include both iron and copper losses, and the motor will approximate to a static transformer with a non-inductive secondary load.

Now the frequency f of the stator supply will also be that

of the rotor circuits when at rest, and if we put $K=1$ in the expression for the running torque (p. 295), since the rotor is now stationary, it will be seen that the starting torque T_0 is given by the relation

$$T_0 = \frac{N_R^2 E_S^2 r_R}{N_S^2 \frac{2\pi f}{p} (2\pi f L_R)^2 + r_R^2}$$

where $\frac{f}{p}=$ speed of the rotating field in revs. per sec. and $\frac{2\pi f}{p}$ its angular velocity, $(2\pi f L_R)^2 + r_R^2 =$ the square of the impedance, and $2\pi f L_R =$ the reactance per phase of the rotor when at rest. From the above relation we see that the starting torque T'_0 is $\propto$ to the square of the stator voltage, *i. e.* to the square of the stator flux, and increases as both the supply frequency and reactance per phase of the rotor decreases, becoming a maximum when $2\pi f L_R = r_R$.

For this last condition—

$$T_0 = \frac{N_R^2 E_S^2 p}{2 N_S^2 2\pi f r_R} = \frac{N_R^2 p}{4 N_S^2 \pi} \times \frac{E_S^2}{f r_R}, \text{ i. e. inversely} \propto \text{ to } r_R,$$

we therefore have the following most important deductions, namely, that for a given supply frequency, the starting torque is a maximum when the resistance and reactance of the rotor circuits are equal and each as small as possible. Since the rotor currents are a maximum at starting, the present test enables the maximum value of the starting resistance to be obtained under either of two conditions: namely, (1) for maximum starting torque, or (2) for maximum safe starting rotor current. In the former, by measuring the values of L_R and r_w per phase winding of the rotor we know that for maximum starting torque $2\pi f L_R$ must $= r_w + r$, whence the external starter resistance must have a maximum value $r = 2\pi f L_R - r_w$ ohms per phase.

In the latter, if $V_R =$ the standstill slip-ring voltage at normal stator volts and frequency, then $\frac{V_R}{\sqrt{3}} =$ the standstill volts per phase winding, whence $r = \frac{V_R}{\sqrt{3} A_R}$ ohms per phase for a maximum safe starting rotor current A_R. The gradation of r

between this maximum value and 0 depends on the number of switch contacts and sections chosen.

Apparatus.—That detailed for the no-load short-circuit test No. 98, using an induction motor having a slip-ring rotor connected to the usual form of three-phase equal variable starting resistance of a current-carrying capacity sufficient to allow the necessary time for taking readings without overheating. In addition, a block brake and lever, preferably similar to that shown in Fig. 95, will be needed to measure the torque exerted by the shaft.

Observations.—(1) Connect up as in Fig. 100, levelling and adjusting to zero such instruments as need it. On starting up see that all lubricating arrangements are feeding properly.

Starting Torque with Rotor Circuit Resistance for a Constant Supply Voltage and Frequency.—(2) Adjust the supply frequency f to the normal value for the motor and the supply voltage V_S to some convenient value, if necessary lower than the normal value for the motor in order to avoid excessive rotor currents and keep both constant. Then read the spring balance and all other instruments *as quickly as possible*, when (r) is moved one contact stud at a time from its "full in" position to such a position nearer that of short circuit at which the rotor current A_R reaches a safe overload value. Finally measuring the resistance of each rotor circuit corresponding to each contact-stud position.

Starting Torque with Supply Frequency for Constant Rotor Circuit Resistance and Supply Voltage.—(3) With the supply voltage V_S and starter resistance r adjusted to convenient values for giving safe maximum rotor currents and kept constant, read the spring balance and all the other instruments *as rapidly as possible* at each of a series of supply frequencies (f) between the maximum and minimum values possible and convenient.

Starting Torque with Supply Voltage for Constant Rotor Circuit Resistance and Frequency.—(4) With supply frequency and starter resistance r adjusted to convenient constant values, read the spring balance and all instruments as rapidly as possible at each of a series of supply voltages V_S between maximum and minimum values giving safe maximum rotor currents, and tabulate all your results as follows—

Motor: No. . . . Type . . . Maker . . .
Full load : Volts = . . . Amps. = . . . Speed = . . . Frequency = . . .
Res. per phase winding of rotor $r_w =$ ohms.
Leverage of Spring Balance from Shaft Centre (l) = ft.

Balance Pull W lbs.	Starting Torque, $T_0 = W.l$ lb.-ft.	Frequency f.	Volts V_S.	Amps. A_S.	True Watts.			Apparent Watts $\sqrt{3}A_SV_S$.	Power Factor $\cos\phi = w/\sqrt{3}\,A_SV_S$.	Angle of Lag ϕ.	Rotor.				Values of $\frac{V_S^2}{fr_R}$.
					w_1.	$\pm w_2$.	Total $W = w_1 \pm w_2$.				Volts V_R.	Amps. A_R.	Starter Res. r.	Total Circuit Res. $r_R = r_w + r$.	

(5) Plot curves having values of A_S, A_R, and T_0 as ordinates, with values of (r_R) from obs. 2 as abscissæ, also curves having T_0 as ordinates with (a) frequency (f) from obs. 3, and (b) stator supply volts (V_S) from obs. 4, (c) stator amps. A_S, and (d) rotor amps. A_R. An additional column for values of $T_0 \div \left(\frac{V_S^2}{fr_R}\right)$ might be added to this table in order to see how nearly this ratio is constant, *i. e.* how nearly the starting torque is $\propto$ to $\frac{V_S^2}{fr_R}$.

Inferences.—From a careful study of the numerical results and curves, carefully point out all that can be deduced.

(105) Determination of the Efficiency, B.H.P., and other Characteristics of Single Phase Alternating Current Commutator Motors.

Introduction.—The comparatively small starting torque of the induction motor to that necessary for electric traction work has led in recent years to the production, improvement, and utilization on an increasing scale of the so-called alternating-current commutator motor. It is well known that any ordinary direct-current series or shunt-wound electric motor will run in one and the same direction whichever way the supply current flows through it, for with every reversal of the supply current, the magnetization of both field and armature will also be simultaneously reversed, and the motor will continue to run as if nothing had been changed. From this it follows that any ordinary D.C. machine will run as a motor when supplied with

A.C., though inefficiently owing (1) to the large eddy-current loss due to heavy eddy currents which would be set up in the solid field system by reason of the rapid reversal of the magnetization, and (2) the demagnetizing effect of such currents on the field. If, however, the field system is well laminated, like the armature of any machine always is, the machine would run with reasonable efficiency on an A.C. supply, but will develop less power than when run with D.C., of the same mean voltage, owing to the smaller current and flux, and to the larger internal losses due to eddy currents and hysteresis resulting from an A.C. supply.

The field magnets of A.C. commutating motors are either bi-polar or multi-polar, whether of the projecting-pole form used in D.C. machines, or of the cylindrical form with uniform air-gap as used in induction motors, and with definite polarity produced by the windings but not otherwise so evident. The armature, however, presents the usual appearance of D.C. forms, although, along with its commutator, embodying features mentioned later and necessary for ensuring satisfactory operation.

These features will be appreciated after a brief consideration of the actions taking place in the machine, but at the outset it should be realized that a single-phase series-wound commutator motor, built on the best possible lines for a given voltage supply, will operate in every way as well, but even more efficiently when run from a D.C. supply of the same voltage. In fact, such motors have to run on A.C. in some parts and on D.C. in other sections of certain tramway undertakings.

Now, considering such a series-wound motor with (for simplicity) a two-pole field, and hence with one pair of brushes, with a D.C. supply, producing a *unidirectional field in the poles and through the armature*, there will be set up a unidirectional *induced* potential difference (P.D.) having its maximum value between the brushes, *i. e.* along the "diameter of commutation" which, with the motor running light, will be coincident with the "neutral axis" and perpendicular to the direction of the fixed field. This induced P.D. (or "back E.M.F.") is set up solely by reason of the forced rotation of the armature conductors across the field, by the supply current flowing in them, and with a given field is entirely due and

directly $\propto$ to the speed (n) of rotation. On the other hand, with an A.C. supply producing an *alternating field in the poles and through the armature*, there will be set up two distinct alternating P.D.s: namely, (1) the induced P.D. having its maximum value between the brushes exactly as mentioned above, and with a given field entirely due and directly $\propto$ to the speed (n) of rotation; it is in phase with the field and also practically with the current, and consequently not in direct opposition of phase with the supply E.M.F., and (2) the self-induced P.D. having its maximum value between two points in the armature winding on a diameter perpendicular to the diameter of commutation. This self-induced P.D. is set up solely by reason of the *transformer action* due to the armature conductors cutting the alternating field, and will lag in phase behind the field flux by an angle of 90°. Its magnitude will depend only on the strength and rate of reversal (*i. e.* the frequency f) of the alternating field, and in no way on whether the armature rotates or is stationary. It has no effect on the action of the motor, nor on the supply; consequently, due to the main field, in the rotating armature of a single-phase commutator motor, there are induced two entirely distinct E.M.F.s—one caused only by and directly $\propto$ to the speed of rotation, the other caused only by transformer action and directly $\propto$ to the supply frequency.

Now, when a current flows through the armature, the latter becomes a powerful electro-magnet, the two halves of the winding in parallel between the brushes producing two similar semi-circular electro-magnets having a consequent north and a consequent south pole situated in the diameter of commutation, and at a distance apart equal to the diameter of the armature core. The flux of this armature magnetization will be in phase with the current, and have a direction therefore perpendicular to the main field flux, or in line with the diameter of commutation, giving rise to the phenomenon commonly known as armature reaction. It will react on the main flux in three ways: (1) by distorting and dragging it round in the direction of rotation, (2) by inducing in the armature conductors, as they rotate through it, an E.M.F. along an axis parallel to the main field, but which will not in any way affect the action of the motor,

(3) by inducing, through transformer action on the armature conductors, an E.M.F. of self-induction 90° in phase behind the current and acting along an axis joining the two brushes. The value of this self-induced or reactance voltage of the armature is $L_a 2\pi fA$ where L_a = coefficient of self-induction of the armature winding carrying a current A, and f = the frequency of the current A, which in this case is that of the supply to the motor. Since the motor is series wound, the same current A will flow in the field-winding which will have a coefficient of self-induction L_F. Consequently the series field coils will introduce into the circuit a self-induced, or back, or reactance voltage $= L_F 2\pi fA$. Thus the total reactance voltage of the motor will $= 2\pi fA(L_F + L_a)$. Now the reactance of the machine has the disadvantage of reducing the power factor of the circuit, and should therefore be minimized as far as possible.

That due to the field coils cannot be reduced, because the chief cause of its existence, viz. the flux, is also necessary for the operation of the machine as a motor.

The reactance of the armature can, and is, compensated for by an additional winding on the field system midway between the main field windings, and producing a flux equal and opposite to the reactance field of the armature, and which is connected either in series with the circuit or short-circuited on itself. In either case the effect is the neutralization of the armature reaction flux and reactance and an increase in power factor. Again, although the self-induced voltage in the armature coils, due to transformer action and main field has no effect on the action of the motor, it has an effect on the commutation. For example, an armature coil undergoing commutation is short-circuited by the brush while inactive, *i. e.* generating no E.M.F. by reason of its rotation across the field. Since, however, in an A.C. motor, the coil by transformer action has, during commutation, a self-induced E.M.F., this will produce in it, when short-circuited by the brush, a heavy current which when broken as the segments leave the brush will cause sparking and the deterioration of the commutator.

Now, the self-induced E.M.F. of a coil decreases with a decrease in the number of turns, and if the circuit of the coil is broken before the current has time to attain its full value the

spark will be decreased. Hence in commercial single-phase commutator motors, sparking is minimized by (*a*) having as few a number of turns per armature coil as possible, (*b*) an increased number of coils and peripherally narrower commutator segments and brushes, (*c*) as small a supply frequency as possible, (*d*) brushes of special composition. The narrower segment in (*b*) reduces the time during which an armature coil is short-circuited and reduces the short-circuit current in it.

The single-phase A.C. motor possesses much the same characteristics as the D.C. form, being a variable speed motor, giving maximum torque on starting which decreases with increase of speed, and is ∝ to armature current, but independent of power factor. It will tend to race in speed on suddenly removing the load. Further, since the current is simultaneously

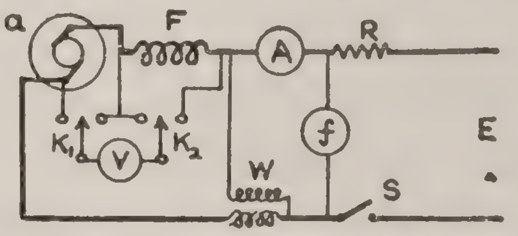


FIG. 106.

reversed in armature and field, the torque will be undirectional though pulsating.

Apparatus.—An A.C. supply *E*, preferably a motor-driven alternator having a speed and field control independently variable between wide limits; ammeter *A*; variable non-inductive rheostat *R*; frequency meter (*f*); voltmeter *V* with two two-way keys K_1 K_2; wattmeter *W*; switch *S*; and single-phase commutator motor, to be tested, of which the series field windings are *F* and (*a*) the armature.

Observations.—(1) Connect up as in Fig. 106, levelling and adjusting to zero such instruments as need it. N.B.—With a town's supply for *E*, the rheostat *R* will be needed to start up *M* and for regulating the current afterwards, otherwise with a motor alternator this may be done by field excitation.

(2) With *R* or the alternator field rheostat full in, and the armature shaft clamped to prevent it rotating, close *S* and adjust the frequency *f* to the normal value for the motor. Now

gradually raise the voltage until A reads the full-load current of the motor, and note the readings of f, A, W and V (when switched by K_1 K_2 across (a), (F), and ($a + F$), giving readings V_a, V_F and V respectively).

(3) For this same value of V and f unclamp the armature shaft so as to see what all the instruments, including tachometer V_a, V_F and V, will read when the speed has risen to a constant value (which must not be excessive), the shaft running quite "light."

(4) With normal frequency, and the motor running perfectly "light," read all the instruments and speed at each of a series of voltages V between 0, 25% above normal value.

(5) Repeat obs. 4 with constant normal voltage and wide variation in frequency.

(6) With the normal frequency (f) and the motor running light, alter V so as to obtain normal speed on the tachometer, and note the readings of f, A, W, V_a, V_F and V.

(7) With this same value of speed and f load up the motor to about 25% above full load in some eight or ten successive steps, noting the readings of all the instruments, the speed being kept constant by raising the voltage V.

(8) With the motor running light at constant normal frequency, obtain the maximum safe speed allowable, note this and also the values of V, A and W; next apply about ten different braking loads up to about 50% above normal, noting the values of the speed V, A and W at each—*V and f being kept constant.*

Tabulate all your results as follows—

Motor Tested: No. = . . . Type . . . Maker . . . Weight = . . .
Full load: B.H.P. = . . . Amps. = . . . Volts = . . . Speed = . . . Frequency = . . .
Resistance: Armature (worm) r_a = ohms. Series coils (worm) F = ohms.

Speed (n) r.p.m.	Frequency f	Brake Pulls: Larger W lbs.	Brake Pulls: Smaller w lbs.	Brake: Leverage r ft.	Nett Torque $T=(W-w)r$ lb.-ft.	Voltages across: Armature V_a	Voltages across: Field V_F	Voltages across: Whole V	Amps. A	Impedance of: Armature V_a/A ohms	Impedance of: Field V_F/A ohms	Impedance of: Whole V/A ohms	Wattmeter: Reading W_1	Wattmeter: True Watts KW_1	Apparent Watts $= AV$	Power Factor $\cos\phi = KW_1/AV$	Total H.P.: Absorbed $H_2 = \frac{KW_1}{746}$	Total H.P.: Developed $H_1 = \frac{2\pi nT}{33,000}$	Efficiency $\Sigma = 100\frac{H_1}{H_2}$ %

(9) Plot the following curves—

From obs. 4 between voltage V as abscissæ with speed A and W as ordinates.

From obs. 5 between frequency f as abscissæ with speed A and W as ordinates.

From obs. 6 and 7 between loads H_1 as abscissæ with values of Σ, cos ϕ, H_1 and A as ordinates.

From obs. 8 between speed as abscissæ with values of Σ and H_1 as ordinates.

From obs. 8 between torque as abscissæ and values of speed and A as ordinates.

Inferences.—State clearly all that can be deduced from the tabular results and curves.

(106) Relation between the Field Excitation and Armature Current, or the "V" and other Curves, of a Synchronous Alternating-Current Motor Running Light or at Constant B.H.P.

Introduction.—All alternators, whether single or polyphase, are reversible machines, and will run as motors synchronously with the periodicity of the A.C. supply to their armatures, the field system being in all cases supplied with a separate source of direct current.

Synchronous motors are, however, *not self-starting*, for at every succeeding rapid reversal of the A.C. supply, the armature coils receive equal impulses but in opposite direction, and hence there is no resultant torque. If, however, the motor is first started up and run by some other driving source of power, at such a speed that any armature conductor passes through the distance between the centres of two poles (*i.e.* the pitch) in half the periodic time of the A.C. supply, then on switching it on to the supply it will continue to run as an efficient A.C. motor in dead synchronism with the supply frequency, irrespective of load, so long as this is not sufficient to pull it out of step with the supply current.

A single-phase synchronous motor therefore develops an alter-

nating armature polarity and torque which reverses with the rapidity of reversal of the A.C. supply, thus producing unidirectional rotation. On the other hand, the currents in the phase windings of a polyphase synchronous motor combine so as to form a constant polarity of fixed position relatively to that of the field, so causing a *unidirectional torque* and rotation.

Apparatus.—Sources of A.C. supply E_1 to synchronous motor M, and of D.C. supply E_2 to starting motor (m) and field of M; switch S; lamps L_1L_2; A.C. ammeter A, voltmeter V, wattmeter W; D.C. ammeter a, voltmeter v. Field ammeter a_f, rheostats rh and r_f, switches S_f and S_m, with starter or main variable resistance (r).

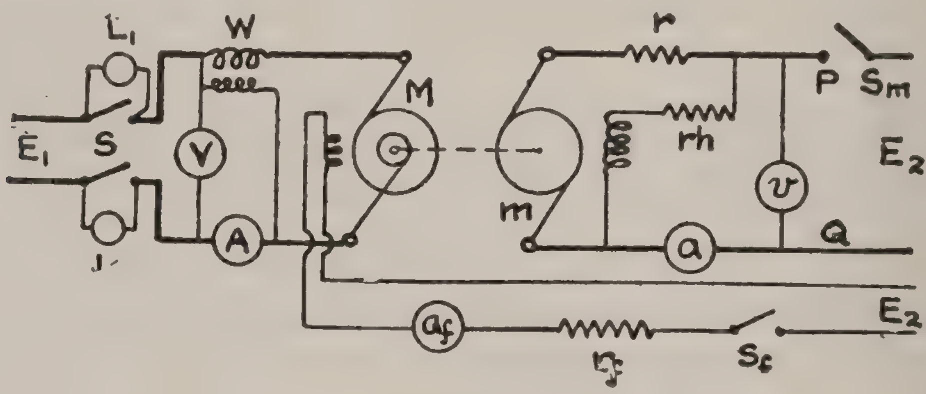


Fig. 107.

Note.—The lamps L_1L_2 should be stamped for a voltage, each equal or even 10% higher than that of M, so as to avoid burning them out while synchronising.

Observations.—(1) Connect up as shown in Fig. 107, levelling and adjusting to zero such instruments as require it. On starting the machines, see that their lubricating arrangements are working properly.

(2) **The Synchronising** or starting up of the A.C. motor under test can be effected as follows: with S and S_f open and (r) off, if a starter, or "full in," if a variable rheostat, close S_m and operate (r) so as to start the machines up to about the normal speed of M; now close S_f and adjust r, rh and r_f until V indicates the same voltage as that of the supply E_1, and the lamps L_1L_2 cease to blink and go out definitely with a slow period. At this moment close S and open S_m, when the A.C. machine M will continue to run as a synchronous A.C. motor at a speed entirely governed by, and directly proportional to, the supply frequency.

Note.—At the above moment of closing S, the back E.M.F. (V) of M will be not only practically equal to, but also exactly opposite in phase with (*i. e.* differ by 180° from), that of the supply E_1.

The starting up may also be effected by one of the special forms of synchroniser now made for the purpose, *e. g.* the rotatory type or synchroscope of Messrs. Everett, Edgcumbe & Co., the characteristics of which are as follows: with the supply and the motor connected to the respective pairs of terminals on the synchroscope, the speed and field of M are adjusted until the frequency of M = that of the supply (indicated by the rotating *pointer coming to rest*), and the voltage of M is equal and opposite in phase to that of the supply (indicated by the *pointer taking the vertical position*); under these conditions, the dial will show a white light and S can be closed. Briefly, therefore, *close main switch when pointer stops vertically and white light shows.* If M is running too fast, the pointer rotates clockwise and a red light shows, whereas if M is running too slow, the pointer rotates counter-clockwise and a green light shows.

Two- and three-phase machines are synchronised by the same single-phase instrument with its 2 pairs of terminals connected across any one phase, either side of the main switch contacts of that particular phase. With the motor M under test running synchronously with the A.C. supply, the following very interesting and important investigations can be made, namely—

(3) With S_m open and r and rh "full in," M will (unless coupled to and released from m by an electro-magnetic clutch) simply be turning it against the small windage, brush, and bearing frictions, and will therefore practically be *running light* itself. For this no-load condition at normal supply frequency and voltage adjust r_f to obtain *minimum reading on A*, and note simultaneously that of V, W, a_f and the speed.

(4) Next vary r_f, and hence the exciting current (a_f), by a series of steps, *above* and *below* the value found in obs. 3, as will raise A to a value *not exceeding* 25% *over load*, in each case noting V, W, a_f, A and the speed at each excitation. The supply voltage V and frequency being kept constant throughout at the value of obs. 3.

(5) Repeat obs. 3 and 4 for constant B.H.P. load outputs

from M of say $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and full load respectively *at the same constant supply volts and frequency*, taking that value of a_f giving *minimum* main current A as the starting point of the "up and down series" of (a_f).

Note.—The brake load can most conveniently be taken up electrically in the coupled D.C. starting motor m by causing it to act as a D.C. generator and send current through a suitable current rheostat to be connected in series with a switch (neither shown in Fig. 107) across the points P and Q. In this case (r) must be short-circuited and *special precaution taken to keep S_m open.*

The product $v \cdot a$ = the power absorbed in the added rheostat, and, if the efficiency of M is known, the actual B.H.P. developed by M is at once obtainable, otherwise with constant excitation of (m), the power absorbed by it will be roughly $\propto$ to the currents (a) developed, and therefore to the B.H.P. given by M. Tabulate all results as follows—

Synchronous Motor: No. . . . Maker . . . Type . . .
Full load (normal): B.H.P. = . . . with Amps. at . . . Volts. and Speed = r.p.m.
D.C. Starting Motor: Full load Amps. = . . . Volts. = . . . Speed = . . .
Efficiency Σ = at . . . load.

Supply.		Exciting Amps. a_f.	Armature.			Power Factor $\cos\phi = W/AV$.	Angle of Phase Difference ϕ.	Speed in r.p.m.	D.C. Starting Motor as load.		
Frequency f.	Voltage V.		Amps. A.	Apparent Watts AV.	True Watts W.				Volts v.	Amps. a.	B.H.P. of $M = \frac{a \times v}{\Sigma}$.

(6) Plot curves for running light and for each load on M having (1) amps A, (2) power factor ($\cos\phi$), and (3) watts W as ordinates with exciting current (a_f) as abscissae in each case.

Inferences.—From a careful study of the shapes of the curves and of the tabular results, state what can be deduced.

(107) Efficiency and B.H.P., with other Characteristics, of a Synchronous Motor run from a Constant Voltage and Frequency Supply at Constant Excitation.

Introduction.—In view of the peculiar relations existing between the excitation and other factors as determined in the last test, and the use of the synchronous machine for raising the average working power factor of a supply system, it is both interesting and important to see what effect load has on the same factors. This is apparent when determining the efficiency-load curve of the machine in the present test.

Apparatus.—Precisely that for test, No. 106.

Observations.—Carry out obs. 1, 2, and 3 of test, No. 106, exactly as stated.

(4) With the *supply voltage V and frequency f each kept constant* at the normal value for the motor M, and with the *excitation* (a_f) *kept constant* at the value noted in obs. 3 (namely that giving minimum armature current A), take a series of brake loads on M rising by about equal amounts up to about 25% over load, noting the readings of V, W, a_f, A speed, and output factors at each load.

Note.—As the heating of a machine is the factor limiting the maximum safe output, it is desirable to estimate the brake loading by roughly equal amounts of main current A up to about 25%, or even 50%, over load if kept on only a few minutes, calculating and taking the B.H.P. corresponding to such current values. The method of loading the motor M may be that indicated in the Note, obs. 5, test No. 106.

(5) Repeat obs. 4 for two or three higher—and two or three lower—*constant* values of excitation (a_f) than that used in obs. 4 above, which gave minimum value of A, and tabulate as per schedule shown on page 308, but adding one extra column for efficiency $\left(=\dfrac{\text{B.H.P. output}}{\text{E.H.P. absorbed}}\right)$ of M at each load.

(6) Plot the following curves, namely, having in every case B.H.P. outputs as abscissae with (1) efficiency, (2) amperes A, (3) watts W, or E.H.P. absorbed, and (4) cos ϕ, as ordinates respectively.

Inferences.—From a careful study of the figures and also of the shape and relative dispositions of the curves, state what can be deduced.

Relations between the Supply Factors of an Alternating Current and the Constants of the Circuits supplied.

General Remarks.—Every electrical circuit possesses three distinct qualities, namely—

(i) Electrical—or ohmic resistance, depending on the length, sectional area and material of which it is made.

(ii) Electrical—or electrostatic capacity, depending on the length, surface, form and the specific inductive capacity of the surrounding dielectric.

(iii) Electrical—or self-inductance, depending on the shape, form and magnetic permeability of the surrounding conducting material.

All of these qualities are always present in every circuit whatsoever, but it may happen that one or more of them are so small as to be negligible from a practical point of view. Thus we are accustomed to speak of some special circuit as possessing *only* one of them, any two, or all three of them at once. It is often of the utmost importance to know the nature of a circuit, with reference to the above qualities, when alternating currents are employed, for the presence of one or more of them in such a circuit may be very troublesome or may be a necessity according to circumstances. Theory dictates that variation in the periodicity of the alternating supply causes, in some cases, a considerable change in the working results of a circuit, and it is with a view to clearly elucidating the effects of variation of frequency on circuits in which one or more of these qualities predominate that the following tests have been devised, and also of determining how such variations affect the power absorbed in the circuit, and also the corresponding variation of temperature (if any). In all cases the power is to be measured by a Wattmeter as nearly *non-inductive* as it is possible to have it, for it will then give a *true* measure of the power absorbed. The results to be expected, as dictated by theoretical considerations, are as follows—

Let $A = \sqrt{\text{mean sq.}}$ value of current in amperes flowing in the circuit.

$V = \sqrt{\text{mean sq.}}$,, ,, voltage acting on the circuit.
R = ohmic resistance of the circuit.
L = its self-induction in henries.
C = its electrostatic capacity in farads or $C \times 10^6$ microfarads.
p = the angular velocity of the alternating supply $= 2\pi \times$ frequency.

Then we have for circuit possessing—

$$\textbf{R only :—} A = \frac{V}{R}$$

and the current is in phase with the voltage.

$$\textbf{R and L only (in series) :—} A = \frac{V}{\sqrt{(Lp)^2 + R^2}}$$

the current now *lagging* in phase behind the voltage by an angle θ such that $\tan. \theta = \frac{Lp}{R}$.

$$\textbf{R and C only (in series) :—} A = \frac{V}{\sqrt{\left(\frac{1}{Cp}\right)^2 + R^2}},$$

the current now *leading* in phase in advance of the voltage by an angle θ such that $\tan. \theta = \frac{1}{CpR}$.

$$\textbf{R, L, and C (in series) :—} A = \frac{V}{\sqrt{\left(Lp - \frac{1}{Cp}\right)^2 + R^2}},$$

where L predominates over C, the current now lagging in phase behind the voltage by an angle θ such that $\tan. \theta = \dfrac{Lp - \frac{1}{Cp}}{R}$.

where C predominates over L, the current leads in phase in front of the voltage by an angle θ and the last two relations become—

$$A = \frac{V}{\sqrt{\left(\frac{1}{Cp} - Lp\right)^2 + R^2}} \quad \text{and } \tan \theta = \frac{\frac{1}{Cp} - Lp}{R}.$$

$$\mathbf{C}\text{ (only)}:-A = \frac{V}{\frac{1}{Cp}} = CpV,$$

the current now being 90° in phase in advance of the voltage.

$$\mathbf{L}\text{ (only)}:-A = \frac{V}{Lp},$$

the current lagging 90° in phase behind the voltage.

$$\frac{V}{A} = \text{Impedic Resistance.}$$

The radical denominators in the three expressions for A are termed the *apparent* or *effective resistances* of the circuit containing those particular qualities, though one of these, namely $\sqrt{L^2p^2 + R^2}$, is very generally termed the *impedance* of the circuit. The terms Lp, $\frac{1}{Cp}$, and $\left(Lp - \frac{1}{Cp}\right)$ are called the *reactances* or reactive resistances of the circuit, and when multiplied by the current give the reactance voltage. The phase relations are shown by the vector diagram OBD, Fig. 108, and

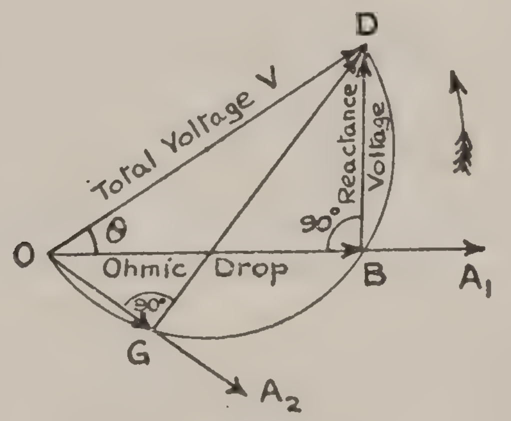


FIG. 108.

if each voltage is ÷ by the current we shall get a proportionate *Scalar* diagram (*i. e.* without arrows) in which OD = apparent resistance, OB = ohmic resistance, and BD = reactance or inductive resistance, while OA = the current. It will now be obvious that if the ohmic resistance is extremely small (for it cannot be zero) the impedance becomes = the reactance, but when the ohmic resistance is large, the impedance is affected by it considerably. Again, since the term for reactance contains $p = 2\,\pi \times$ frequency, it is directly $\propto$ to frequency, while ohmic resistance is independent of frequency.

Further, impedance depends on the remaining component factor, namely, self-induction, which latter in most cases is due to a coiled circuit surrounding iron. Now, the self-induction of a circuit depends on the linkage of turns with magnetic field, increasing directly with the latter and with the square of the former. Thus it depends on the current, which in turn decides the degree of magnetic saturation of the core. The self-induction of the armature of an alternator is really only an average of several values obtained for different positions of the armature coils relatively to the field poles, and it affects the "wave form" of the voltage generated. Since the self-induction L is directly $\propto$ to the mean permeability (μ) of the magnetic circuit, it follows that in cases where R is small compared with the Lp, the impedance will vary nearly in direct proportion to μ, consequently a curve between impedance and current will approximate to the orthodox permeability curve of the core.

On the other hand, with a low resistance winding, the voltage absorbed in it, due to the term AR, is so small that the voltage at the terminals is practically that due to self-induction only, and hence directly $\propto$ to the core flux. Thus a curve between terminal voltage and current will have the shape of that part of the magnetization curve between the origin and "knee," the higher parts of the curve being absent owing to the low degree of magnetic saturation used in the cores of A.C. plant.

Turning now to considerations relative to capacity, the fundamental definition of an electrical condenser being that of two conductors (called the coatings), separated by an insulator (called the dielectric), it follows that on connecting the coatings to a source of E.M.F., positive and negative quantities of electricity will flow on to them in raising them to the same difference of potential as that of the source. The attraction between these two quantities sets up a corresponding stress in the dielectric and causes them to remain "bound" after the charging source is removed. The charge of the condenser is measured in coulombs, and is the quantity Q (which = the current in amps $\times$ time of flow in seconds) necessary to raise the voltage between the coatings to the value V of the source. Thus, if C denotes the capacity of the condenser, we have $Q = CV$, or the capacity $C = \frac{Q}{V}$ = a *constant* for any condenser and for all charging

currents as distinguished from the self-induction of a coiled circuit which is not constant, when containing magnetic material, but varies with the magnetic saturation of the core, and hence with the current. If the source of E.M.F. is an alternating one, the state of charge of the condenser will follow exactly the change of voltage, reaching $a + ^{ve}$ and $- ^{ve}$ maximum, each once in every period of the supply. While, therefore, the current flowing in an A.C. circuit containing a condenser is actually a *charge* and *discharge current alternately*, and does not flow continuously through, owing to the impassable dielectric insulation, an A.C. ammeter placed in the same circuit, by its steady reading and inability to follow the rapid pulsations of current, makes it appear as if the current really passed *through* the condenser, though it does not do so.

Again, the internal or ohmic resistance of a self-inductance affects the corresponding impedance, while the internal resistance of a given condenser has no such effect on the corresponding impedance. From the relation already given for the current in amperes $A = CpV \div 10^6$, where $C =$ the capacity in micro-farads and $V =$ terminal pressure in volts, it will be seen that at the smaller pressures of 100 volts or so at about 50 ~ per sec., a considerable value of (C) will be needed to give an appreciable current. Since, therefore, the capacity available is usually well under 110 mfd.s, only a small current will result. In this case, care should be taken that the voltmeter used does not affect the voltage across the points between which it is applied, a condition fulfilled by the use of an electrostatic voltmeter.

The tests which immediately follow are arranged to show the variation of the quantities indicated with the factors composing them, only one of which must be varied at a time in order to test its influence on the main quantity.

(108) Determination of whether a Resistance is truly Non-Inductive at any Frequency and Current.

Introduction.—As in nearly all laboratories there is usually a shortage of rheostats, more particularly those of a non-induc-

tive nature, which are essential in the majority of A.C. tests, the present determination is both instructive and useful.

It is obvious that any inductive resistance must possess *some* ohmic resistance, while a so-called ohmic resistance usually exhibits *some* slight inductiveness. Carbon plate, liquid and glow-lamp rheostats are usually taken to be non-inductive for all practical purposes, which they are, especially the two first named. Since carbon filament lamps are usually employed in lamp rheostats in pure parallel combinations, the inductiveness of one lamp with its filament of one or more turns is finite though very small, and hence that of any combination is still smaller and practically nil.

Liquid rheostats usually consist of two or more metal plates dipping into a container of water, the conductivity of which is increased to any desired extent by the addition of a little common washing soda or aluminium sulphate. Such rheostats are undoubtedly less convenient than the carbon or lamp types, because, although electrolytic action with A.C. is negligible, they froth and alter in resistance considerably with rise of temperature, due to the absorption of the power in them.

Wire-wound rheostats, whether composed of wire spirals wound in a continuous spiral or non-inductively, are usually prominent in most test rooms. How far such rheostats, whether wound with high-resistance alloys (usually non-magnetic) or with iron (which is highly magnetic) are non-inductive, is the object of the present investigation. With the former, the self-induction would be constant for all current densities, but would vary with the frequency. Further, the effective or apparent resistance increases for increase of cross-sectional area of wire with alternating current, and the self-induction varies as the (number of turns)^2 $\times$ sectional area of spiral $\div$ length of spiral.

Apparatus.—Alternator D, capable of being driven at a wide range of speeds, so as to obtain a corresponding range of frequency at *constant* voltage V by varying the exciting circuit (not shown); Siemens electro-dynamometer, hot wire or other A.C. ammeter A unaffected by frequency; electrostatic voltmeter V; non-inductive wattmeter W; switch S, and *resistance* R to be tested.

Observations.—(1) Connect as in Fig. 109, and adjust all the

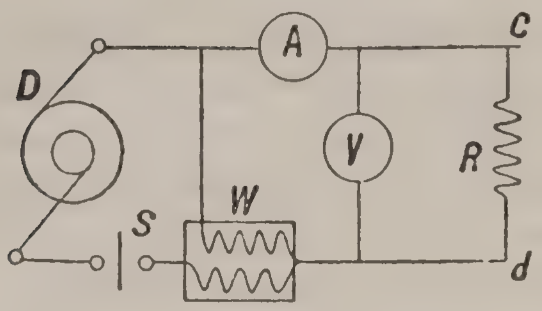


FIG. 109.

instruments to zero. Then start D, seeing that the lubricating arrangements feed properly.

(2) By field regulation adjust the voltage V across the terminals of R, that a convenient current flows through it as indicated by A.

(3) Obtain about ten or twelve different speeds of D from the smallest to the largest practicable and safe, varying the excitation so as to keep A constant throughout. Note simultaneously the readings of A, V, W and speed.

(4) With the speed, and hence the *frequency* adjusted to some convenient value to be kept *constant*, vary the voltage by field regulation so as to obtain 8 or 10 different currents through R between 0 and maximum safe value, noting the readings of A, V, W, and speed at each, and tabulate your results as follows—

NAME . . . DATE . . .

Alternator: Periods per Revolution $K = . . .$ Resistance $R = . . .$ ohms.
Constants: Wattmeter $= . . .$ and $A = . . .$ Nature of Resistance R . .

Speed Revs. per min. (N).	Frequency $\sim$ per sec. $F = \frac{KN}{60}$	Volts. V.	Scale Reading of		Amps. a_A.	True Watts $W_1 = K_1 W$.	Apparent Watts $a_A V$.	$a_A{}^2 R$.	$\frac{V}{a_A}$	$\cos\theta = \frac{W_1}{a_A V}$	$\theta°$.
			A.	W.							

(5) Plot curves having values of F and a_A as abscissæ in each case, and V, $\cos\theta$, and $\frac{V}{a_A}$ as ordinates.

Inferences.—State clearly all that you can deduce from your experimental results and your curves.

(109) Measurement of Power Factor in Alternating Current Circuits.

Introduction.—Alternating-current ammeters and voltmeters measure the *mean* or average value of the current or voltage in

an A.C. circuit, and the product of their readings is, therefore, the product of two mean values. If the circuit is non-inductive, this product of the mean or average values is the *true mean* or average value of the power in watts given to the circuit. If the circuit is inductive and possesses self-induction or capacity or both, the above product does not give the true mean power, but only what is commonly called the volt-amperes of *apparent power* in watts. The true mean or average power in this case is given by the mean or average value of the product (amperes $\times$ volts) in the circuit, for *the mean product of two periodic functions representing current and pressure is not equal to the product of their mean values.*

Now, the mean value of the product, which thus represents the *true power* in watts, can be measured directly by a wattmeter, and the ratio

$$\frac{\text{true power in watts}}{\text{apparent power in watts}} = \frac{\text{wattmeter reading}}{\text{amps.} \times \text{volts}}$$

is called the *power factor* of the circuit, which in practice can vary only between the two extreme values 0 and 1.

This limiting variation, together with the difference observed between the true and apparent power in watts in an inductive circuit, is explained by the fact that the current and voltage in such a circuit are not in phase, as will be understood by a reference to the so-called vector diagram (Fig. 110). Let the voltmeter reading be represented in magnitude (on some convenient scale) by the length of a straight line OV and the direction of action of the pressure or voltage by the arrow head, *i. e.* from O to V.

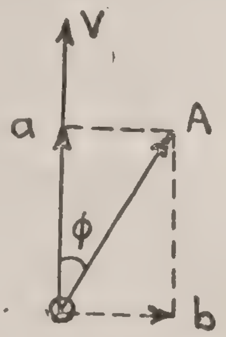


FIG. 110.

Thus OV is the voltage vector. Similarly let OA be the current vector for the ammeter reading, differing in phase from OV by an angle ϕ.

Now OA can be resolved into two component currents at right angles to one another—the one Oa in line with OV, the other (Ob) at right angles to it. Then Oa is called the useful energy- or *load-current*, and Ob the useless-, idle-, or *wattless-current*, connected solely with the periodic charge and discharge of energy in the circuit due to its inductance OaA is therefore

a triangle of currents for the inductive circuit in which OA is the resultant (or ammeter current) of two other currents, namely, an energy current Oa and an idle current aA always differing in phase by 90°. Then the product of the ammeter and voltmeter readings $= OA \times OV =$ the apparent watts, the wattmeter will give a reading $= OV \times Oa =$ the true, or useful, watts, while the wattless power will be given by $OV \times Ob$ watts, which does *no work* in the circuit.

Thus from the geometry of the figure we have the ratio

$$\frac{\text{true watts}}{\text{apparent watts}} = \frac{OV \times Oa}{OV \times OA} = \frac{Oa}{OA}$$

$= \cos \phi =$ *the power factor* of the circuit.

Although obtainable in other ways (vide p. 381), this ammeter, voltmeter, wattmeter method is by far the best and most direct one for measuring power factor, and is almost invariably employed.

The evaluation of the power factor $\cos \phi$ in the case of single-, two-, and three-phase A.C. inductive circuits, by this direct method, is given on p. 388 *et seq.*, and in the following test we shall restrict ourselves to single-phase circuits.

Apparatus.—Precisely that detailed for test No. 111.

Observations.—(1) Connect up as in Fig. 112, levelling and adjusting to zero such instruments as need it. The extremities T_1T_2 of the combination of C and r are the terminals of the circuit of which the power factor (P.F.) is required. As, however, it is sometimes necessary in A.C. testing work to obtain either an electrical load at varying P.F. or a variable load at constant P.F. with a choker and resistance, it is both useful and instructive to determine the effect on the value of the P.F. of changing (A) the ohmic resistance, (B) the inductance (whether by change in current strength or in disposition of magnetic circuit), and (C) the frequency—one at a time.

(2) A.—Note the readings of all the instruments, for each position of the two-way key K, for some eight different values of (r)—the frequency F and current A being kept constant throughout.

(3) B.—Note the readings of all the instruments, for each position of the two-way key K, for some eight different values of current A, covering the range of current utility of the circuit

or appliance in use—the frequency F and resistance r being kept constant throughout.

(4) C.—Note the readings of all the instruments for each position of the two-way key K, for some eight different values of frequency F—the resistance r and current being kept constant throughout.

Tabulate all your results as follows—

Coil (C):—Length = . . . No. of turns = . . . Resistance (R) = . . . ohms
Core: Length = . . . Sections = . . .
Non-Inductive Resistance (r): Nature. . . .

Ohmic Resistance				For Inductive Circuit XZ				For Non-Inductive Circuit XY			
Non-Inductive $r = \frac{V_r}{A}$.	Total $RT = R + r$.	Current A.	Frequency F.	Volts V.	Apparent Watts AV.	True Watts W.	Power Factor $\cos\phi = \frac{W}{AV}$.	Volts V_r.	Apparent Watts AV_r.	True Watts W_r.	Power Factor $\frac{W_r}{AV_r}$.

(5) Plot curves having values of power factor $\frac{W}{AV}$, as ordinates, with values of R_T in obs. 2; current A in obs. 3; and frequency F in obs. 4—respectively as abscissæ in each case.

Inferences.—State clearly all that can be deduced from the tables of results and the curves.

(110) Determination of the Effect of Frequency on the temperature of a given Circuit containing *Self-Induction and Ohmic Resistance only.*

Introduction.—The present test is devised with a view to ascertaining whether change of frequency materially alters the temperature of any appliance possessing self-induction and ohmic resistance, and for the success of the investigation the coils of the appliance used should have a low ohmic resistance, so that transference of heat due to the term C^2R to the core in which any alteration of temperature is to be observed, may be as small as possible.

Apparatus.—Alternator D capable of having its speed varied between wide limits, and whose voltage can be regulated by the exciting circuit (not shown); ammeter A (Fig. 251); hollow solenoid C to test, having a bundle of fine soft iron wires, smaller than the internal hollow, to allow of a thermometer being inserted as well as the bundle.

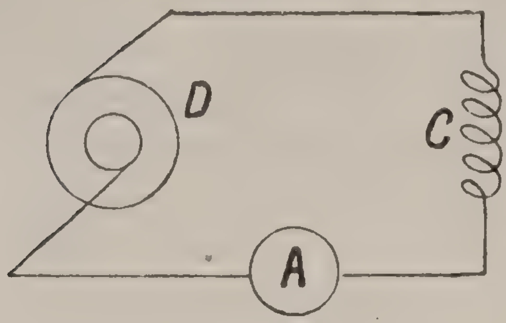


Fig. 111.

Observations.—(1) Connect up as in Fig. 111, and insert the core and thermometer in the solenoid, covering up the ends with cotton wool to prevent external cooling effects due to the air, etc. Note the temperature when steady.

(2) Adjust the speed and excitation of D so that the frequency has the lowest value practicable, and the current A some convenient value not high enough to heat the coils much.

(3) The speed and current A being kept constant, take the temperature $T°$ on the thermometer at successive noted intervals of time (t) from switching on until it remains constant, and note also the temperature of the room at intervals.

(4) Repeat 2 and 3 for the maximum speed allowable, and for one intermediate between this and the first-named, the core having been allowed to cool down in between each distinct set of observations, and the current A being the same. Tabulate your results as follows—

Name . . . Date . . .

Nature of Coil tested . . . Form of Core . . .
Periods of alternator per Revol. K = . . .

Temperature of Room.	Temperature of Core for Start $T°$ C.	Time from start t.	Speed Revs. per min. N.	Frequency $\sim$ per sec. $=\frac{KN}{60}$

(5) Plot a curve for each frequency on the same sheet, having temperatures $T°$ C. as ordinates, and (t) minutes as abscissæ.

Inferences.—What can you infer from your experimental results and the curves?

(111) Variation of Impedance with (*a*) Self-Induction, (*b*) Frequency, and (*c*) Ohmic Resistance in Circuits having Self-Induction and Ohmic Resistance only in Series.

Introduction.—In this test it will be necessary to vary one factor only at a time, keeping the remaining two constant. It should also be remembered that, as the coefficient of self-induction of any coiled circuit is the number of lines of force linked with it when unit current flows through it, any change in the current will alter the permeability of the magnetic path when this is composed partly or wholly of magnetic material, and hence also the self-induction. With an air-core the self-

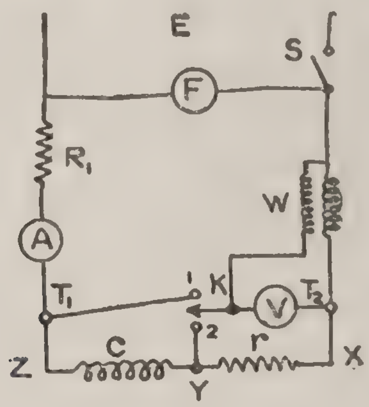


FIG. 112.

induction will be constant for all values of current. The variation of impedance and self-induction of an iron-cored solenoid with the position of the core has already been investigated in test No. 118, and the present determination can be conveniently made for fixed positions of the core P (Fig. 123).

Apparatus.—Source of A.C. supply E, preferably a motor-driven alternator, the excitation and speed of which are independently variable within wide limits; switch S; frequency meter F; ammeter A; voltmeter V; wattmeter W; two-way key K; two non-inductive variable rheostats R_1 and r (*e.g.* banks of lamps or carbon-plate rheostats); solenoidal choker C with movable iron core.

Note.—R_1 is needed (only if the supply E is from town mains) for keeping the current A constant, as r is varied. T_1T_2 are to be taken as the terminals of the impedance.

Observations.—(*a*) *Impedance with Self-induction at Constant Frequency and Ohmic Resistance.*

(1) Connect up as in Fig. 112, levelling and adjusting such instruments to zero as need it. Start the alternator with field regulator "full in," and see that the lubricating arrangements are working properly.

Note.—The following tests, Nos. 2 and 3, can be made on a public A.C. supply instead, if desired.

(2) With the core P clamped centrally in the coil C, and the speed adjusted to give a frequency F (= no. pairs of poles $\times$ revs. per min. $\div$ 60) of 50 $\sim$ per sec., close S, and by field regulation obtain some eight different currents on A, rising by about equal increments from 0 to the maximum safe value for the coil C, and note the readings of F, A, W and V at each, the frequency F and resistance r being constant throughout.

(3) Readjust the field regulator to "full in," and with the core P removed altogether, repeat 2 for the same *constant frequency* and range of currents.

(*b*) *Impedance with Frequency for Constant Self-induction and Ohmic Resistance.*

(4) Fulfil obs. 1 above; a variable speed alternator now being a necessity.

(5) With P clamped centrally, adjust the speed and consequently the frequency F to the lowest convenient value, and the current A (by field regulation) to, say, half the maximum value for the coil, to avoid much change of ohmic resistance by heating. Note the readings of F, A, W and V at each of some eight different values of F between the lowest and highest convenient, obtained by speed regulations, the above value of *current* A being kept *constant* throughout by field regulation.

(6) Repeat 5 with the core removed altogether, and for the same constant current, and tabulate all the results of obs. 2 to 6, as shown.

(*c*) *Impedance with Ohmic Resistance for constant Self-induction and Frequency.*

(7) With the core of C clamped centrally and a constant frequency F of, say, 50 $\sim$ per sec. adjust the current A to about half the maximum safe value for C (to minimize heating) and keep it *constant* throughout (by varying R_1 with town

supply for E or by field regulation with an experimental alternator for E) for some eight different values of r between 0 and the highest convenient, noting at each the readings of F, A, W and V, when the latter is connected by K across T_1, T_2 and r respectively.

(8) Repeat (7) with the core of C removed altogether for the same current and frequency, and tabulate as follows—

Coil (C): Length = . . . No. of turns = . . . Resistance (R) = ohms.
Core: Length = . . . Section =
Non-inductive Resistance (r): Nature . . . $p = 2\pi F =$ . . .

Frequency F.	Amps. A.	Position of Core of Choker C.	Voltage across		Impedance	Ohmic Resistance		Wattmeter Reading across T_1 T_2.	Total True Watts W.	Apparent Watts AV_T.	Power Factor	Angle of Lag $\theta°$.	Re-actance $2\pi fL =$	Coefficient of Self Induction $L =$
			Impedance V_T.	Non-Inductive Resistance V_r.	$\frac{V_T}{A} = \sqrt{(Lp)^2 + R_T^2}$.	$r = \frac{V_r}{A}$.	Total $R_T = (r + R)$.				$\cos\theta = \frac{W}{AV_T}$.		$\sqrt{\left(\frac{V_T}{A}\right)^2 - R_T^2}$.	$\frac{1}{p}\sqrt{\left(\frac{V_T}{A}\right)^2 - R_T^2}$.

(9) From obs. 2 and 3 plot curves having values of impedance, as ordinates, with values of A, L, and Lp as abscissæ in each case; also between V and L as ordinates and A as abscissæ.

From obs. 5 and 6 plot curves having values of impedance, as ordinates, with values of F and (Lp) as abscissæ.

From obs. 7 and 8 plot curves between impedance as ordinates and R_T as abscissæ.

Inferences.—From a careful study of the tables of results and forms of curves state clearly all that can be inferred therefrom.

(112) Numerical and Phase Relations between the Voltages and between Voltage and Current in Circuits containing Capacity only and when in Series with Ohmic Resistance.

Introduction.—The numerical relations between the various voltages in a circuit such as is now under consideration, seem at first sight to be so impossible that it is necessary to consider them in relation to phase. This can be done by a reference to the vector diagram (Fig. 113). In this, the total or *resultant*

voltage, as indicated on the ammeter A, is set off in magnitude and direction $= OD$. The *energy voltage* or ohmic drop OG is set off at an angle ϕ_1 in advance of OD, while GD will be the condenser, idle, or *reactive voltage* in magnitude and direction.

Comparing this with Fig. 108 for a self-inductive circuit, we see that capacity causes the current and its vector OA_1 to lead in front of the voltage OD by an angle ϕ_1, instead of to lag behind as shown in Fig. 108 for self-inductive circuits, both diagrams being supposed to rotate about O in the $+^{ve}$ direction (counter-clockwise). The angles B and G being right angles, lie (by geometry) on a semicircle, which is consequently the loci of the point of intersection of the energy and reactive voltages (which always differ by 90° in phase) between their limiting

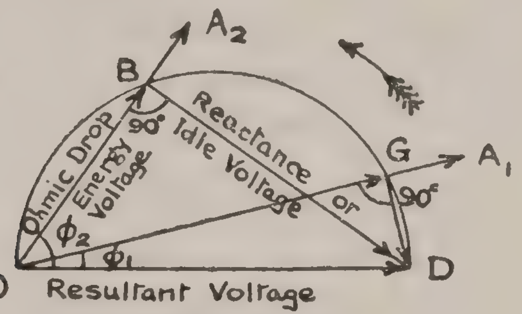


FIG. 113.

values of resistance R and capacity C respectively, namely, that of $R =$ maximum with $C = O$, for which $\phi_1 = O$, $OG = OD$, and $GD = O$; and that of $R = O$ with $C =$ maximum, for which $\phi_1 = 90°$, $GD = OD$, and $OG = O$. If for the triangle of voltages OGD we divide each of the voltages by the current A_1, the sides will represent the corresponding resistances in circuit, while the voltage vectors, as given if divided by these resistances respectively, will give the triangle of currents. The idle or wattless current equals $A_1 \sin \phi_1$, and the energy current equals $A_1 \cos \phi_1$.

Apparatus.—Source of A.C. supply E of, say, constant frequency, such as town mains; switch S; variable non-inductive resistances r and R; ammeter A; electrostatic voltmeter V; two two-way keys K_1K_2; condenser C.

Observations.—(1) Connect up as in Fig. 114, levelling and adjusting to zero A and V, if necessary.

(2) With R adjusted to O close S, and by varying R obtain

some six or eight values of current A, rising by about equal increments from O to the maximum possible, noting the values of V_1, V_2, and V_3 on V by means of K_1 and K_2.

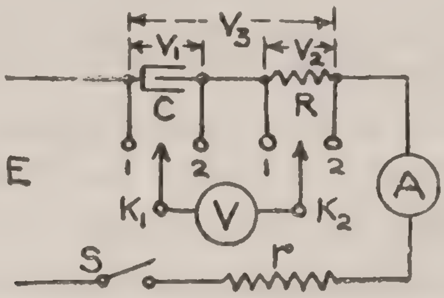


FIG. 114.

Note.—A convenient form of non-inductive resistance to use for R would be an 8 C.P., 16 C.P., and 32 C.P. glow-lamp, each of the same voltage as that of the supply, and arranged so as to be paralleled in any combination, thus giving seven possible different resistances of wide range. Tabulate your results as follows—

Capacity used : Type and Maker . . . Value = . . . mfds.
Non-Ind. Res. used : Nature . . . Frequency (constant) f = . . . ∼ per sec.

Voltage across			Amps. A.	Ohmic Resistance $R = \frac{V_2}{A}$.	Impedance $\frac{V_3}{A}$.	Power Factor $\cos\phi = V_2/V_3$.	Angle of Lead ϕ.
Supply V_3.	Condenser = Idle volts $(A/2\pi fC)$ V_1.	Non-Inductive Resistance = energy volts (AR) V_2.					

(3) Plot curves having values of V_1, as ordinates, with V_2 and C as abscissæ respectively. Also between $\cos\phi$ and impedance, as ordinates, with values of R as abscissæ. Compare V_3 with the algebraical sum $(V_1 + V_2)$.

Inferences.—State clearly all that can be deduced from your results.

(113) Variation of Impedance with (*a*) Capacity, (*b*) Frequency, and (*c*) Ohmic Resistance in Circuits having Capacity and Ohmic Resistance only in Series.

Introduction.—While the term impedance is applied almost universally to denote the apparent resistance of an alternating current circuit containing self-induction and ohmic resistance only, it is also used here to denote the expression $\sqrt{1/C^2p^2 + R^2}$ for the apparent resistance of an A.C. circuit having capacity (C) and ohmic resistance (R) only, the angular velocity p of the current being $= 2\pi \times$ frequency. As therefore it contains three variable factors, it will be necessary to vary one only at a time, keeping the remaining two constant.

When C and p are respectively the variables, we may make $R = 0$ and determine the effect of each on the remaining term, $\sqrt{\frac{1}{C^2p^2}} = \frac{1}{Cp}$, called the reactance or reactive resistance of the circuit.

Apparatus.—Source E of A.C. supply, preferably a motor-driven alternator, the speed and voltage of which can be varied

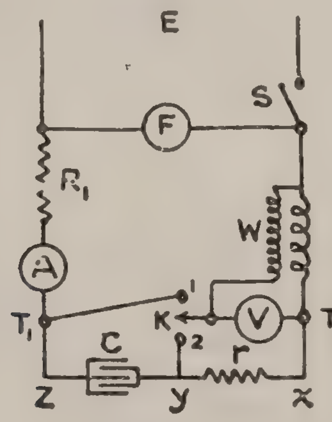


FIG. 115.

within wide limits; ammeter A; voltmeter V; wattmeter W; frequency meter F; switch S; variable capacity C; variable non-inductive resistance r; two-way key K; variable non-inductive rheostat R_1 (only needed if E is town mains).

Observations.—(*a*) *Impedance with Capacity at Constant Frequency and Ohmic Resistance.*

(1) Connect up as in Fig. 115, levelling and adjusting to zero such instruments as need it. Start the alternator with field regulator "full in," and see that all lubricating arrangements in use are working properly.

(2) With the alternator up to maximum speed, and therefore giving maximum frequency F (= No. of pairs of poles $\times$ rev. per min. $\div$ 60), close S, and adjust C to some six or eight different capacities, rising by about equal increments from the smallest to largest available, noting the reading of F, A, C and of W and V on K_1K_2 at *constant frequency* F and *resistance* (r).

(3) With $r = 0$ repeat obs. 2 for the same values of C, noting W and V on K_1, in addition to F, A, C, for the same constant frequency.

(*b*) *Impedance with Frequency for Constant Capacity and Ohmic Resistance.*

(4) A variable speed alternator being available, make C a maximum and give r some convenient fixed value. Now read all the instruments for both positions of K at each of some six or eight frequencies differing by about equal amounts between the maximum and minimum values obtainable by speed regulation.

(5) With $r = 0$ repeat obs. 4 for the same constant value of C and range of frequencies.

(*c*) *Impedance with Ohmic Resistance for Constant Capacity and Frequency.*

(6) Give C and the frequency their maximum possible values, and take the readings of all the instruments with K on studs 1 and 2 for some six or eight values of r, differing by about equal amounts between maximum and zero values.

Tabulate all your results as follows (where $p = 2\pi f$)—

Frequency f.	Amps. A.	Capacity C.	Volts across Impedance $T_1T_2 = V_I$.	Volts across non-Ind. Res. $r = V_r$.	Ohmic Res. r.	Impedance V_I/A ohms $= \sqrt{1/C^2p^2 + r^2}$.	Reactance $1/Cp$ ohms.	Wattmeter across $T_1T_2 = W_I$ Watts.	Wattmeter across $r = W_r$ Watts.	Watts absorbed in $r = AV_r$.	Apparent Watts $= AV_I$.	Power Factor $\cos\phi = W_I/AV_I$.	Angle of Lag ϕ.

(7) From obs. 2 and 3 plot curves having values of impedance $\left(\frac{V_I}{A}\right)$, reactance $\frac{1}{Cp}$ and amps. A as ordinates with values of capacity C as abscissæ.

From obs. 4 and 5 plot $\frac{V_I}{A}$, $\frac{1}{Cp}$ and A as ordinates with frequency (f) as abscissæ, and from obs. 6 plot $\frac{V_I}{A}$ as ordinates with ohmic resistance (r) as abscissæ.

Inferences.—State clearly all that can be deduced from the above results.

(114) Variation of Impedance with (*a*) Self-induction, (*b*) Capacity, (*c*) Ohmic Resistance, (*d*) Frequency in Circuits having Characteristics *a*, *b* and *c* in Series.

Introduction.—It has been stated that self-induction L causes the current to lag behind the voltage, while capacity C causes it to lead in front of the voltage. A circuit possessing both L and C may therefore cause the current to lag behind, lead in front of, or be in phase with, the voltage, depending on the relative mag-

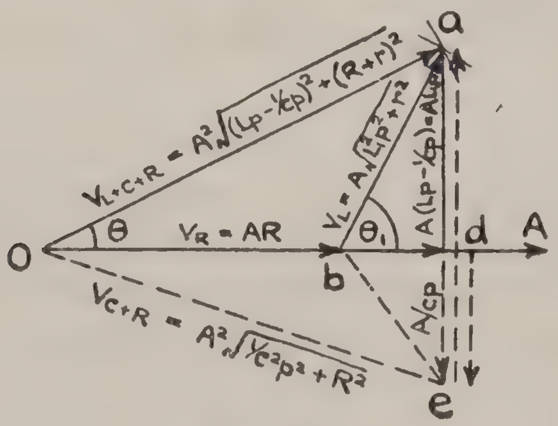


FIG. 116.

nitudes of L and C for a given ohmic resistance and frequency. Each of the two last named will in turn affect such phase relations, and hence in the present investigation we have four possible variables composing the *impedance* or "apparent" resistance of the circuit, only one of which must be varied at a time with the remaining three kept constant.

The phase difference between current and voltage will be less than that which would be caused by the same value of either L or C alone, and, as stated above, may even be zero. The above remarks will be better understood by a reference to Figs. 116 and 117.

Let OA be a vector representing, in magnitude and direction, the current A. Set off $Ob =$ the voltage $V_R (= AR)$, which is an energy or useful voltage in phase with A, along OA.

With centre (b) and radius $ba = V_L$ describe an arc of a circle, and with centre (O) and radius $Oa = V_{L+R}$ describe an arc

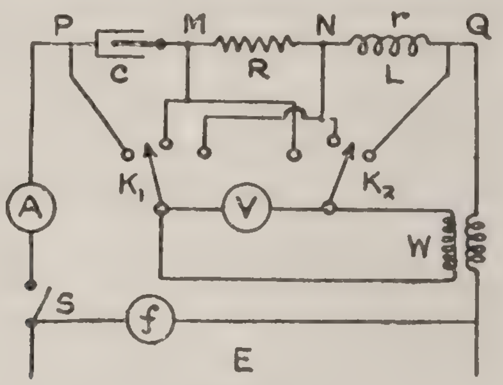


FIG. 117.

of a circle (Fig. 116), intersecting the other arc in the point (a).

From (a) drop a perpendicular to OA meeting it in (d) and produce it to (e) so that $de = V_C = \dfrac{A}{Cp}$ the condenser voltage. This will be 180° out of phase, *i. e.* in direct opposition to the voltage overcoming the self-induced idle voltage $(ea) = LpA$. Consequently the nett or effective idle voltage of the circuit $da = A\left(Lp - \dfrac{1}{Cp}\right) = AL_1p$, where $L_1 =$ the effective or nett inductance of the whole circuit PQ, and is of a self-inductive nature causing an effective angle of lag θ in the circuit. In the triangle Ode, $Ar = O$, since the condenser C is not considered to have any ohmic resistance r like the self-inductance L has. Therefore $Od =$ the energy voltage AR for the portion PN, Od is therefore also the useful or load component of the total voltage of the circuit PQ.

A most important deduction, affecting the calculation of the rise of pressure in cables and sometimes the breakdown of their insulation, now follows, namely, if the reactances of the self-

inductive and capacity portions are equal, *i.e.* if $Lp = \frac{1}{Cp}$ where $p = 2\pi f$ and $f =$ the supply frequency, the idle voltages of L and C will be equal and opposite in sign, and each may have much greater values than that of the supply. This condition in such a *series* combination, shown in Fig. 117, is called *pressure resonance.* The above condition may be written $2\pi fL = \frac{1}{2\pi fC}$, from which we get $f = \frac{1}{2\pi\sqrt{LC}} \sim$ per sec., but only when the ohmic resistance of the circuit is negligible is the periodicity of the supply equal to the *natural* periodicity of oscillation. The natural period of the circuit $\frac{1}{f} = 2\pi\sqrt{LC}$ seconds. The periodicity giving maximum resonance in a circuit of appreciable ohmic resistance R is $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{r^2}{4L^2}}$, which is not the *natural* periodicity of oscillation of the circuit.

Either of the above values of the critical frequency (f) giving maximum resonance is usually much greater than that of the supply voltage.

Apparatus.—Source of A.C. supply E, preferably a motor-driven alternator having a wide range of speed and excitation; ammeter A; wattmeter W; switch S; voltmeter V; and two three-way keys; a capacity C, self-induction L, and ohmic resistance R, each capable of variation; frequency meter (f).

Observations.—(*a*) *Impedance with Self-induction at Constant Frequency, Capacity and Resistance.*

(1) Connect up as in Fig. 117, levelling and adjusting to zero such instruments as need it. Start the alternator with field regulator "full in," and see that the lubricating arrangements are working properly.

(2) With the self-induction L, resistance R, and capacity C adjusted to convenient values, and the speed to give a frequency F (= No. of pairs of poles × rev. per min. ÷ 60) of 50 ~ per sec., close S and by field regulation obtain some eight different currents on A (and hence values of L) rising by about equal increments from 0 to the maximum safe value, noting the values of F, A, W, V and C at each, F, R and C being constant throughout.

(*b*) *Impedance with Capacity at Constant Frequency, Self-induction and Resistance.*

(3) Repeat obs. 2 for some eight different values of capacity C between 0 and the maximum possible, F, R and A (i. e. L) being constant throughout.

(*c*) *Impedance with Ohmic Resistance at Constant Frequency, Capacity and Self-induction.*

(4) Repeat obs. 2 for some eight different values of resistance R between 0 and the maximum possible, F, C and A (i. e. L) being constant throughout.

(*d*) *Impedance with Frequency at Constant Self-induction, Capacity and Resistance.*

(5) Repeat obs. 2 for some eight different frequencies between the minimum and maximum values possible, C, R and A (i. e. L) being kept constant.

Tabulate all your results as follows—

Self-Induction used : Nature or type . . . Ohmic Res. r = . . . ohms. . . .
Ohmic Res. (R) used : Nature or type . . .
Capacity (C) used : Type . . . $p = 2\pi F$.

Frequency F.	Amps. A	Capacity C.	Voltage across						Impedance	Ohmic Res.		Wattmeter Reading across PQ.	Total True Watts given to $PQ = W$.	Apparent Watts $A, V_{CRL} = W_a$.	Power Factor $\cos\theta = \frac{W}{W_a}$	Angle of Lag θ.	Total Reactance	Self-Ind. $L = \frac{1}{p}\left(K + \frac{1}{Cp}\right)$.
			PQ. V_{CRL}.	PM. V_C.	MN. V_R.	NQ. V_L.	PN. V_{CR}.	MQ. V_{RL}.	$\frac{V_{CRL}}{A} = \sqrt{\left(Lp - \frac{1}{Cp}\right)^2 + R_T^2}$.	$R = \frac{V_R}{A}$.	Total $R_T = R + r$.						$K = \left(Lp - \frac{1}{Cp}\right) = \sqrt{\left(\frac{V_{CRL}}{A}\right)^2 - R_T^2}$.	

(6) Plot curves having values of impedance as ordinates with each of the variables A (or L), C, R_T, and F in obs. 2–5 as abscissæ on the same curve-sheet.

Inferences.—State clearly all that can be deduced from the results of the test.

Note.—The numerical and phase relations between the voltages across C, R and L, and the combinations of these can

be studied in the above table with advantage and are highly instructive. From them the student should draw to scale the diagram shown in Fig. 116 above, for, say, two extreme values of the overall voltage V_{CRL}, and see how the angle of phase difference θ compares with that calculated in the above table.

(115) Numerical and Phase Relations between Main and Branch Currents in Circuits containing Ohmic Resistance in Parallel with either Self-induction or Capacity.

Introduction.—The determination of the above relations between the main and branch currents in a circuit comprising ohmic resistance and self-induction in parallel is effected in detail in test No. 134, which should be done for the present test.

The numerical relations are at once seen in the table of results, while the phase relations are best seen from the diagram (Fig. 142) constructed for any particular set of simultaneous currents. It will be obvious that the relations will differ according to whether the self-induction branch PQ possesses appreciable ohmic resistance or practically none. Fig 141 presumes the former condition, but if the latter obtains, then the current A_1 in the non-inductive branch, being *in phase* with the voltage, will be given by OC, while that in the inductive branch A_2 (having no resistance) lags just 90° *behind the voltage*, and will now be given by Ca (perpendicular to OC), instead of by ba as in Fig. 142.

The present test should also be operated with capacity C substituted for the self-induction shown, when the student should have no difficulty in modifying both the tabular form of entry and the vector diagram to suit the new condition of capacity in parallel with ohmic resistance. If no resistance is purposely added to the condenser branch, the current in this will lead just 90° *in advance of the voltage*. Thus, the main or resultant current will be given by the diagonal of a parallelogram, the

sides of which will be at right angles and represent the branch currents.

(116) Variation of Impedance and Phase Relations between the Currents in a Circuit containing Capacity and Self-Induction in Parallel.

Introduction.—The combination of self-induction in parallel with capacity is an extremely important one, and has some interesting and highly useful applications in electrical engineering which will be mentioned later. Referring to Figs. 118

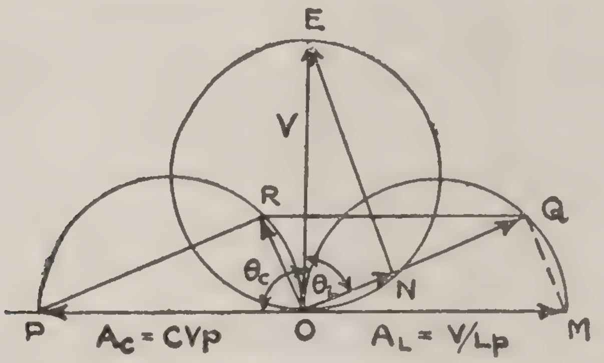


FIG. 118.

and 119. Let $OE =$ the supply voltage V, and if the ammeter A_C and switch S_C add a negligibly small resistance to the condenser branch, the current A_C in this branch will $= CVp$ and $\tan\theta_C = \dfrac{1}{Cpr_c} = \infty$, whence A_C will make an angle of phase difference $POE = \theta_C = 90°$ in advance of V.

Similarly, if the ohmic resistance of the self-inductive branch is negligibly small, the max. current A_L in this branch will $= \dfrac{V}{Lp}$ and $\tan\theta_L = \dfrac{Lp}{r} = \infty$, whence A_L will lag behind V by an angle $MOE = \theta_L = 90°$. OP and OM are therefore the max. values of the respective branch currents. If, however, the self-ind. branch possesses a resistance (r) ohms in addition to self-ind. L, its current will be given by $A_L = \dfrac{V}{\sqrt{L^2p^2 + r^2}} = OQ$ lagging behind V by an angle $\theta_L = QOE$, such that $\tan\theta_L = \dfrac{Lp}{r}$

(less than before). The total current A being the resultant OR of OP and OQ and making an angle $ROE = \theta$ (seen to be one of advance in this case) with the voltage V.

The semi-circles ORP and OQM are the loci of the vectors representing the branch currents A_C and A_L, and it will be seen that the smaller the resistance (r), the nearer will Q approach M, and the smaller will be the resultant or main current OR ($= A$) from the supply and the more nearly will it be in phase with OE ($= V$).

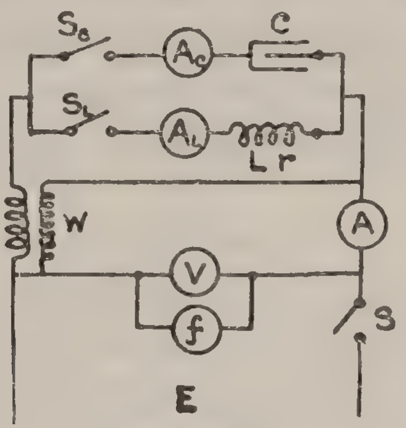


FIG. 119.

Thus at the *practical* limit, r will be very small, Q very close to M, A_L nearly $= A_C$ and nearly 180° out of phase with it, and OR very small and nearly *in phase* with V. Under these conditions *current resonance* is said to prevail in distinction to pressure resonance explained on p. 330 for series circuits. With current resonance in such a parallel circuit, the local current circulating in the loop may be many times greater than the main supply current A—a condition obtained when the idle or wattless components of the branch currents are practically equal, but of opposite sign when the wattless or idle component of the main current A will be less than that of either branch. Equal and opposite wattless currents in the branches will be obtained when

$$\frac{1/Cp}{1/C^2p^2 + r_c^2} = \frac{Lp}{L^2p^2 + r^2},$$

but as explained for series circuits this condition can only truly be called resonance when both r_c and r are negligibly small.

In practice we see capacity used for starting up alternating current motors; for nullifying the effects of the idle currents in a distributing system, thereby raising the power factor, and so increasing both the efficiency and economy of operation. The

capacity effect in this case is produced by an *over-excited* synchronous motor connected to the same supply.

Apparatus.—Source E of A.C. supply preferably a motor-driven alternator variable in speed and excitation within wide limits; frequency meter (f); voltmeter V; wattmeter W; ammeters A, A_C and A_L; switches S, S_C and S_L; variable capacity C; variable self-induction L of ohmic resistance (r).

Observations.—(*a*) *Impedance with Self-induction at Constant f, C and r.*

(1) Connect up as in Fig. 119, levelling and adjusting to zero such instruments as need it. Start the alternator with field regulator "full in," and see that the lubricating arrangements are working properly.

(2) With f, C and r (if alterable) adjusted to convenient values, close S, and then S_C only, taking the readings of all the instruments.

(3) With f, C and r as in obs. 2, close S, and then S_L only and take the readings again.

(4) With f, C and r again the same, close all 3 switches and by field regulations obtain some 8 different currents on A_L (and hence values of L) rising by about equal increments up to a safe max. value, noting the readings of all instruments.

(*b*) *Impedance with Capacity at Constant f, A_L* (i. e. *L*) *and r.*

(5) Repeat obs. 4 for some 8 different values of capacity C at constant f, r and A_L (i. e. L).

(*c*) *Impedance with Ohmic Resistance* (r) *at Constant f, A_L* (i. e. *L*) *and C.*

(6) Repeat obs. 4 for several values of (r) if this is variable at constant f, L and C.

(*d*) *Impedance with Frequency* (f) *at Constant A_L* (i. e. *L*), *C and* r.

(7) Repeat obs. 4 for some 8 different values of frequency (f) at constant A_L (i. e. L), C and r.

(8) By varying C, L, f, find the minimum value of A obtainable, noting the readings of all instruments at this, and tabulate all your results as follows—

Frequency (f).	Voltage V.	Wattmeter Reading.	True Watts W.	Amps. A.	Amps. A_C.	Amps. A_L.	Apparent Watts AV.	Power Factor $\cos\theta = W/AV$.	Angle of Phase Diff. θ.	$\cos\theta_L = \frac{r}{\sqrt{L^2p^2+r^2}}$	θ_L.	Capacity $C = A_C/pV$.	Self-Ind. $L = \frac{1}{p}\sqrt{\left(\frac{V}{A_L}\right)^2 - r^2}$.	Impedance V/A_C.	Impedance V/A_L.	Impedance V/A.

Inferences.—State clearly all the inferences which can be deduced from the above results.

(117) Determination of the Load and Wattless Currents in an Inductive Alternating Current Circuit.

Introduction.—While the present investigation is bound up with that of power factor, considered in test No. 109, p. 316, the whole subject has such a vastly important bearing on the economical and efficient generation, transformation, and distribution of electrical energy, that a further consideration of it will be an advantage.

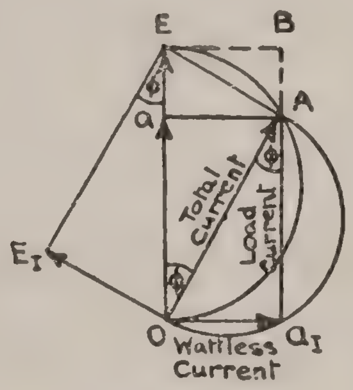


FIG. 120.

It is well known that of the power in watts, given by the product (amperes × volts) "apparently" supplied to an inductive A.C. circuit or one containing self-inductance, or capacity, or both, only a portion constitutes an actual or useful load or power and does useful work in the circuit, while the other portion represents no load at all, and is said to be wattless or idle power, doing no work in the circuit.

The *useful power* is given by the product of that portion of the current and voltage in phase with each other, and is usually called the *true power*, while the wattless or idle power is given by the product of those portions of the current and voltage which are in quadrature, as it is termed, *i. e.* differ in phase by 90° or a quarter period, the average value of the latter product being always zero. The useful and wattless powers are each given by a product of *amperes* × *volts*, and may be arrived at in either of two ways as follows: let a voltage OE and a current OA differ in phase by an angle ϕ. Resolve OA into two components, one Oa along and in phase with OE, the other Oa_1 perpendicular to it. Then $OaAa_1$ is a rectangle, and the corner

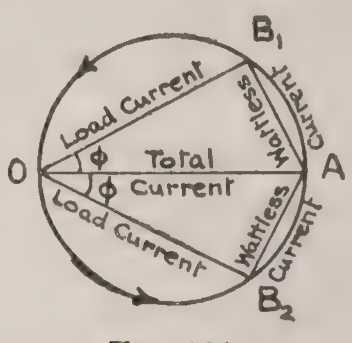


FIG. 121.

a_1 will lie on a semi-circle Oa_1A drawn on OA as diameter. We now have the true power $= OE \times Oa$, the wattless power $= OE \times Oa_1$, and the apparent power $= OE \times OA$. Oa_1A is, therefore, the triangle of currents of which OA is the total or resultant or ammeter current, a_1A the load or useful current, and Oa_1 the idle or wattless current, always perpendicular to a_1A. The power factor $\cos\phi = \dfrac{OE \times Oa}{OE \times OA} = \dfrac{a_1A}{OA} = \cos O\hat{A}a_1$.

Again, resolve OE into two components—one OA along and in phase with the current vector OA, the other OE_1 perpendicular to it. Then OE_1EA is a rectangle and the corner A will lie in a semi-circle OAE drawn on OE as diameter. We now have the true power $= OA \times OA$, the wattless power $= OA \times OE_1$. OAE is therefore the triangle of voltages of which OE is the total or resultant or voltmeter voltage, OA in phase with the current, the load or useful or energy voltage, and OE_1 the idle or wattless voltage always perpendicular to OA. The power factor $\cos\phi =$

$$\frac{OA \times OA}{OA \times OE} = \cos A\hat{O}E.$$

Now, the wattless powers in the two cases are $OE \times Oa_1$ and $OA \times OE_1$ respectively, which are equal, since the areas of the rectangles ($OE \times Oa_1$) and ($OA \times OE_1$) are equal. Mathematically, therefore, it is immaterial from which point of view the matter is treated, as both lead to the same result, namely—

True power = total voltage $\times$ useful current = $V \times A \cos \phi$,
" " = total current $\times$ " voltage = $A \times V \cos \phi$.

In practice, however, it is more convenient and general to consider the total current A to be made up of two components, respectively $A \cos \phi$ *in phase* with, and $A \sin \phi$ *in quadrature* with, the voltage, and termed the energy, useful, or load current and the idle or wattless current. These are related geometrically, as seen in Fig. 121, by the equation

$$(\text{Total current})^2 = (\text{useful current})^2 + (\text{wattless current})^2,$$

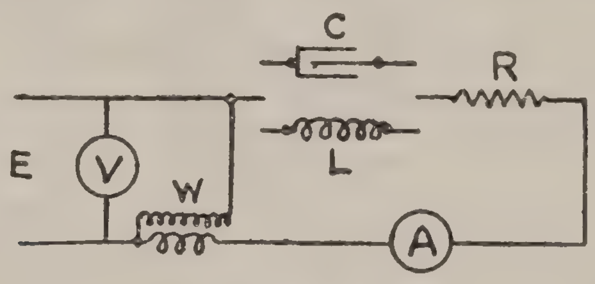


FIG. 122.

from which the wattless current measurement of the present test is deduced. This can be made with the two possible circuit conditions, namely, self-induction with ohmic resistance, for which the total current *lags* in phase behind the load current by an angle ϕ, and capacity with ohmic resistance, for which the load current lags in phase behind the total current by an angle ϕ, *i. e.* the total current *leads* in front of the load current. This is shown in the single diagram, Fig. 121, though more commonly by two separate ones split along the line OA.

Apparatus.—Source E of alternating current; voltmeter V; ammeter A; wattmeter W; variable non-ind. resistance R; capacity C; self-induction L.

Observations.—(1) Connect up as in Fig. 122, levelling and adjusting to zero such instruments as need it.

(2) With C only connected in circuit, note the readings of V, W, and A for some five or six values of current A between O and the maximum possible by varying R.

(3) With L only connected in circuit, repeat obs. 2 and tabulate as follows—

Nature of Circuit.	Voltage V.	Watts W.	Currents.			Power Factor $\cos\phi = W/AV$.	Angle of Phase Diff. $\phi°$.
			Total (OA) $= A$.	Load $= W/V$.	Wattless $= \sqrt{A^2 - (W/V)^2}$.		

(4) Check one or more of the tabular readings by diagram, such as Fig. 121.

Inferences.—State all that can be deduced from the results of the test.

(118) Variation of Impedance, Reactance and Self-Induction with Position of Movable Core in Solenoidal Choker.

Introduction.—This test is intended to show the principle underlying the action of the so-called dimmer, which is so commonly used now in theatres and picture-halls for raising and

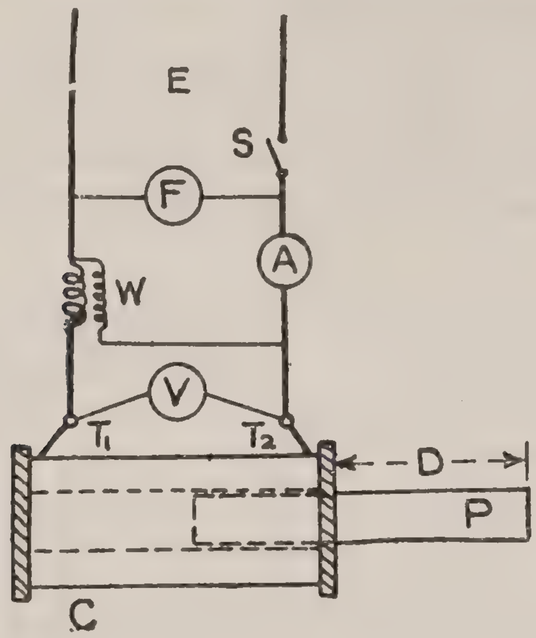


Fig. 123.

lowering the lighting when the supply thereto is alternating current, also the range of regulation of a choking coil for working on arc lamp circuits of different voltages. It has the great advantage over a variable "line" resistance of preserving completely the electrical continuity of the circuit, while introducing

a back E.M.F. of self-induction to the supply depending on the position of the movable core.

Apparatus.—Source of A.C. supply E, preferably an experimental motor-driven alternator, the excitation and speed of which are independently variable within wide limits; switch S; frequency meter F; ammeter A; voltmeter V; wattmeter W; and the movable core solenoidal choker C.

Observations.—(1) Connect up as shown in Fig. 123, levelling and adjusting such instruments to zero as need it. Ensure that the lubricating arrangements are working properly on starting up.

(2) With the field regulator of E full in and the machine supplying *constant frequency* F, close S and adjust the alternator field excitation so as to give the max. safe current A through C with the centres of P and C coinciding (i. e. $D = O$), then note the readings of F, A, W and V.

(3) Take the readings of F, A, W, V and D with the *same constant values of F and A* for each of a series of clamped positions of P between $D = O$ and $D =$ full length of P, with P finally removed to a distance.

(4) Repeat obs. 2 and 3 for the *same constant frequency* F, but with V *now maintained constant*, at such a value as will prevent the current rising above the max. safe value when P is removed to a distance, and tabulate all your results as follows—

Choker coil: Length = . . . No. of turns = . . . Res. R = . . .
Core: Length = . . . Cross Section = . . .

Frequency F.	Amps. A.	Volts V.	Distance D.	Wattmeter Reading.	True Watts W.	Apparent Watts AV.	Power Factor $\cos\theta = \frac{W}{AV}$	Angle of Lag $\theta°$.	Impedance $V/A = \sqrt{L^2p^2 + R^2}$.	Reactance $Lp = \sqrt{(V/A)^2 - R^2}$.	Coeff. of Self-Ind. $L = \frac{1}{p}\sqrt{\left(\frac{V}{A}\right)^2 - R^2}$.

(5) Plot curves for obs. 2 and 3 having values of D as abscissæ with values of V, W, $\cos\theta$, V/A, Lp and L as ordinates respectively, and for obs. 4 having values of D as abscissæ with values of A, W, $\cos\theta$, V/A, Lp and L as ordinates respectively.

Inferences.—State all you can deduce from the table of results and curves.

(119) Effect of Length of Air Gap in a Closed Magnetic Circuit on Impedance, Reactance, Self-Induction, Current and Power.

Introduction.—The present test is a very important one, inasmuch that it is a direct proof of fundamental theory, and has an important bearing on the use and range of regulation of all kinds of choking or reactance coils for adjusting the current in arc lamp circuits at different voltages, while at the same time emphasizing the relative merits of "closed" and "open" magnetic circuits. The factors of an alternating current supply being voltage, current, and frequency, with the last named usually

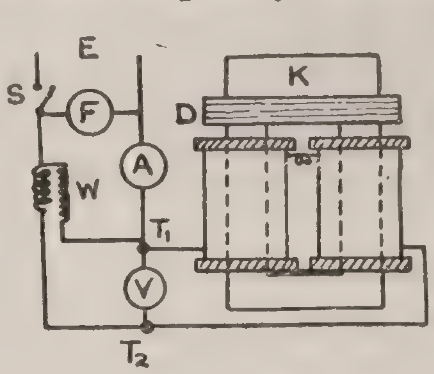


FIG. 124.

constant, it follows that the present investigation can be carried out in at least two ways, namely—

(*a*) With *constant* current and varying voltage at *constant* frequency.

(*b*) With *constant* voltage and varying current at *constant* frequency.

In the former method of supply, and with a series wound and connected choker, any effects observed by varying air-gap must be due to this alone. In the latter method, owing to change of current strength (in all but saturated magnetic circuits) causing a change of magnetic flux and induction density, any effects otherwise due to change in length of air-gap may be seriously vitiated.

Apparatus.—Experimental magnetic circuit with adjustable

air-gap; switch S; frequency meter F; ammeter A; voltmeter V; wattmeter W; source of A.C. supply E, preferably from an experimental motor-driven alternator, the voltage and speed (*i. e.* frequency) of which are independently variable within wide limits.

Observations.—(1) Connect up as in Fig. 124, levelling and adjusting to zero such instruments as need it. Now start the motor-alternator, observing that the lubricating arrangements are working properly.

Note.—If a constant voltage and frequency town supply is used, a suitable non-inductive resistance must be connected in series with one of the mains *on the supply side* of W for regulating the current.

(2) Remove all the non-magnetic distancing strips D and clamp the laminated iron keeper K down on to the poles by means of the wing-nut clamp (not shown in Fig. 124). With the field-regulating resistance of the alternator full in, close S and obtain a frequency of 50 or 60 ~ per sec. on F, to be kept *constant* by driving the alternator at the requisite *constant speed*, where the frequency $(f) =$ No. of pairs of poles $\times$ revs. per min. $\div$ 60.

(3) Raise the A.C. voltage by field regulation to the maximum value possible, so long as the current produced does not exceed the safe maximum for the choker winding. Then note the readings of F, A, W, V, and that the air-gap is zero.

(4) Unclamp K, and carefully raise it *just sufficiently only* to slide one distance strip D in between it and the poles, and re-clamp K.

Now, lower the voltage (by field regulation) until A *has the same value* as before—the *frequency being also the same.* Then read F, A, W and V.

(5) Repeat (4) for about ten different air-gaps, increasing by one distance strip at a time, and finally with K removed altogether, *i. e.* air-gap = max.

(6) **Employing supply condition** (*b*), mentioned in the introduction above, with K removed altogether, adjust the field regulator of the alternator so as to give such a voltage as will send the max. safe current through the choker winding at the same frequency as before. Now note the values of F, A, W and

V, and that the air-gap = max. This voltage and frequency is to be kept constant in future.

(7) Next take the readings of F, A, W and V for each of a series of air-gaps, ranging from that given by all the distance strips clamped together between K and the poles to none in at all, by one at a time, and tabulate all your results as follows—

Form of Inductive Circuit tested . . .
Section of: Core = . . . Yoke = . . . Keeper = . . .
Distance Strips each = . . . Thick: Resistance of Choker winding R = . . . ohm.

Frequency F.	Amps. A.	Volts V.	No. of Strips in use.	Length of Air Gap D.	Wattmeter Reading.	True Watts W.	Apparent Watts AV.	Power Factor $\cos\theta = \frac{W}{AV}$	Angle of Lag $\theta°$.	Impedance $V/A = \sqrt{L^2p^2 + R^2}$.	Reactance $Lp = \sqrt{(V/A)^2 - R^2}$.	Coeff. of Self-Ind. $L = \frac{1}{p}\sqrt{\left(\frac{V}{A}\right)^2 - R^2}$.

(8) Plot curves from obs. 3–5 having values of D as abscissæ with V, W, $\cos\theta$, V/A and L as ordinates respectively. Also from obs. 6 and 7 plot curves having D as abscissæ with A, W, $\cos\theta$, V/A and L as ordinates respectively.

Inferences.—From a careful study of the above table and curves state clearly all that can be deduced.

(120) Investigation of Mutual Inductive Effects due to the Relative Positions of Two Coiled Circuits.

Introduction.—The object of this investigation is to find out to what extent, and in what way, two neighbouring electromagnetic fields may react on one another when in different relative positions.

Qualitative and quantitative results are obtainable which are both interesting and instructive, in view of how little the average student realizes the possibilities of interaction between neighbouring magnetic fields and apparatus with the prejudicial effects often resulting. For the investigation, two solenoidal movable iron core choking coils (preferably similar in all respects) may be used, connected in series.

Apparatus.—Two similar chokers; ammeter A; voltmeter V; wattmeter W; frequency meter F; switch S; non-inductive resistance R (such as a bank of lamps) for regulating the current, if the supply E is from the town. If from an experimental motor-driven alternator, R can be omitted and A adjusted by field regulation on the alternator.

Observations.—(1) Connect up as in Fig. 125, where T_1T_2 are the terminal extremities of the two coils connected permanently in series. Level and adjust to zero such instruments as need it.

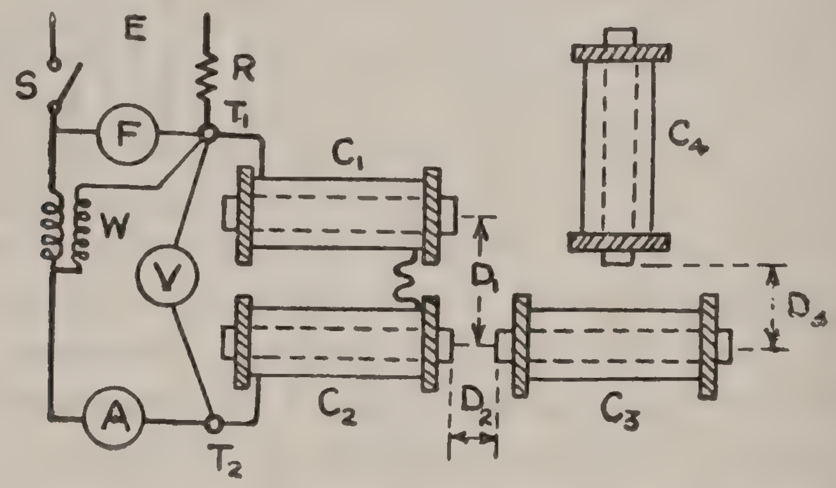


FIG. 125.

(2) With coils touching side by side in arrangment C_1C_2 (*i.e.* D_1 = minimum) and cores clamped centrally, take the readings of F, W, V, A and D_1 for *constant* full-load current A and frequency $F = 50$ for each of a series of values of distance D_1 up to a convenient maximum, the coils being parallel at each.

(3) Take a copy of an iron filing diagram for the position D_1 = a min.

(4) Repeat obs. 2 and 3 with one coil reversed or turned through 180°.

(5) Repeat obs. 2 to 4 for the position of coils shown at C_3C_4 (*i.e.* with magnetic axes of cores perpendicular) at different distances D_3.

(6) Repeat obs. 2 to 4 for the position of coils shown at C_2C_3 (*i.e.* axes in line) at different distances D_2, and tabulate as follows—

Coil C_1 used : Length = . . . No. of turns = . . . Res. R = .
Core length = . . . Cross section = . . .
Coil C_2 used : Length = . . . No. of turns = . . . Res. R = . . .
Core length = . . . Cross section = . . .

Relative Positions of Coils.	Distances D_1, D_2 and D_3.	Frequency F (Constant)	Current A (Constant)	Volts V.	True Watts W.	Apparent Watts $A \times V$.	Impedance V/A.	Power Factor $\cos\phi = W/AV$.

(7) In addition to the copies of the respective iron filing diagrams, plot the following curves having values of distance D as abscissæ, with volts V and impedance $\frac{V}{A}$ respectively, as ordinates in each case.

Inferences.—From the table of results, diagrams and curves state all that can be deduced.

(121) Measurement of Magnetic Permeability by the Permeameter.

Preliminary.—The following method, devised by Prof. S. P. Thompson, is a simple and convenient workshop one for rapidly measuring the magnetic permeability (μ) of any material. It is quite distinct from the ballistic, direct magnetometric, or optical methods of measuring (μ), and is based upon the law of magnetic traction, viz. that the tractive force over a given area of contact is proportional to the square of the magnetic flux through the junction. This and all other traction methods are not capable of giving very accurate measurements of (μ), for both the tensile stress and the place chosen for contact between specimen and block may affect the results somewhat, as in the latter case the distribution of the induction is not very uniform at this point. Now, since in the permeameter the magnetizing coils remain fixed, the pull on the specimen core will be due to $(B-H)$ lines, where B = induction per sq. cm. through the junction, and H = magnetizing force producing it. If S = sectional area of the junction in sq. cms. the force of attraction between core and block, *i. e.* Pull $(P) = (B-H)^2 S \div 8\pi$ dynes $= (B-H)^2 S \div (8\pi \times 453{\cdot}6 \times 981)$ lbs. $\therefore B = 1317 \times \sqrt{\frac{P}{S(\text{sq. in.})}} + H$ C.G.S. lines. Where P = pull in lbs. to detach.

If the magnetizing coil consists of T turns, carrying a current A amps., and its length between ends $= l$. Then $H = \frac{4\pi AT}{10\,l} = KA$, in C.G.S. measure. (The constant $K = 4\pi T \div 10\,l$.)

An advantage with this method of measuring permeability is that the specimen of the material to be tested is in the form of a simple straight rod of circular cross section, and in this form it can generally be very easily obtained. Further, owing to no delicate ballistic galvanometer being used in the method, the test can be performed even in fairly close proximity to dynamos or near other magnetic fields without especially vitiating the results. The permeameter illustrated is fitted with a slip coil $C\,C$ for measuring μ ballistically if desired. For a more detailed description of the instrument *vide* p. 615.

Apparatus.—Permeameter (Fig. 280); Salter's spring balance; ammeter A; rheostat r_2 (p. 599); battery b; switch K_1; Pohl's commutator or reversing switch S (p. 584); specimen or rod R to be tested.

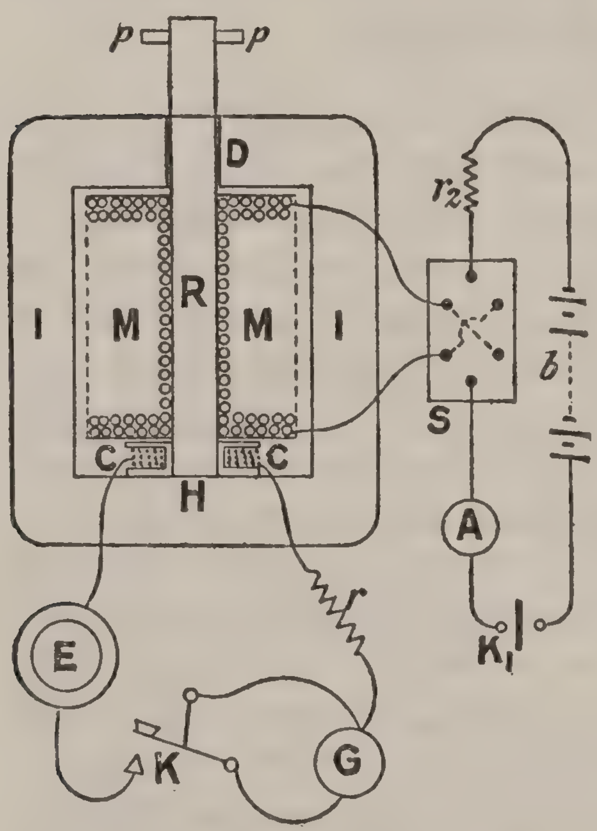


Fig. 126.

Tests.—(1) Connect up as indicated in Fig. 126, omitting the ballistic circuit shown at the lower part of the Fig. Insert a specimen in the coil after having cleaned the end and demagnetized it. Then attach the spring balance by means of its double hook to the pin pp in the present case.

(2) With r_2 full in, adjust the current to a small value (say 0·02 amp.), and note the force P lbs. required to detach. This should be repeated two or three times, and the mean noted.

(3) Repeat 2 for about fifteen different currents up to the maximum (about 7 amps.), rising by such amounts as will give about equal increments of pull on the balance.

(4) Repeat 2 and 3 for different specimens, and tabulate as follows—

NAME . . . DATE . . .

Specimen tested.				Magnetizing.		Pull P lbs.	Induction B.	Permeability $\mu = \frac{B}{H}$
No	Nature.	Diameter d (ins.)	Section (S) $= \frac{\pi d^2}{4}$ Sq. in.	Current A amp.	Force $H = KA$.			

(5) Plot two curves, one having H as abscissæ and B as ordinates, the other having B as abscissæ and μ as ordinates.

(122) Measurement of Magnetic Permeability (by Hopkinson Permeameter).

Introduction.—The present test is very similar to the preceding one (No. 121), except that a slightly different form of

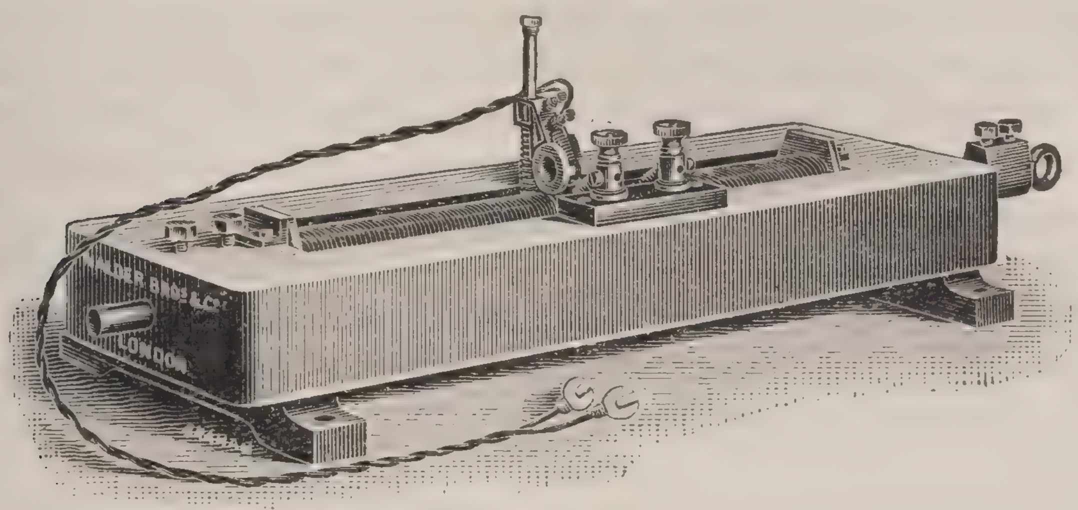


FIG. 127.

permeameter, devised by Professor Hopkinson, is provided. This instrument consists (as seen in Fig. 127) of a heavy wrought-iron yoke with two magnetizing coils, one having a fixed core and the other a movable one. On the movable core is a small coil of wire which, when the core is withdrawn, is jerked up by a spring; this coil is connected to a ballistic galvanometer, and the throw is proportional to the lines in the core; by this means cores made of various samples may be compared for permeability. The apparatus and connections, except for the differ-

ence in the actual form of permeameter, and the addition of a ballistic galvanometer G, key K, adjustable standard known resistance (r), and earth inductor E, or preferably a standard solenoid, are precisely those indicated in Fig. 126, and the student should read the introductory remarks of test No. 121.

The evaluation of the throws on the ballistic galvanometer G is exactly as given in the introduction to test No. 78, and therefore need not here be repeated.

Observations.—These consist in taking the galvanometer first throw at the moment when the small slip-coil springs out, for each of a series of exciting currents ranging from 0 to the safe maximum permissible, and tabulating all the readings and evaluated results in a convenient form.

Note.—The first throw should be repeated, *for each excitation*, by replacing the slip-coil two or three times, when the mean throw only at each current should be tabulated. Further, a preliminary trial must be made, before taking the above series of observations, by adjusting the resistance (r) until maximum permissible exciting current gives a mean first throw, not exceeding full-scale deflection. Lastly, the increments of current in the series must be smaller during that part of the range where the magnitude of the mean throws appears to be differing considerably.

A curve should be plotted between values of induction density B found, as ordinates, with values of magnetizing force H as abscissæ, and also one between permeability μ as ordinates, and values of B as abscissæ.

(123) Measurement of Magnetic Hysteresis and Eddy or Foucault Currents in Samples of Magnetic Material. (By Single Phase Alternating Currents.)

Introduction.—The determination of magnetic hysteresis in magnetic materials by some of the most important methods is given in considerable detail in *Practical Electrical Testing* by the author, and the reader is referred to this book for further particulars of these tests. It is of course well known that an iron

core, magnetized by an alternating current of electricity, is the seat of two distinct losses of power, (1) from hysteresis, and (2) from eddy currents generated in the transverse section of the conductor.

The former depends on the induction density in the iron, the frequency of the alternating current, and on the volume and nature of the magnetic material in question. No amount of lamination will get over this loss.

The latter source of power loss is dependent on the extent of lamination of the iron and on the transverse section of each individual portion of the core.

Now by employing sufficient iron in a test specimen, *well laminated*, the eddy current loss can be made small compared with the hysteresis loss, whence any measurement now made of the total core or iron loss will for all practical purposes represent the hysteresis loss simply. This is the principle upon which the present method is based, but if greater accuracy be desired the results so obtained can easily be checked by one of the methods given in the above-mentioned work.

It will thus be at once evident from the foregoing remarks that the iron employed in all electrical engineering appliances, but more particularly in alternating current ones, should be tested for hysteresis loss prior to being used in the construction of such appliances. The present method is one of the simplest and most expeditious ways of measuring the hysteresis loss in different samples of iron which may be to hand, and it is accurate enough for most practical purposes.

Probably the most important direction in which the preceding remarks find an application is in transformer, alternator, and alternating current motor work. As the magnetic circuits of such appliances are built up of stampings-out of thin soft sheet iron, this latter is the form in which samples to be tested usually come to hand. Assuming therefore that a few large sheets of the material to be tested, the thickness of which usually varies from 0·35 m.m. to 0·5 m.m. for transformers and up to about 1 m.m. for alternators, etc., is at hand, the first thing to do is to prepare the material for testing by constructing a small transformer out of it thus—

Preparation of Iron Samples for Test.

Cut such a number of strips out of the sheet, each about 12″ × 2″ as will make four equally thick piles, each containing the same number of strips, placed on the top of one another like the leaves of a book, to a thickness of, say, ½″ and each weighing about 2 lbs.

Now remove any burr from the edges of each by means of a file, and weigh all the strips, noting the total weight of iron W which should preferably be 8 or 10 lbs.

Next varnish *one side* of *each strip* with thin shellac varnish, and when dry assemble into four equal piles with varnished faces all pointing one way. Bind each pile, to within 1½″ of each end, with a layer of thin prepared tape, when each will be ready to receive the magnetizing coils. It will be noticed that each strip is insulated from the next by the equivalent of one layer of thin varnish, which is all that is needed. Next make a thin rectangular cardboard tube about 4″ to 4½″ long for each pile and capable of just slipping easily over it. Wind each of these with two distinct coils of, say, No. 18 double cotton-covered copper wire, each coil consisting of two layers and the two coils wound one over the other. Place the four bundles of strips with their coils in position so as to form a rectangular frame of iron with adjacent ends *interleaved*, so to speak, and clamped together so as to form a compact joint of low resistance. Join the four coils of each set together so that they would help one another in magnetizing the ring and the specimen is then ready for test. Note the total number of turns N_P and N_S on both primary and secondary coils respectively, also the cross section S sq. c.ms. of iron in the frame, *i.e.* thickness of strip × by number side by side × width of strip, and the mean length of the path of a line of force right round.

Apparatus.—Iron core on frame I to be tested and wound with the two distinct (closely-wound) primary and secondary coils P and S. Siemens electro-dynamometer or Parr direct reading dynamometer ammeter A (Fig. 577); non-inductive Wattmeter W; non-inductive rheostat R (p. 597); switch K; electrostatic voltmeter V; Pohl's commutator D (p. 584), or other suitable change-

over switch for throwing V in quick succession across P or S. Source of alternating current supply E, preferably one the frequency of which is under control; a tachometer will be required in this latter case.

Observations.—(1) Connect up as indicated in Fig. 128, and adjust the pointers of A, V and W to zero if they require it and levelling them where necessary. If the alternator is under control see that all the lubricating cups in use feed slowly and properly.

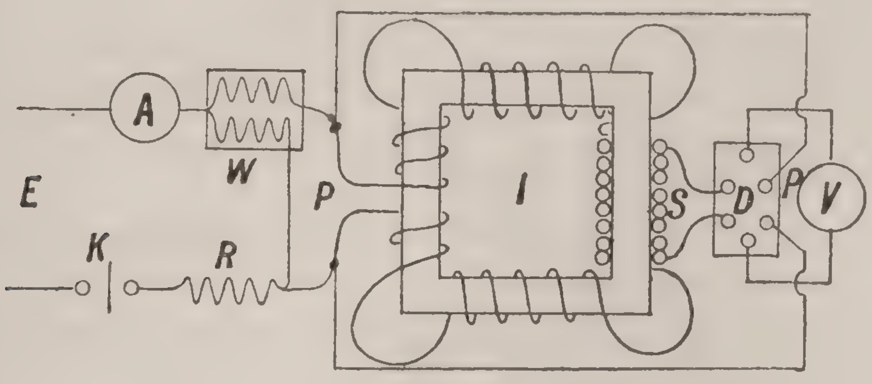


Fig. 128.

(2) Start the alternator up to its highest desirable speed, which is to be kept constant, then with R at its full close K and alter R and the excitation to give the smallest readable current on A. Note simultaneously the readings on A, W and V in quick succession when across P and S by turning D to P or S as the case may be, and the speed.

(3) Repeat 2 at the same speed for eight or ten different currents A, rising by about = increments to the highest desirable.

(4) Adjust the current A to some convenient value, preferably one that will produce an induction of B = about 4000 lines per sq. cm. in I and keep this constant.

(5) Now take a series of readings of W and V for about eight or ten different speeds, ranging from the greatest down to the smallest, noting the value at each.

(6) Measure the resistances of the primary and secondary windings by means of a Wheatstone Bridge set, and tabulate all your results as follows—

NAME . . . DATE . . .

Alternator: Periods per Revoln. $K = \ldots$ $p = 2\pi n = \frac{2\pi KN}{60}$ ∼ per sec.

Resistances (hot) Primary $R_P = \ldots$ ohms. Section of iron Core $S = \ldots$ sq. cms.

„ („) Secondary $R_S = \ldots$ „ Weight of „ „ $W = \ldots$ lbs.

No. of Turns: Primary $N_P = \ldots$ Secondary $N_S = \ldots$ Thickness of Strips = . .

Wattmeter Constant = . . .

Speed of Alternator N revs. per min.	Frequency $n = \frac{KN}{60}$ ∼ per sec.	Current.		E.M.F.		Power.		Approx. loss in Primary A^2R_P.	Mean Core Loss Total = $W - A^2R_P$ = H approx.	Hysteresis loss per lb. of iron H/W.	Mean Core Induction $\overline{B} = \frac{V_S}{N_S S_P}$ per sq. cm.
		Deflection on A.	True Amps. (A).	Primary V_P.	Secondary V_S.	Reading on W.	True Watts (W).				

(7) Plot the following curves—

(*a*) Between H and $\overline{B}$ having $\overline{B}$ as ordinates and hysteresis loss H as abscissæ.

(*b*) Between H and n having n as ordinates and hysteresis loss H as abscissæ.

(*c*) Between H and A having A as ordinates and hysteresis loss H as abscissæ.

Note.—$\overline{B}$ varies from 3000 to 5000 C.G.S. lines in ordinary transformers. With good iron H/W should not exceed $\frac{1}{2}$.

Inferences.—State very clearly what you can infer from the results of your tests.

(124) Separation and Measurement of Iron Losses in the Cores of Alternators, Transformers, Motors and other Electro-magnetic Appliances. (Alternating Current Frequency Method.)

Introduction.—The iron losses taking place in the cores of alternating current plant (*e. g.* alternators, transformers, motors, etc.) consist of those due to magnetic hysteresis and eddy or Foucault currents respectively.

The Hysteresis Loss depends on the induction density in the iron core, the periodicity of the supply current, and on the volume and quality of the iron used, but in no way on the extent

to which the lamination of the core is carried and increases with, but more rapidly than, the induction.

Steinmetz gives the empirical equation for the work done on account of hysteresis as $w = \eta \hat{B}^{1\cdot 6}$ ergs per cycle of current and magnetization where η equal the hysteretic constant which may vary from 0·001 to 0·003 for soft, annealed core plates, and $\hat{B}$ = maximum value of induction density in lines per sq. cm.

If (n) = periodicity of the supply or number of complete periods per second, the effective loss W_H due to hyteresis (per cub. cm. of core) will be

$$W_H = \eta n \hat{B}^{1\cdot 6} \text{ ergs per sec.} = \eta n \hat{B}^{1\cdot 6} 10^{-7} \text{ watts,}$$

on the assumption that the hysteresis loss per cycle is independent of the rate of cycle which the latest research shows to

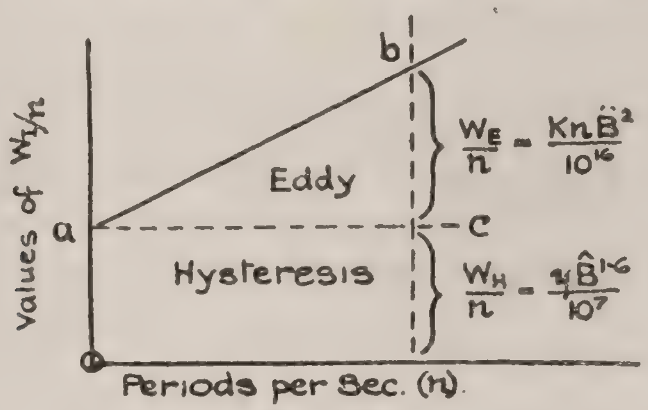


FIG. 129.

be not quite the case, though sufficiently so for practical purposes. Actually the hysteresis loss per cycle increases slightly with increase of periodicity, and from the above relation we see that for a given core, run at constant induction density W_H is $\propto n$.

The Eddy Current Loss depends on the strength of the induced eddy currents set up in the thickness of each lamina composing the core, and hence, by Ohm's law, will vary as the square of such strength. The eddy currents are due to the varying flux through the core, and will depend on the rate of variation of this flux, *i.e.* on the periodicity (n). Thus the eddy current loss will vary in proportion to n^2, and the effective loss W_E due to eddy currents (per cub. cm. of core) will be

$$W_E = Kn^2 \hat{B}^2 10^{-16} \text{ watts,}$$

where K = a constant taking into account the specific electrical resistance of the iron and the thickness of plate lamina.

Eddy currents tend to reduce the flux in a core due to their demagnetizing action and also cause a non-uniform distribution over the sections of the laminæ. Due to this, the hysteresis loss will be further increased with increasing values of (n), and will appear as an increase in the eddy current constant (K) in the present method over and above the calculated value. Especially will this be the case if the insulation between core laminæ is not all effective.

If $W =$ the total power in watts absorbed by any electro-magnetic appliances

and $W_o =$ the watts absorbed or expended in the exciting coil, then the nett iron losses

$$= W_I = W - W_o = W_H + W_E = \eta n \hat{B}^{1 \cdot 6} 10^{-7} + K n^2 \hat{B}^2 10^{-16}$$
watts.

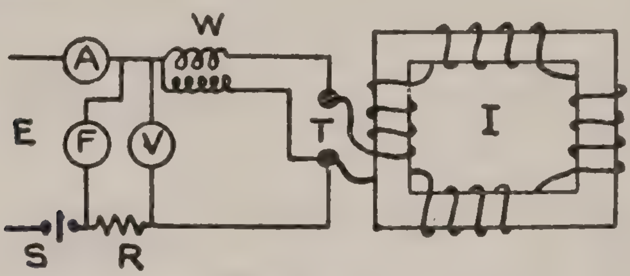


FIG. 130.

Now the coefficients η and K can be found experimentally by testing the appliance at constant induction density $\hat{B}$ with alternating current at variable periodicity (n). This can be done by *varying the speed* of an alternator running at *constant excitation*, for then the voltage V varies $\propto$ to the periodicity (n), so that V/n, and hence the flux, remains constant. On plotting the values $\frac{W_I}{n}$ as a function of the induction $\hat{B}$, the straight line ab is obtained corresponding to one particular value of $\hat{B}$ and of V/n. From the curve ab and this value of $\hat{B}$ the coefficients η and K can be calculated, and hence the hysteresis and eddy current losses respectively.

Apparatus.—Electro-magnetic core I to be tested; low-reading alternating current ammeter A; wattmeter W_1 and voltmeter V, each independent of periodicity; frequency meter F, or, failing this, a tachometer for measuring the speed of the supply alternator E; switch S and non-inductive variable resistance R.

Observations.—(1) Connect up as shown in Fig. 130, levelling and adjusting to zero such instruments as require it. Start up

the experimental alternator and see that the lubricating arrangements are working properly.

(2) With a suitable ammeter, variable regulator, and switch in series with the alternator field across the D.C. supply, close the field switch TS, and adjust the speed to give the lowest readable value of periodicity (n) on F (or smaller value by tachometer), and adjust the field regulator to give some suitable reading on V. Now note the readings of all the instruments, and particularly the value of V/n for future use.

(3) Note the readings of all instruments for each of a series of periodicities (n) up to the highest permissible, taking care that the value of V/n (and therefore the value of the induction $\hat{B}$ in I) is the same at each periodicity.

(4) Repeat the above for the same range of periodicity, but for each of two other widely different values of V, giving corresponding values of the ratio V/n (and hence inductions $\hat{B}$) *kept constant* throughout each range of $\sim$ variation.

Note.—If the appliance tested will safely stand, say, 150 volts, then three values of the constant V/n might be used, viz.—$\frac{150}{50}$, $\frac{100}{50}$ and $\frac{50}{50}$ or 3, 2 and 1 by suitable variation of field excitation, thus giving three corresponding values of $\hat{B}$ in the core. Further, since $\frac{W_I}{w}$ is not likely to exceed 1·5 or 2·0 watts per lb. in modern iron cores, a low-reading wattmeter will be required, unless the core is a heavy one.

Tabulate as follows—

NAME . . . DATE . .

Supply Alternator: Periods per rev. $P =$ . . .

Core tested: Form or type . . . Material . . .

Mean length of magnetic circuit $l =$ cms.	No. of magnetizing turns $T =$	
Net cross section of iron $s =$ sq. cms.	Res. of magnetizing turns $r =$	ohms.
Net weight of iron $w =$ lbs.	Thickness of core laminations =	
Net volume of iron $v =$ c.c.	Width of core lamination =	

Alternator Speed in r.p.m. N.	Periodicity $n = \frac{PN}{60} \sim$ per sec.	Volts. V.	Calculated Constant v/n.	Amps. A.	Max. Ind. Dens. $\hat{B}$.	Total Watts. W.	Copper Loss A^2r.	Total Iron loss. $W_I = W - A^2r$.	Iron loss per lb. of core W_I/W.

(5) Plot curves having values of (n), as abscissæ, with values of V, A and W_1 respectively, as ordinates, for each constant V/n taken.

(6) Determine the hysteresis and eddy current coefficients, and from O draw a straight line, tangent to the curve relating W_1 and n, separating the hysteresis and eddy losses, when ordinates of this line will represent losses due to hysteresis $\propto n$, while the ordinate intercepts between it and the watt curve will represent losses due to eddy currents $\propto n^2$.

Inferences.—State clearly what you can infer from the above results.

(125) Measurement of Magnetic Hysteresis by Ewing's Hysteresis Tester.

This instrument, a general view of which is seen in Fig. 131, and for a full description of which see Professor Ewing's Paper in the *Journal of the Institution of Electrical Engineers*, April 25, 1895, has been designed to meet the want which has been felt of a means of testing the magnetic hysteresis of sheet-iron or steel in a simple and expeditious way suitable for workshop as well as laboratory use. A few strips of the sheet metal to be tested are cut or stamped, five-eighths of an inch wide and three inches long. They are filed to the exact length when clamped in a gauge, which is provided with the instrument, and are then inserted in a carrier which is made to revolve by turning a handle. The carrier turns between the poles of a permanent magnet, which is suspended on a knife-edge. In consequence of the hysteresis of the specimen the magnet is deflected, and the amount of its deflection is observed by means of a pointer and scale. From this deflection the hysteresis of the specimen is determined. The magnetic induction is practically the same in all specimens, notwithstanding differences in the permeability of the iron, on account of the comparatively large air-gap between the specimen and the magnet poles.

Two standard samples are provided with the instrument,

having stated amounts of hysteresis. The test of any other specimen is made simply by comparing the deflection produced by it with the deflections produced by the standard samples. This serves to determine the hysteresis of any specimen in absolute measure.

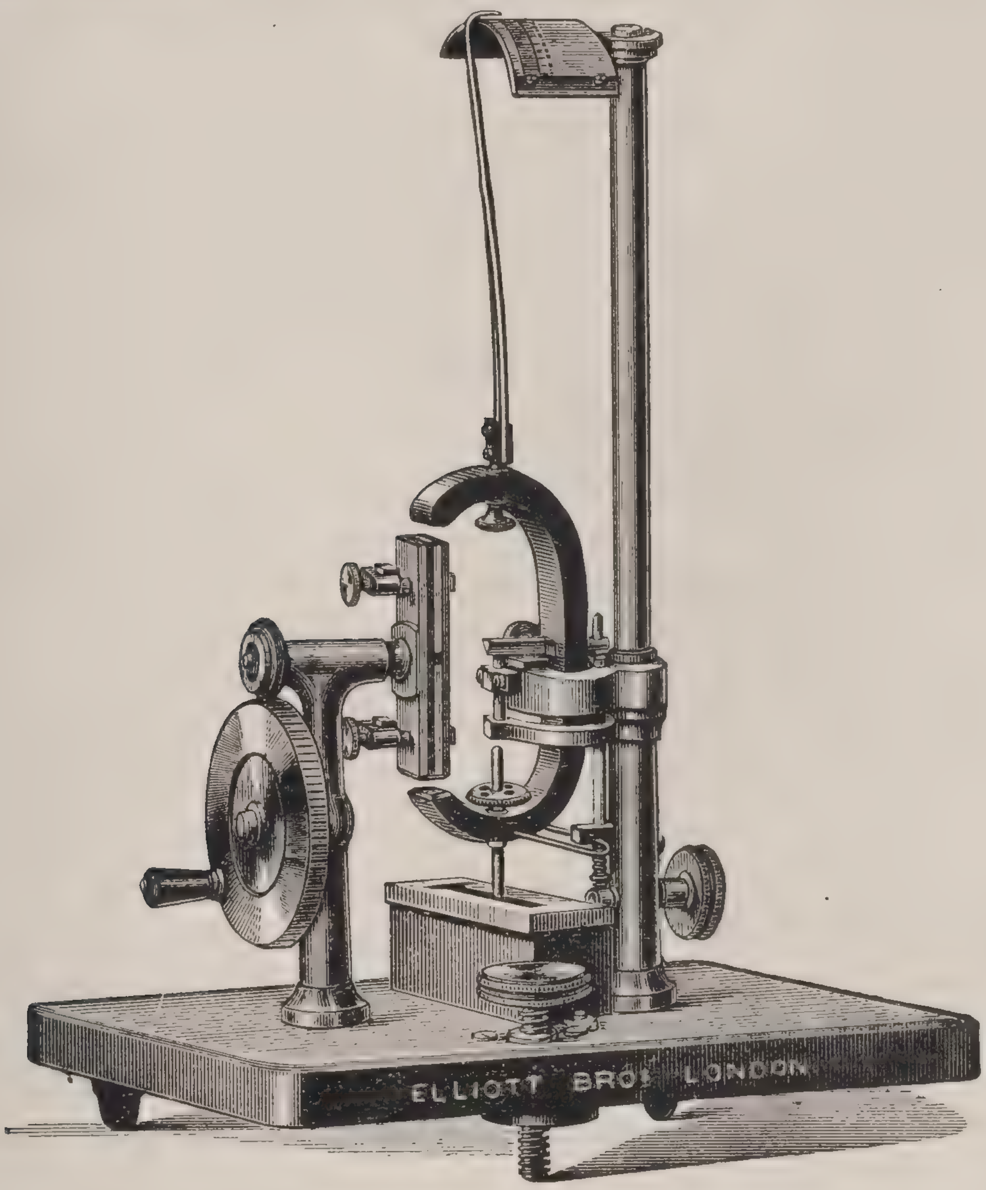


Fig. 131.

The operation of the instrument is entirely mechanical, and requires no knowledge of electrical testing.

(126) Measurement of the Impedance, Reactance, and Self-Induction of Alternator Armatures, Motor Stators, Transformer and other windings (by Alternating Currents).

Introduction.—The following is a simple and an approximate method of finding the self-induction L of an inductive circuit. It depends on the fundamental relation subsisting between current (A) and impressed E.M.F. (V) in such a circuit, namely—

$$A = \frac{V}{\sqrt{L^2p^2 + R^2}}$$

or, as it may otherwise be written,

$$\text{Impedance} = \sqrt{L^2p^2 + R^2} = V/A,$$

where the angular velocity of the current $p = 2\pi n$, n being its frequency in $\sim$ per sec.

Since by definition, the coefficient of self-induction L of any coiled circuit is $= \frac{N}{A}$ where N is the total magnetic flux threading the coil and produced by a current A, it follows that if the inductive circuit encloses, and is surrounded by, a non-magnetic medium, the value of L calculated will be the same for all values of A. If, however, it encloses an iron core, any variation of A will produce variations in the permeability of, and consequently the flux in, the core, and the value of L will vary with A.

Thus the impedance and self-induction of the primary of a static transformer, and of the stator winding of an induction motor will decrease as the secondary load of the former and B.H.P. developed by the latter increases, owing in each case to alteration of current and core flux. The same effect occurs with the armature of an alternator or of a synchronous motor, the impedance and self-induction of which will vary with—

(1) The armature current, since the core flux and permeability will vary inversely together as the current changes.

(2) The magnetization of the core due to any variation of the field-magnet strength.

(3) The type of winding used, *i.e.* whether "distributed" or

"concentrated," the former having small self induction owing to the circuits being partly in both favourable and unfavourable positions for linking with the flux, the latter having a large self-induction, due to all the circuits in certain positions in the revolution linking up simultaneously with the flux.

(4) The reciprocal of the length of air-gap between armature core and field poles.

(5) The exact position of the armature relatively to the field poles, especially with windings concentrated into single slots.

(6) The induced currents in the pole pieces and field windings due to the armature current.

In view of the above considerations, it will therefore be obvious that the value of the impedance or self-induction of the armature of an alternator available for calculation can only be a mean value as obtained in the manner indicated in the present list.

Two methods of procedure are possible, according as to the mode of obtaining the Ohmic resistance R.

(*a*) R may be measured in the usual way on a Wheatstone Bridge either before or after the test, in which case a measurement of the current at a known voltage, or *vice versâ* together with (n), at once gives the self-induction L.

(*b*) R may be obtained by Ohm's Law in terms of a continuous current and pressure when this latter is available, and therefore no Wheatstone Bridge is necessary. This method of procedure, which is the one adopted in the present instance, has the further advantage that in cases where R is liable to heat up, due to the current, its value will be obtained correctly, which would not be so if obtained by the bridge.

The following precautions should, however, be carefully observed, and are practically the same as appear in the measurement of the resistance of an electric glow lamp while running (*vide* p. 47).

If the voltmeter is shunted across the terminals of L, then its reading is correct, but the *true current through* L which is required = *ammeter reading* − *voltmeter current.* If, therefore, an electrostatic voltmeter is employed this correction does not occur, but if a hot wire voltmeter is used, the correction should be made, as the voltmeter current is not usually negligibly small compared with the main current.

If the voltmeter is across the ammeter and L combined, then the *true voltage across* L = *voltmeter reading* − *voltage absorbed in ammeter.* Owing to the low resistance of the last-named usually, this correction is negligible, but must be made if the ammeter resistance is considerable. In this arrangement we also have—

True self-induction of coil = *calculated* L − *self-induction of ammeter.*

The test can be performed either using the same voltage from the direct and alternating sources and noting the relative currents, or employing the same current and observing the relative direct and alternating volts necessary to send this current through the circuit, the frequency in either case remaining the same.

In the present instance the latter way will be adopted as being more readily applied.

Apparatus.—Inductive circuit L to be tested; alternating current voltmeter V, preferably electrostatic; Siemens electro-dynamometer or A.-C. ammeter A (Fig. 251); variable non-inductive resistance R (p. 598); change-over switch S (p. 582); alternator P, with its tachometer; direct current dynamo or secondary battery D.

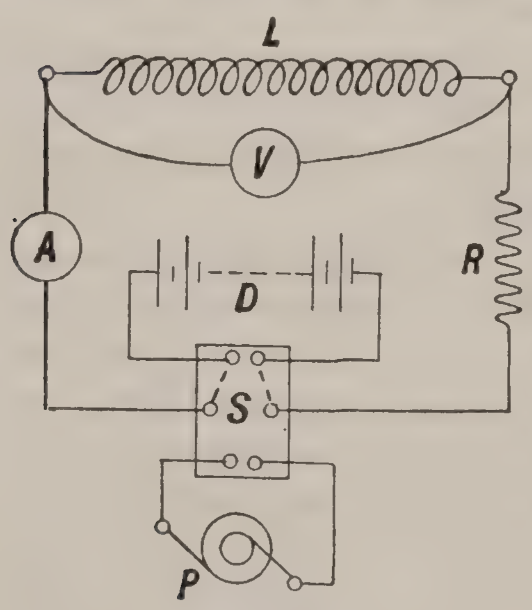


FIG. 132.

Observations.—(1) Connect up as indicated and adjust the pointers of A and V to zero. Before starting see that all lubricators in use feed slowly.

(2) Make R as large as possible and switch S over to D, adjusting A to the maximum current which L will carry. Note the current A amps. and the volts V_D across L.

(3) Turn S over to P and adjust R and the speed of P so as to again obtain the same current A amps. Note the volts (V_a) and the speed of alternator.

(4) Open S and make R as large as possible again. Repeat 2 and 3 for about ten different decreasing values of current to the smallest convenient, and keep the speed of P constant throughout, its excitation being varied, if necessary.

Note.—If the double pole change-over switch S is not available, the common circuit containing A, L and R can be closed to D through a single pole switch, and a series of pairs of values of A and V first taken in order to obtain the ohmic resistance $\left(\frac{V}{A}\right)$ of L. P can then be substituted for D, and a similar series taken with alternating currents of the same scale values on A as before between 0 and the maximum L will carry at constant frequency.

(5) If the inductive circuit has a removable magnetic core, as, *e. g.*, in the central movable core open-magnetic-circuit type of choking coil. Then operate obs. 2–4 above, or the alternating current series only of observations mentioned in the *Note* above, with the core (*a*) central in the coil, (*b*) removed altogether away from the coil.

(6) If the inductive circuit L (Fig. 132) consists of the armature of an alternator the impedance and self-induction of which is required, the A.C. supply should have the same periodicity as the normal value for the machine under test. Then, with the field magnets of the machine under test, *unexcited* vary R so as to obtain about a quarter, half, three-quarters and full-load currents (A) through the armature, noting the corresponding readings of V *at each* of a series of positions of the armature throughout a fraction of a revolution equal to half the polar pitch.

(7) Repeat (6) with normal field excitation and tabulate as indicated.

(8) Plot curves for each fraction of full-load current having impedance and self-induction respectively as ordinates with positions of armature throughout the half-polar pitch as abscissæ.

Inferences.—State clearly all that can be deduced from the results of the test, and find the average value of the impedance and self-induction, and tabulate as follows—

NAME . . . DATE . .

Alternator: Speed $N = \ldots$ *R.p.m.* Periods per Revolution $K = \ldots$

Frequency $n = \frac{KN}{60} = \ldots \sim$ per sec. $p = 2\pi n = \ldots$

Nature and form of coil tested . . .

Position of Core or Armature.	Field Amps. For test of Armature.	Current. Amps. A (true).	Voltage		Ohmic Resistance $R = \frac{V_D}{A}$ ohms.	Inductive Resistance Lp.	Impedance $\frac{V_a}{A} = \sqrt{L^2p^2 + R^2}$.	tan. $\theta = \frac{Lp}{R}$.	Angle of Lag $\theta°$.	Time Constant of Coil $T = L/R$.	Self-Ind. $L = \frac{1}{pA}\sqrt{V_a^2 - V_D^2}$.
			Direct V_D.	Alternating V_a							

(6) Plot curves having values of L and V_a as ordinates and the corresponding currents A as abscissæ in each series.

Inferences.—What can you infer from your experimental results? On what does the self-induction of an alternating current circuit depend? Show how the formula given for L can be obtained.

Self-induction by Rowland's Alternating Current Method.

General Remarks.—Every electrical conductor possesses three qualities, namely, (1) *Electrical resistance*, which depends on the size and material of the conductor.

(2) *Electrical capacity*, depending on its surface and form, and on the specific inductive capacity of the surrounding media (*i.e.* dielectric).

(3) *Electrical inductance*, which depends on the shape and form of the conductor and on the magnetic permeability of the surrounding media.

This last-named property may be of one or other of two kinds, namely, either the self-induction of the conductor on itself, or the mutual induction of the conductor and a neighbouring circuit on one another.

The quality (1) above is usually easily obtained, except perhaps in the case of electrolytic liquids, and this only in one or two

methods; the other qualities are much more difficult of determination, and almost numberless methods have been devised for obtaining them.

In general it may be remarked that relative or comparative measurements are more accurate than absolute ones, though the final results might be completely vitiated by comparing with an inaccurate standard. The former remark results in the difficulty experienced in accurately measuring an alternating current, and from the fact that its E.M.F. wave may differ considerably from that of a sine curve.

In employing condensers in methods of measuring self and mutual induction, considerable difficulty is usually met with in the phenomena of electric absorption. Professor H. H. Rowland has found that this can be represented by a resistance placed in series with the condenser, which resistance is a function of the square of the current period.

(127) Absolute Measurement of Self-induction (by Alternating Currents).

Introduction.—The following method of measuring the self-induction of a coil, due to Professor Rowland, necessitates the employment of ordinary single phase alternating currents of electricity with an electro-dynamometer specially constructed, so as to be as sensitive as possible. It is possible to make such an instrument, having its fixed and moving coils connected up to two distinct pairs of terminals, that it will detect 0·0001 of an ampere with a self-induction in the suspended coil not exceeding 0·00075 henries, and in the fixed coil of not more than 0·0006 henries, capable of carrying about 0·1 ampere comfortably.

Such an instrument obviates the necessity for using large currents in order to obtain accuracy and sensibility. If (d) = the deflection of the swing coil from zero when its plane was perpendicular to that of the fixed coil, C_1 and C_2 = strengths of the alternating currents flowing through the movable and fixed coils and having an angle of phase difference θ. Then $d \propto C_1 C_2 \cos. \theta$.

The principle of the present method consists in adjusting C_1 and C_2 to a phase difference of 90°.

In deflection methods cos. θ is greater than 0, while for zero methods cos. $\theta = 0$. In the former the self-induction is obtained in terms of resistance and the angular velocity of the current $p = 2\pi \times$ frequency, which consequently require that (n) the frequency should be constant and accurately known to at least 1%, in which case the results will probably agree to within about the same amount.

Apparatus.—The electro-dynamometer, of which (f) is the fixed, and (m) the moving coil; non-inductive resistance r; source of alternating current; and the self-induction L to be measured; switch S; rheostat Rh (non-inductive).

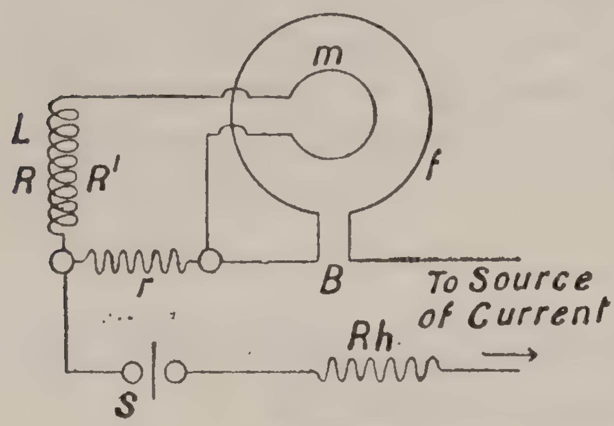


FIG. 133.

Observations. — (1) Connect up as indicated in Fig. 133, placing L and a *non-inductive* resistance R in series with the moving coil (m), and the combination across the terminals of a resistance (r) in the main circuit in which the fixed coils are also placed.

(2) With the moving coil adjusted to zero, Rh at a maximum, close S, and obtain a convenient deflection d by adjusting Rh and the non-inductive resistance R in the moving coil circuit. Note this deflection (d) and the speed (N) or periodicity (n) of the alternator and the added resistance R in circuit with L.

(3) Remove the self-induction (L) which is being tested, and add a non-inductive resistance to the swing coil circuit such that the same deflection (d) as before is reproduced. Note the new resistance R' in the circuit, the frequency (n) $\sim$ per sec. being the same as before.

(4) Calculate the self-induction L tested from the relation

$$L = \sqrt{\frac{(R'-R)\,(R+r)}{p^2}} = \frac{1}{2\pi n}\sqrt{(R'-R)\,(R+r)} \text{ secohms},$$

and tabulate as follows—

NAME . . DATE . . .

Alternator: Periods per revolution $K = \ldots$ $p = 2\pi n$.

Self-induction tested: Nature . . .

Speed of Alternator N revs. per. min.	Frequency $n = \frac{KN}{60}$ ∼ per. sec.	Resistances.			Deflection (d).	Self-induction.	
		r.	R.	R'.		L.	Mean L.

(5) Repeat 2—4 for different deflections (d) at constant frequency. Also for different frequencies with the same deflection.

N.B.—Great care must be taken to keep the frequency *constant* throughout any pair of readings.

(128) Comparison of Two Coefficients of Self-induction (by Alternating Currents).

Introduction.—When an accurate standard known self-induction is available the value of an unknown induction can be more accurately determined by comparison, for in this case the measurement is independent of frequency. The following method due to Professor Rowland is a zero one, and is similar in many respects to the preceding method, though this was a deflection method. The effects of induction and electrostatic action of the various parts of the circuit on one another must be carefully avoided as much as possible, and in this connection it should be remembered that a twisted twin lead possesses the latter quality.

Apparatus.—The unknown self-inductions L_1 to be compared, with a standard L (known); non-inductive resistances r, R, R' and rheostat Rh; switch S; source of alternating current E; electro-dynamometer of which (f) is the fixed and (m) the moving coil.

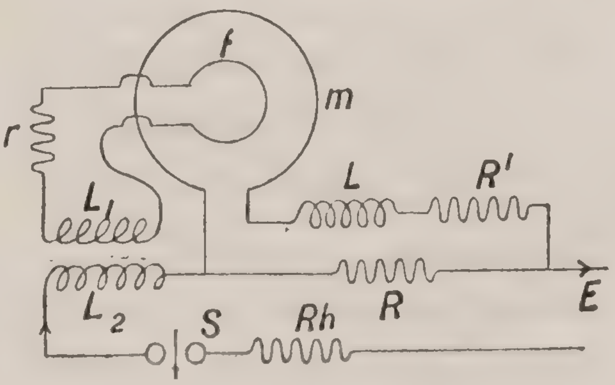


FIG. 134.

Observations. — (1) Connect up as in Fig. 134, the coils L_1 L_2 being together, and adjust the moving coil to zero.

(2) With Rh large close S, and adjust the current to a

convenient value, and adjust R, R' and r so as to get *no* deflection of the moving coil for the currents in f and m.

(3) Calculate the self-induction in terms of the standard from the relation $\frac{L}{L_1} = \frac{R + R'}{r}$, and tabulate as follows—

NAME . . . DATE . . .

Alternator: Periods per Revolution $K = \ldots$ $p = 2\pi n$.

Self-induction tested: Nature ∼ . . .

Speed of Alternator (N) revs. per min.	Frequency $n = \frac{KN}{60}$ ∼ per sec.	Known Self-ind. L.	Resistances.			Ratio L/L_1.	Unknown Self-ind.	
			R.	R'.	r.		L_1 secohms.	Mean L_1.

(4) Repeat 2 and 3 for different values of L, R, R' and r at constant frequency (n). Also for the former constant at different frequencies.

Notes.—When equal self-inductions are being compared it is found that the accuracy depends only on the sensitiveness of D to changes in ($R + R'$), and this instrument may be such that it detects differences or changes of 0·01%.

If it is noticed that increase of frequency causes a diminution of the resulting value of L_1, then the electrostatic capacity of the turns of the coils on one another is asserting itself and cannot be avoided. Considerable care should be taken to avoid this source of vitiation as much as possible, and also that due to heating of the conductors, etc. To minimize the former error, use short small wire leads, some distance apart, and not twisted twin lead.

(129) Self-Inductions in Series and Parallel. Experimental Determination of Laws of Combination.

Introduction.—An electrical circuit may contain any or all of the three qualities—self-induction, capacity, and ohmic resistance. The last-named is always present, and may be combined with one or both of the former.

Let us suppose that no capacity is present, then the circuit, whether consisting of several distinct portions either in series or

parallel, or a combination of these, each having its own particular self-induction and ohmic resistance, possesses on the whole *one* definite effective value of induction and resistance, which may be termed the "*combined, equivalent,* or *effective*" self-induction and ohmic resistance of that circuit composed of such detailed portions.

In alternating current work it is of great importance to know the way in which various combinations of self-inductions and ohmic resistances will affect the working conditions of a circuit. The present test is devised with a view to elucidating these points for the three different forms of circuits or combinations, as follows—

(α) The self-inductions and ohmic resistances in *simple series* only.

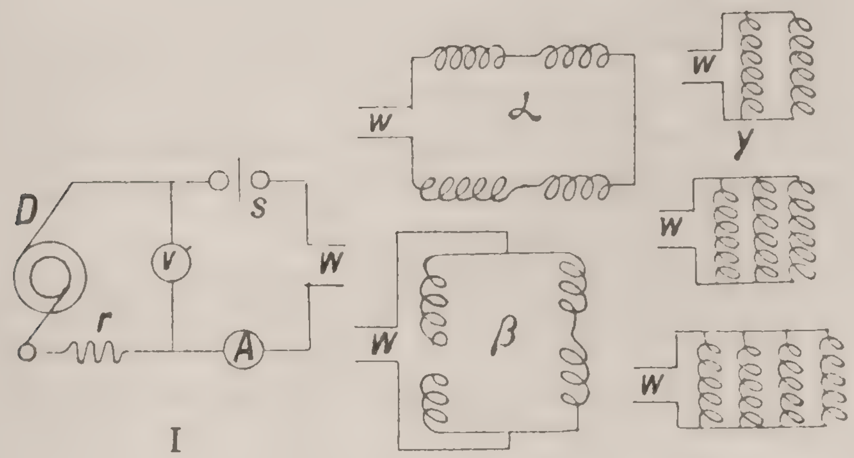


FIG. 135.

(β) Self-inductions and resistances *partly in parallel and in series.*

(γ) " " " " *in parallels only.*

Apparatus.—Source of alternating current, preferably an independently driven alternator D, the exciting circuit (not shown) and the speed of which are under control; Siemens electro-dynamometer (Fig. 251) or Parr direct-reading alternating current ammeter A (p. 572); hot wire or electrostatic voltmeter V; non-inductive rheostat r (p. 598); switch S; speed indicator; four coils A—D to be experimented upon, as nearly alike as possible, and capable of being used with or without iron cores, which latter are also similar in all respects.

Observations.—(1) Connect up as in Fig. 135 I., and adjust the pointers of all the instruments to zero, levelling such as require it.

(2) Connect coil A (only) to W (I.) and remove its iron core.

See that all lubricating cups are feeding slowly, then S being *open*, start D up to a convenient speed, N revolutions per minute, which *must be kept constant* throughout all the tests.

(3) With (r) at its maximum, and the excitation low to start with as a precaution, close S, and adjust r and the excitation so as to give $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and full or maximum safe current through coil A successively at constant speed. Note the reading of the voltmeter V and ammeter A simultaneously at each, then open S.

(4) Repeat 3 for each of the coils B, C and D singly without their cores.

(5) Repeat 3 for the coils connected 2, 3 and 4 in series respectively, *i. e.* as in α.

(6) Repeat 3 for the coils connected 2 in series and 2 in parallel, *i. e.* as in β.

(7) Repeat 3 for the coils connected 2, 3 and 4 in parallel respectively, no iron cores being used in any of the cases above.

(8) Repeat 2–7 with iron cores in all cases, if possible.

(9) Repeat 2–8 with an entirely different speed, and therefore frequency of alternating current.

(10) Measure the ohmic resistance of each of the coils A—D by a Wheatstone Bridge in the usual way, and calculate the self-induction (L_c) of the coil or combination of coils from the relation

$$A = \frac{V}{\sqrt{L_c^2 p^2 + R_c^2}} \text{ amperes,}$$

where R_c = combined or effective ohmic resistance of the coil or combination of coils obtained by employing the ordinary rules for the combined resistance of coils in series and parallel. Tabulate your results as follows—

NAME . . . DATE . . .

Alternator: Speed $N = . .$ *r.p.m.* Periods per Revoln. $K = . .$

Frequency of the Current $\frac{KN}{60} = . . . \sim$ per sec. $p = 2\pi n = 2\pi \frac{KN}{60} = . . .$

Arrangement of Coils measured.	Current.		Volts V.	Effective ohmic Resistance of Circuit tested R.	Inductance $L_c p$.	$\tan. \theta = \frac{L_c p}{R_c}$.	Angle of Lag $\theta°$.	Impedence.		Self-induction.	
	Dynamometer reading.	Amps. A.						Measured $V/A = \sqrt{(L_c p)^2 + R_c^2}$.	Calculated.	Measured $L_c = \sqrt{\frac{(V/A)^2 - R_c^2}{p^2}}$	Calculated.

I. For pure series combinations such as (α) Fig. 135, show that
$$\frac{V}{A} = \sqrt{L_c^2 p^2 + R_c^2} = \sqrt{(L_1 + L_2 + L_3 + \ldots)^2 p^2 + (R_1 + R_2 + R_3 + \ldots}$$
or generally that $\frac{V}{A} = \sqrt{(\Sigma L)^2 p^2 + (\Sigma R)^2}$, and therefore that the total effective self-induction L_c = sum of the individual self-inductions composing the circuit.

II. For series-parallel combinations such as (β) Fig. 135, show that
$$\frac{V}{A} = \sqrt{L_c^2 p^2 + R_c^2}$$
$$= \frac{\sqrt{(L_1 + L_2)^2 p^2 + (R_1 + R_2)^2}\ \sqrt{(L_3 + L_4)^2 p^2 + (R_3 + R_4)^2}}{\sqrt{(\Sigma L)^2 p^2 + (\Sigma R)^2}}$$

III. For pure parallel combinations such as (γ) Fig. 135, show that
$$\frac{V}{A} = \sqrt{L_c^2 p^2 + R_c^2} = \sqrt{\frac{1}{\left(\Sigma \frac{L}{I^2}\right)^2 p^2 + \left(\Sigma \frac{R}{I^2}\right)^2}}$$
where I is the impedence of each parallel branch.

N.B.—In the present test it is assumed that the coils are incapable of having any mutually inductive action on one another, and consequently they must be arranged not to have such when making the test.

The Electrostatic Capacity of Electrical Wires and Cables.

General Remarks.—The condition for obtaining an electrostatic capacity is the passage of a quantity of electricity into one of two conducting bodies which are separated by an insulator. Such an arrangement constitutes what is commonly termed an electrical condenser, the two conducting bodies being called the "coatings," and the separating insulator the "dielectric" of the condenser. Now it will be obvious that any insulated electrical wire or cable in contact with earth or its equivalent will form a condenser, the inner conductor or wire and earth being the two coatings, and the insulation of the cable the dielectric. In the case of an insulated cable possessing only one core, whether

consisting of one wire or a strand of wires, we shall obtain one particular definite capacity with a definite position of the cable. In other words, the capacity will depend on the geometrical form of the wire, so that if it was coiled up in a tank of water the capacity would not be the same as if it was laid out straight on the ground.

The actual value will depend in addition on the length and size of cable, and on the thickness of the insulation and its specific inductive capacity. The latest forms taken by cables, in which one conductor completely envelops another, but is insulated from it by a fairly uniform stratum of insulating material between the two, possess this property of having an electrostatic capacity in a more marked degree than the simple form mentioned above. Such a concentric cable, as it is termed, possesses a definite capacity per unit of length, and which is independent of how the cable is placed, *i. e.* whether coiled or straight. For continuous currents the capacity of cables or wires is of no practical importance, but for *intermittent* or *alternating currents* the case is otherwise. In submarine telegraphy the cable has naturally a very considerable capacity, while the current is intermittent; consequently when the circuit is closed so as to send a message, the cable has first to be charged by the sending battery before any current arrives at the receiving end for actuating the receiving appliances. This may take some seconds, depending on the length of cable, *i. e.* on its capacity. Thus the effect of capacity in such an instance is a detrimental one, giving rise to what is called "*inductive retardation,*" and diminishing the speed of signalling. Here, in the above instance, we have the case of a single cable stranded conductor of which the copper core forms one coating, the iron sheathing and water the other.

With concentric cables, it has already been remarked that their capacity is greater than with single cables for equal lengths and section of conductors in the two cases; but the former possess the advantage that whereas the "outward" and "return" leads are very close together, in fact one encircling the other, their inductive action on telegraph and telephone wires in the vicinity is practically *nil*, as the external magnetic field produced is very small. This is of great value in alternating current distribution, for since the magnetic field produced by such currents alternates rapidly in

direction with the alternations of current, the inductive action of alternate current cables on such wires would otherwise be great. The capacity of a concentric cable can at once be calculated from the analogy to a cylindrical condenser as follows—

Assuming the two conductors to be both concentric and cylindrical, let R = radius of the inner surface of the outer conductor and r = radius of the outer surface of the inner one, and also let L = length of cable in centimetres. Then its capacity in farads—

$$C_F = \frac{2{\cdot}413}{10^{13}} \times \frac{KL}{R - \log_{10} r}$$

where K = specific inductive capacity of the dielectric or insulating material, which for paper = 1·86 about, for india-rubber (pure) 2·34, vulcanized 2·94, for gutta-percha 4·2, and resin 2·55 about.

It may be noticed that since $(\log_{10} R - \log_{10} r) = \log_{10} \frac{R}{r}$ the units in which the radii are measured is quite immaterial, so long as the same is employed for each; the diameters D and d corresponding to R and r, may be used instead if we like, whence we shall have the

$$\text{Capacity} = \frac{2{\cdot}413}{10^{13}} \times \frac{KL}{\log_{10} {}^{R}/_{r}} \text{ Farads} = \frac{2{\cdot}413}{10^{7}} \times \frac{LK}{\log_{10} {}^{D}/_{d}} \text{ Microfarads}$$

reducing this to Mfds. per mile (statute) which = 160,933 cms.

$$\therefore \text{ Capacity} = \frac{2{\cdot}413 \times 160{,}933}{10^{7}} \times \frac{K}{\log_{10} {}^{D}/_{d}} = \left.\frac{K}{25{\cdot}75 \log {}^{D}/_{d}}\right\} \begin{array}{l}\text{Mfds.}\\ \text{per mile.}\end{array}$$

The capacity can readily be measured by means of the "method of mixtures" due to Lord Kelvin, and which is one of the best for the purpose. A complete digest of this and other kindred methods will be found in *Practical Electrical Testing*, p. 182, by the author, and they will not therefore be repeated here.

It may, however, be remarked that in testing the capacity of electric light and other cables by this method of mixtures the E.M.F. employed may conveniently be about 100 volts, and referring to Fig. 82, p. 184, of the above-mentioned work, the resistance ADH might be 100,000 ohms, and D connected to earth or tank if a single conductor cable is being tested. If it is a concentric cable this will not be immersed, and its two conductors at one end must be carefully insulated, while their other ends will form the two terminals of the capacity to be tested.

(130) Measurement of the Electrostatic Capacity of Concentric—or Ordinary—Cables and Condensers. (Alternating Current Method.)

Introduction.—When an alternating-current E.M.F. is placed across a condenser or, say, a concentric cable, a certain measurable alternating current flows into the condenser or cable, even though in the latter the two conductors are quite free of all connections to lamps or any other appliance throughout their entire length. Moreover this current, which is called the "*capacity current*" of the cable, is not in phase or step with the periodic impressed E.M.F., but *leads* in *advance* of it, and constitutes what is called a *Wattless* or *idle current*, to distinguish it from the *load* or *useful* current which would flow in the cable when lamps or other appliances were switched on. In other words, it represents waste energy in the copper of the mains so far as the utility of the current is concerned, and the effect is always present with alternating currents. Thus it becomes of importance to know this idle or capacity current in order that its flow in the cable may not be mistaken for leakage current when no apparatus is connected to the cable.

It should also be noted that this current is out of step or phase with the main current.

The value of the capacity current in any cable can be deduced when certain constants are known. Thus—

Let A = virtual or $\sqrt{\text{mean square}}$ value of the capacity current in amperes,

V = virtual or $\sqrt{\text{mean square}}$ value of the E.M.F. impressed in volts,

between the two conductors.

C = capacity of the cable in farads,

and p = $2\pi n$ where n = frequency of the alternating current in periods ($\frown\smile$) per second,

$$\text{then } A = \frac{V}{\frac{1}{Cp}} = CpV \text{ amperes,}$$

$\frac{1}{Cp}$ being the effective resistance to the passage of the current or *reactance* of the cable or condenser.

If C is in microfarads per mile and L = length of cable in miles,

$$\text{then } A = \frac{CLVp}{10^6} = \frac{2\pi nCLV}{10^6} \text{ amperes,}$$

$$\text{whence } C = \frac{10^6 A}{LV2\pi n} \text{ mfds. per mile.}$$

The present test is a very practical one and can nearly always be applied if the working pressure V is available.

Apparatus.—Cable or condenser to be tested (C); either a Siemens electro-dynamometer, hot wire or Parr ammeter A (p. 577), each of which will correctly measure the $\sqrt{(\text{mean})^2}$ value of the current; an electrostatic or hot wire voltmeter (V), preferably the former; tachometer for measuring the speed of the alternator D, and from it deducing the value of (n); switch S.

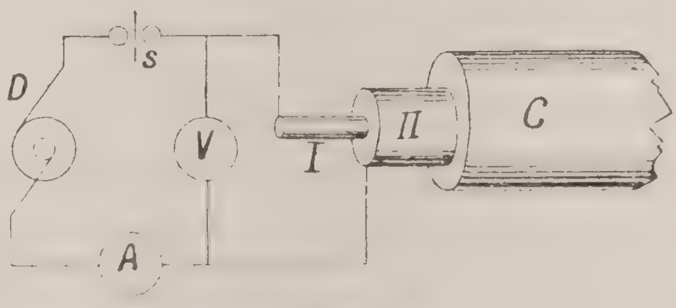


FIG. 136.

Observations.—(1) Connect up as shown in Fig. 136, and adjust the pointers of V and A to zero, carefully levelling them if necessary.

(2) Carefully "*free*" and *insulate* the *far end* of the cable, and prepare the *near end* so as to make contact with the two conductors I and II.

Note.—If a condenser is being tested I and II will now be its terminals.

(3) Close S, and with D running at constant speed take some six or eight widely different values of V by altering the excitation of D, and note the corresponding values of A simultaneously with V.

(4) Next run D at six or eight different speeds, keeping V constant by altering the excitation, and note the corresponding reading on A for each speed.

(5) Calculate the capacity tested from the relations—

$$C = \frac{10^6 A}{V2\pi n} \text{ mfds. for a condenser,}$$

$$C = \frac{10^6 A}{LV2\pi n} \text{ mfds. per mile for a cable,}$$

and tabulate as follows—

NAME . . . DATE . . .

Cable tested: Type . . . Maker . . . Size . . . Insulation . . .
Condenser: Type . . . Maker . . . Dielectric . . .

Length of Cable (L) miles.	Speed of Alternator.	Frequency (n) $\sim$ per sec.	Volts V.	Inductive Reactance $\frac{1}{Cp}$.	Current.		Capacity of	
					Reading on Instrument.	Amps. A.	Condenser in Mfds.	Cable Mfds. per min.

(131) Measurement of the Electrostatic Capacity of Short Lengths of Submarine and Electric Light Cables. (Kelvin Dead Beat Multicellular Voltmeter Method.)

Introduction.—The effects of electrostatic capacity in a cable on the intermittent or alternating currents flowing in it have already been mentioned (*vide* p. 370), consequently it is desirable to obtain the value of its capacity. The following is a convenient and accurate method of measuring the capacity of any insulated conductor comprising a short length of submarine, telephone, telegraph, or electric light cable, and its great advantage lies in the fact that it is applicable to short lengths.

The method, which is very analogous to the "Siemens subtraction method," except that only one deduction is made, consists in charging a standard known condenser to a measured potential and observing the fall of this on connecting the cable as a condenser in parallel with it. The condition for maximum accuracy, *i.e.* when a slight error in reading the diminished value of potential has least effect on the final result, has been shown to be when the standard capacity is equal to that of the unknown, or when the diminished potential = half the original value.

For accurate work it is necessary to employ two or three small corrections—one arising from the multicellular voltmeter

possessing a small capacity itself which varies with the deflection of the suspended needle vanes, being less for smaller deflections. It is of the order of about 10^{-6} mfd., and in most cases can be neglected in comparison with the capacity of the standard and cable to be tested, at least when these are of the order of 0·01 mfd. or greater. When, however, extreme accuracy is required, the potential-capacity curve of the voltmeter, which is supplied by the makers, must be referred to, and the capacities of it, for the deflections obtained, taken into account.

Another correction is for loss of charge due to leakage occurring in the voltmeter, condenser, and cable; since during the time taken for the needle of the multicellular to come to rest after cutting off the battery or charging E.M.F. and putting the cable in parallel, the potential may have fallen owing to leakage.

It may therefore be necessary to determine the leakage of the voltmeter, cable, and condenser, which can be done as follows—

(*a*) Charge the voltmeter to some conveniently large potential, and take readings of *potential* and *time* after disconnecting the charging source. Then the curve plotted with potential as ordinates and time as abscissæ shows the *rate of fall* of potential at any time after the disconnection of the charging source.

(*b*) Join the voltmeter and standard condenser in parallel, and repeat the preceding operations. Then the leakage from the condenser will be given at any time by the difference of the ordinates of the curves in *a* and *b*.

(*c*) Join the voltmeter and cable in parallel and again repeat. From these results leakage of the cable can be found at any time by subtracting the ordinates of the curves in *a* and *c*.

Preparation of Cable Ends.—The *free ends* of the cable should be bared of the outer insulation down to the pure rubber for a space of some $2\frac{1}{2}''$, and the rubber itself pared or tapered with a clean sharp knife for a length of about $1\frac{1}{2}''$ from the end; the ends should be carefully dried over a spirit lamp. One end should then be repeatedly painted with melted paraffin-wax (for some $3\frac{1}{2}''$ from the end) heated to a temperature not exceeding 100° C. by means of boiling water. The other end of the cable after having a short well-insulated gutta-percha wire soldered to the copper core should be treated in a similar manner.

This method of preparation if carried out carefully will prevent all end leakage.

Apparatus.—Kelvin dead beat multicellular electrostatic voltmeter V (Fig. 240); Kelvin standard air leyden or condenser C (p. 616); cable L to be tested immersed in a metal-lined water-tank T; highly insulated two-way key K (p. 586); battery B giving an E.M.F. of something like 100 volts; and a fine fuse F to act as a safeguard in case of accidental short circuit in the condenser or voltmeter.

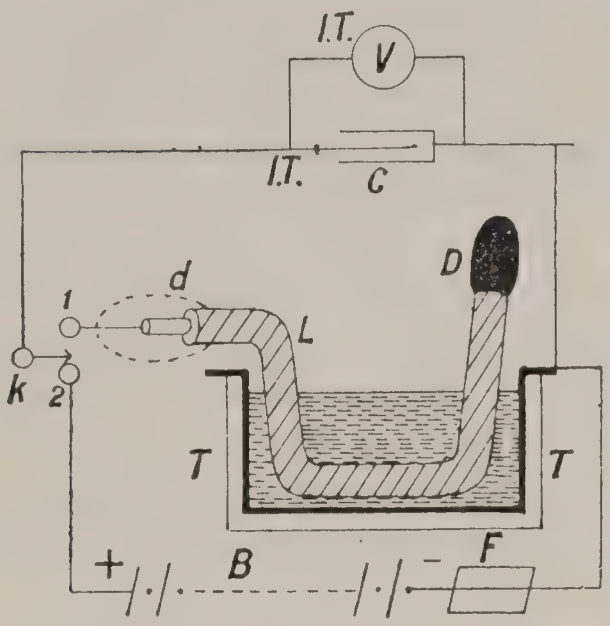


FIG. 137.

Observations. — (1) Carefully prepare the ends of the cable as indicated above and immerse the cable in the tank T, taking care not to allow the prepared ends D and d to get wet; these must be trained up out of the water.

(2) Connect up as in Fig. 137, taking care that the insulated terminal $I.T.$ of both V and C are joined as shown. Adjust the pointer of V to zero if necessary and see that a fine fuse is in F.

(3) Press K to 2 to charge V and C, and note the reading V_1 on the voltmeter when steady and then release K. Now observe the reading for one or two minutes to see if there is any sensible loss due to imperfect insulation in V or C, and if so, whether it is small enough to neglect.

(4) Close K to 1, and in fifteen to twenty seconds, which is usually long enough, note the steady diminished reading V_2 on the voltmeter.

(5) Repeat 3 and 4 with different charging E.M.F.s, and calculate the capacity of the cable L from the relation—

$$C_L = \frac{C_a(V_1 - V_2) + V_1 K_1 - V_2 K_2}{V_2} \text{ mfds.}$$

where C_a = standard capacity, in this case 0·0025 mfd., and $K_1 K_2$ the capacities of the voltmeter at potentials V_1 and V_2.

NAME . . . DATE . . .

Standard Capacity : Type . . . Capacity = . . . mfds.
Cable tested : Type . . . Length (l) . . . Insulation . . .

E.M.F. used to Charge.	Voltages.		Capacity of Voltmeter.		Capacity of Cable.	
	Initial V_1.	Final V_2.	K_1 at V_1.	K_2 at V_2.	C_L Mfds.	$\frac{C_L}{l}$ Mfd. per mile.

(132) Measurement of the Electrostatic Capacity of a Concentric Cable Ballistically. (Standard Magneto Inductor Method.)

Introduction.—When some standard form of magneto inductor is available, the form devised by Dr. W. Hibbert being a very convenient and easily manipulated one, the capacity of a concentric or other electric light cable can be readily determined, providing a few other additional pieces of apparatus are available. The reader should note the general introductory remarks on p. 369 concerning the capacity of cables in general, and also those of the alternating current method of measuring the capacity of cables.

The present test can be employed for finding the capacity of cables in tanks and of ordinary and concentric mains. As, however, the former are best tested by the "method of mixtures" (p. 371), we shall here only consider the test of a concentric cable by this inductor method.

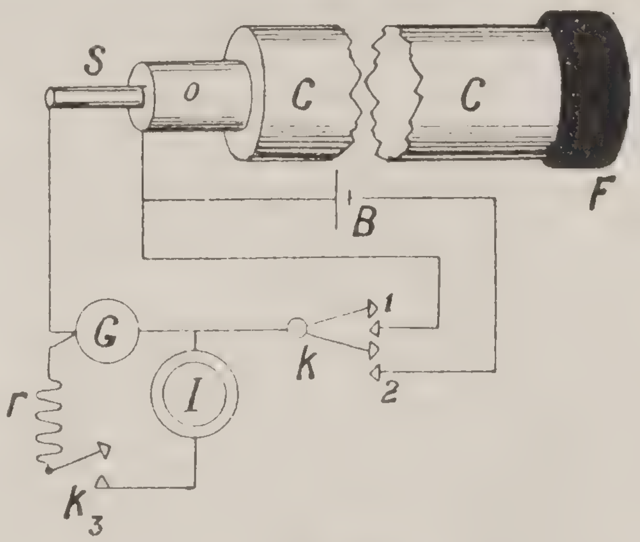


FIG. 138.

Apparatus. — Standard inductor to be tested I (Fig. 138); sensitive ballistic galvanometer G; concentric cable to be tested C, of which F is the free and well-insulated end, S the inner conductor, and O the other; box of known resistances r; battery B, of known E.M.F., or, if this is unknown, a standard voltmeter to measure the P.D.; two-way spring

tapping-key K (p. 586); ordinary spring tapping-key K_3; damping-coil with its cell and key.

Observations.—(1) Connect up as in Fig. 138, and adjust the galvanometer needle to zero, carefully prepare the free (far) end of the cable, viz. F, in the manner described on p. 375, and also the (near) end as well; by so doing the vitiation of the results from leakage across the cable ends will be avoided.

(2) $K1$ and $K2$ being open, adjust r to a low value, such that pressing K_3 nearly a full-scale throw d_1 is obtained on slipping down I. Note this value of d_1 and the box resistance r ohms.

(3) K_3 being open, adjust the voltage of the battery B to such a value that on closing $K2$ for two or three seconds, then opening it, and immediately closing $K1$, a first throw d_C is obtained on discharging C, as nearly as possible equal to the former.

N.B.—Two or three throws should be taken in both 2 and 3, and the means noted as being more accurate.

(4) Obtain the mean throw on the charge in a similar way by first closing $K1$ for a few seconds so as to completely discharge C, and then opening it and closing $K2$ afterwards.

Note.—Care must be taken that C is each time discharged *before* taking the charge throw.

(5) If possible employ three or four different voltages and repeat 2 and 3 with each of them, keeping the deflections d_I and d_C about equal to one another, preferably by varying r to suit.

(6) Calculate the capacity of the cable tested from the relation

$$C = \frac{FN}{100\ VR} \cdot \frac{d_C}{d_I} \text{ microfarads,}$$

where F = total magnetic flux in the air-gap of the inductor and R = total resistance in ohms of the inductor circuit.

Tabulate as follows—

NAME . . . DATE . . .

Standard inductor . . . ; turns N = . . . ; resistance r_E = . . . ohms.
Galvanometer resistance G = . . . ohms; Total Flux F = . . . C.G.S. lines.
Cable tested: Type . . . Maker . . .
Length of Cable L = . . . miles. Section (for reference only) = . . . sq. ins.

Mean first throws.		Resistance in Ohms.		P.D. if variable V.	Capacity in Microfarads C.	Mean Capacity C.	Capacity of Cable in Mfds. per mile C/L.
d_1.	d_C.	In box r.	Total $R = r + r_E + G$.				

Inferences.—Show how the relation given in 6 can be obtained, and state any assumptions made in obtaining it. Is any correction required for greater accuracy in the relation for C?

(133) Measurement of the Electrical Power absorbed in Alternating Current Inductive Circuits. (Three-voltmeter Method.)

Introduction.—The measurement of alternating current power depends on the nature of the external circuit. Thus, if this *circuit* is *non-inductive*, then the true power $W =$ amps. $(A) \times$ volts (V), the former flowing in the external circuit at a terminal potential difference (V).

If the circuit is inductive, and every circuit *is* to some slight degree, then $W = AV \cos \theta$, where $\theta =$ angle of phase difference of the current A behind the voltage V and the product $(A \times V)$ is called the *apparent power* absorbed.

The measurement, therefore, of this electrical power accurately is more difficult than that in the case of a direct-current circuit, owing to the effects of self and mutual induction and capacity which appear in alternate-current (A.-C.) working. In such a case a Wattmeter may be used, but it must be practically non-inductive to give accurate results. Another method to employ, which will give accurate results even though most of the circuit is highly inductive, is that known as the "*three-voltmeter method*," and it has the advantage that only one A.-C. voltmeter is required, though three similar ones may be used if available. By it the *true power* absorbed by the circuit may be obtained with any degree of accuracy desired by using an accurately graduated voltmeter, and by carefully repeating the readings two or three times and noting the mean in each case.

The three-voltmeter method, which was simultaneously suggested by Prof. Ayrton, Dr. Sumpner and Mr. Swinburne, gives a true measure of the power given by any current, whether harmonic or otherwise, to any circuit, inductive or not. It has the disadvantage that, as the differences of squares of quantities is being taken, a small error in the quantities themselves may make a considerable error in the final result, especially if the angle of lag θ is large.

The voltmeter used must be such as will not alter the voltage across the points to which it is applied. In other words, it must have a high resistance compared with that between these points.

An electrostatic voltmeter most accurately fulfils this condition, but if a hot-wire voltmeter is used (of relatively low resistance), the main current must be *large* compared with its own current, or an error will thus be introduced.

With this apparatus we are in a position to investigate the following important characteristics of an inductive circuit formed

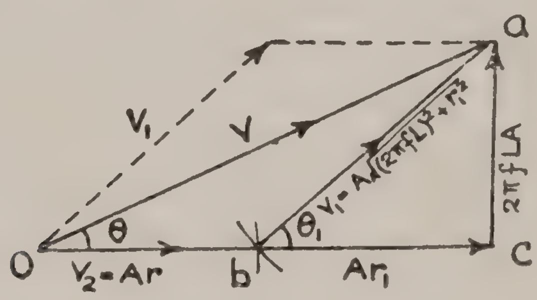


FIG. 139.

by, say, a choking coil or the primary of a static transformer, etc., namely—

(1) The *true power* absorbed in the whole and each part of the circuit.

(2) The angle of phase difference between the current and both the supply and choker voltages.

(3) The impedance, ohmic, and inductive resistances, and self-induction of the choker.

The vector diagram for the circuit PR is that shown in Fig. 139, and is constructed as follows: set off a vector oa equal to the total voltage V across PR to any convenient scale. With radii $ob = V_2$ and $ba = V_1$ and centres o and a respectively, draw arcs intersecting at b, join b to o and a and produce ob to meet a perpendicular from a in the point c. Then oba is the triangle of E.M.F.s for PR, and bca that for PQ where r_1 and L are the ohmic resistance and self-induction of the inductive portion PQ. Since QR is non-inductive, the current A and voltage V_2 are always in phase, and hence by Ohm's law $V_2 = Ar$ where (r) is the ohmic resistance of QR, and oc will be coincident with the current vector. Thus θ will be the angle of phase difference

between the current (A) and total voltage V, while θ_1 will be that between A and the voltage V_1.

Note.—Errors in V, V_1 or V_2, or in the graduation of the voltmeter scale, will have least effect on the result when $V_1 = V_2$, which is the condition for maximum accuracy, and the resistance r of QR should first be adjusted if possible to obtain this condition.

Should the non-inductive resistance QR not be accurately known, or be likely to alter in value through heating due to the passage of the current A, then its equivalent in terms of V_2 and A can be substituted in the formula. Hence, if A is the $\sqrt{\text{mean square}}$ current in amps as given by a Siemens dynamometer or other direct reading alternating current ammeter, we shall have

$$W = \frac{A}{2V_2}\left\{V^2 - V_1^2 + V_2^2\right\} \text{ Watts,}$$

for the true mean power given to the whole circuit PR, QR may consist of a bank of electric glow-lamps, as the resistance r of QR can vary if it likes with the different mean currents.

It can easily be shown, in like manner, that the true mean power given to the inductive portion PQ of the circuit is

$$W = \frac{1}{2r}\left\{V^2 - V_1^2 - V_2^2\right\} = \frac{A}{2V_2}\left\{V^2 - V_1^2 - V_2^2\right\}$$

The method is not based on any assumptions as to the nature of the current (whether periodic or otherwise) or of the circuit, which may contain either self or mutual induction, and capacity, or all three. It is based solely on the difference in phase between the current and voltage.

If θ = angle of phase difference or lag of the current behind the voltage, then if both are sine functions

$$\cos.\ \theta = \frac{V^2 - V_1^2 - V_2^2}{2V_1V_2}$$

Apparatus.—Alternator D and its exciting circuit; inductive portion PQ of the circuit in series with a strictly non-inductive portion QR; two 2-way keys (K_1 and K_2) (p. 587); an A.-C. voltmeter (V); main switch (S); A.-C. ammeter (A). For comparison of methods, A may be used, and also a non-inductive Wattmeter W for measuring directly the power used up in PR. Frequency meter f connected across the supply D.

In the electrical circuit of the motor M may be used a voltmeter V_E; ammeter a_E; switch S_E; rheostat R_E; source of continuous current E.

Experiments.—(1) Connect up as shown in Fig. 140. Adjust the pointers of all the instruments to zero, levelling such as need it.

(2) See that all lubricators in use feed properly, then start D, running slowly.

(3) Adjust its speed so as to get $\frac{1}{4}$ of the max. $\sim$ per sec., at the same time varying the excitation of D to alter its voltage (V), so

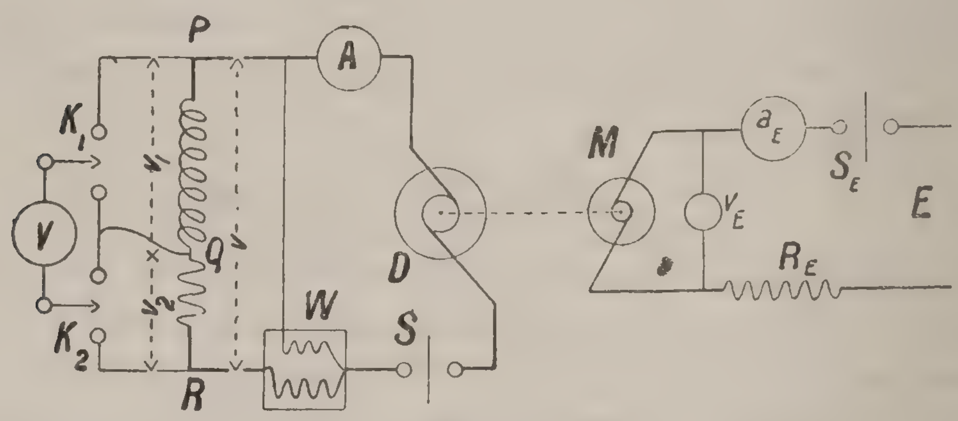


Fig. 140.

as to send a convenient current (A) through PQ. Note A, and in quick succession (the speed being constant) the voltages V, V_1 and V_2, across PR, PQ, and QR. (See Note above.)

(4) Repeat 3 for about 5 frequencies between the max. and min. values possible, using the *same current* A in each case by suitably altering the excitation.

(5) Repeat 3 at a *constant frequency* of about normal for five different current values, rising by equal increments up to the maximum allowed, by varying the excitation.

Tabulate your results as follows—

Form of Inductive Circuit PQ tested . . .
,, Non-Ind. resistance QR used . . .

Frequency (f).	Amps. A.	Voltages.			Power in Watts absorbed in						Power Factor.		From Math. Tables			Ohmic Res.		Coeff. of Self-Ind. (PQ)	Reactance of $PQ = 2\pi fL$ ohms.	Impedance of PQ
					PR.	*PQ.*	*QR.*	*PR.* By Calculation.			*PR.*	*PQ.*	Angle of Lag in *PR*	Angle of Lag in *PQ.*		*PQ.*	*QR.*			
		V.	V_1.	V_2.	Wattmeter W.	$w_1 = \frac{A}{2V_2}(V^2 - V_1^2 - V_2^2)$.	$w_2 = AV_2$.	$w = w_1 + w_2$.	$w = AV\cos\theta$.	$w = \frac{A}{2V_2}(V^2 - V_1^2 + V_2^2)$.	$\cos\theta = \frac{V_2 - V_1^2 + V_2^2}{2VV_2}$.	$\cos\theta_1 = (V^2 - V_1^2 - V_2^2)/2V_1V_2$.	$\theta°$.	$\theta_1°$.	$\tan\theta_1$.	$r_1 = r \times \frac{bc}{ob}$.	$r = V_2/A$.	$L = r\tan\theta_1/2\pi f$.		$V_1/A = \sqrt{(2\pi fL)^2 + r_1^2}$.

(6) Plot curves having values of A as abscissæ with values of V_1, cos θ_1, L, $\frac{V_1}{A}$ and w_1, respectively, for the inductive portion PQ as ordinates.

(7) Draw the vector diagram (Fig. 139) for the maximum current used.

(8) Compare V with the algebraical sum $(V_1 + V_2)$, also W with w.

Inferences.—Prove the formula in column 7, and state any assumptions made in deducing it. What can be inferred from the results of the test and from the curves?

(134) Measurement of the Electrical Power absorbed in Alternating Current Inductive Circuits. (Three-Ammeter Method.)

Introduction.—This method, though inferior to that of the Wattmeter, is nevertheless instructive, and therefore a brief *résumé* of it will be given here. As will be seen, it is very similar to the 3-voltmeter method of measuring power, the formulæ in the two cases being strikingly similar. There is, however, one chief difference between the methods, namely, that practically three ammeters are necessary for a satisfactory test, as large errors

may occur if only one ammeter is employed and interchanged between the circuits, while in the case of the allied method one voltmeter can easily be made to do and no appreciable error need be introduced. The actual arrangement is shown in Fig. 141, in which PQ represents the circuit in which it is desired to measure the power taken up, A, A_1 and A_2 are three non-inductive ammeters, at least A_1 should be of this nature, while (r) is a non-inductive resistance connected as shown, and which is large compared with that of A_1. Greatest accuracy will be obtained when $A_1 = A_2$, and under these conditions it will be seen that (r) consumes about as much power as Q. Hence twice as much power has to be available at the source for operating this method as is taken up in PQ, but practically no excess voltage is needed in this case as it was in the 3-voltmeter method. If w_2 = the power in Watts absorbed by PQ, then

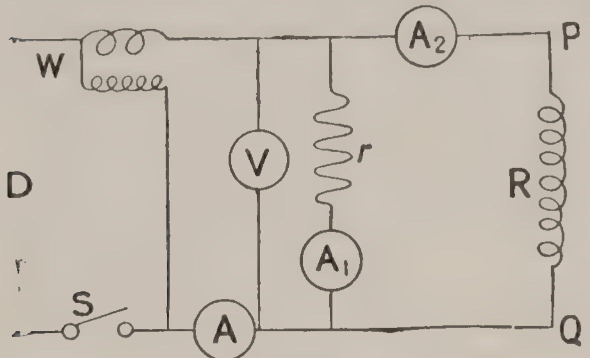


FIG. 141.

$$w_2 = \tfrac{1}{2}r\{A^2 - A_1^2 - A_2^2\}$$

and
$$\cos.\ \theta_1 = \frac{A^2 - A_1^2 - A_2^2}{2A_1A_2}$$

where θ_1 = angle of lag of the current A_2 in the inductive circuit PQ behind the terminal voltage.

It will thus be seen that the method is based on the difference of phase of the various currents, and, as in the 3-voltmeter method, a small error in observing the currents introduces large errors in the answer. The possibility of such occurring can be minimized by using accurately calibrated non-inductive ammeters and taking the mean of three or four similar readings at each value of, say, A. If the value of non-inductive resistance (r) is not accurately known, or if it is liable to alter through heating due to the passage of the current, then its equivalent value $\frac{V}{A_1}$ may be used instead in the formula, which will therefore become—

$$W = \frac{V}{2A_1}\left(A^2 - A_1^2 - A_2^2\right) \text{ Watts,}$$

where V = the voltage across the extremities of PQ. There is no objection to using a bank of incandescent lamps for (r), since the resistance may vary if it likes with the different mean current strengths. It will be observed that if the resistance of A_2 is appreciable, an amount of power may be absorbed in it which is comparable with that in PQ. In such cases the former must be deducted from the result as given by the above relation in order to obtain the true power absorbed in PQ alone.

If (r) is accurately known we may dispense with A_1 and put (r) directly across the mains, then on placing a voltmeter (preferably an electrostatic one) across the mains as in the last instance, we may substitute the value of A_1 in the first formula, when we shall have

$$W = \tfrac{1}{2} r \left\{ A^2 - \left(\frac{V}{r}\right)^2 - A_2{}^2 \right\}$$

The preceding remarks will be understood more clearly from the vector diagram, Fig. 142, for the circuit of Fig. 141, constructed

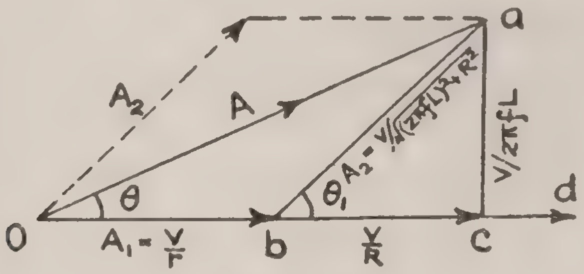


FIG. 142.

as follows: set off a vector oa equal to the total current A in the main line to any convenient scale with radii $ob = A_1$ and $ba = A_2$ and centres o and a respectively, draw arcs intersecting at b. Join b to o and a and produce ob to meet a perpendicular from a in the point c. Then oba is the vector triangle of currents for the main and both branches altogether, while bca is that for the inductive branch only. Since r is non-inductive, its current A_1 and voltage are always in phase, and hence by Ohm's law $A_1 = \frac{V}{r}$. If R equals the ohmic resistance of PQ and L the self-induction, then the energy or magnetizing component of the current A_2 in PQ which is in phase with the voltage od across it, is $bc = \frac{V}{R}$, while the idle or wattless component of the current A_2 in quadrature with the

voltage V is *ca.* The angle of phase difference between the main current A and V will be θ, and that between A_2 and the same voltage V across PQ will be θ_1.

From the geometry of Fig. 142 we see that

$$A_2^2 = A^2 + A_1^2 - 2AA_1 \cos \theta,$$

but $A_1 = \frac{V}{r}$ by Ohm's law, and

$$\therefore \quad A_2^2 = A^2 + A_1^2 - 2A\frac{V}{r} \cos \theta,$$

and $AV \cos \theta =$ total power given to the whole parallel circuit.

$\therefore$ the total power absorbed in the whole circuit is

$$w = V \times oc = \frac{r}{2}(A^2 + A_1^2 - A_2^2) \text{ watts},$$

the power absorbed in the non-inductive branch

$$w_1 = A_1^2 r = A_1 V \text{ watts},$$

and the power absorbed in the inductive branch

$$w_2 = V \times bc = \frac{r}{2}(A^2 - A_1^2 - A_2^2) \text{ watts},$$

where the power factor for the whole circuit =

$$\cos \theta = \frac{A^2 + A_1^2 - A_2^2}{2AA_1} = \frac{r(A^2 + A_1^2 - A_2^2)}{2AV},$$

and the power factor for the inductive circuit PQ =

$$\cos \theta_1 = \frac{A^2 - A_1^2 - A_2^2}{2A_1A_2} = \frac{r(A^2 - A_1^2 - A_2^2)}{2VA_2}.$$

In this three-ammeter method the non-inductive parallel branch is equivalent to an *added current*, while in the three-voltmeter method the non-inductive series resistance means an *added voltage.* Both methods, therefore, require the supply of practically twice as much power as that needed for the circuit under test. Further, the losses in the ammeters, voltmeter, and wattmeter may cause serious errors in the results if the currents are small. For the above reasons, no one would use either method for measuring power if a wattmeter was available, except from a purely scientific interest. While the power absorbed and phase difference may be calculated in each method from the vector diagram, constructed for each set of readings, it would usually be obtained from the respective formulae.

Apparatus.—That indicated in Fig. 141, where D is an adjustable source of alternating current, preferably a motor-driven alternator, the frequency, current and voltage of which can be

varied independently. A voltmeter V is connected across the parallel combination, and a wattmeter inserted so as to measure the total watts absorbed in the parallel combination merely for the comparison of the three-ammeter and wattmeter methods.

Observations.—(1) Connect up as in Fig. 141, levelling and adjusting to zero such of the instruments as need it.

(2) See that all lubricating arrangements are in operation on starting up.

(3) Adjust the speed to get maximum frequency, and also the voltage V of the alternator (by varying its excitation) so as to send the maximum safe current A_2 through PQ, and note in rapid succession the readings of A, A_1, A_2, W and V.

Note.—If possible adjust the non-inductive resistance r so as to obtain the conditions for maximum accuracy (other things being the same) of $A_1 = A_2$.

(4) Repeat (3) for about six different frequencies between the maximum and minimum values possible, using the *same current* A_2 in each case by suitably adjusting the speed and excitation of the alternator.

(5) Repeat (3) at *constant maximum frequency* for about six different values of current A_2 between the maximum and minimum values possible by varying the excitation, and tabulate your results as follows—

Form of Inductive Circuit PQ tested . . .
,, Non.-Ind. Resistance r used . . .

		Currents.			Power in Watts absorbed in						Power Factor in		From Math. Tables			Res. of					
					$PQ + r$.	r.	PQ.	$(PQ + r)$ By Calculation.			$PQ + r$.	PQ.	Angle of Lag in			r.	PQ.				
Frequency f.	Voltage V.	A.	A_1.	A_2.	Wattmeter W.	$w_1 = A_1 V$.	$w_2 = \frac{V}{2A_1}(A^2 - A_1^2 - A_2^2)$.	$w = w_1 + w_2$.	$w = AV \cos \theta$.	$w = \frac{V}{2A_1}(A^2 + A_1^2 - A_2^2)$.	$\text{Cos } \theta = \frac{A^2 + A_1^2 - A_2^2}{2AA_1}$.	$\text{Cos } \theta_1 = \frac{A^2 - A_1^2 - A_2^2}{2A_1A_2}$.	$\theta°$.	$\theta_1°$.	Tan θ_1.	$r = \frac{V}{A_1}$	R.	Coeff. of Self-Ind. of (PQ) $L = \frac{R \tan \theta_1}{2\pi f}$.	Reactance of $PQ = 2\pi fL$ ohms.	Impedance of PQ $V/A_2 = \sqrt{(2\pi fL)^2 + R^2}$.	Impedance of Parallel Combination V/A.

(6) Plot curves having values of A_2 as abscissæ with values of V, $\cos\theta_1$, L, $\frac{V}{A_2}$ and w_2, respectively, for the inductive portion PQ as ordinates.

(7) Draw the vector diagram (Fig. 142), for the maximum current used.

(8) Compare the value of A with the algebraical sum $A_1 + A_2$; also W with w.

Inferences.—What can you infer from the results of the test and from the curves?

(135) Measurement of Power in Three-Phase Alternating Current Circuits.

Introduction.—The measurement of the electrical power absorbed in a 3-phase alternating current circuit might well at first sight appear somewhat complicated. In reality, however, it is very little more so than in the case of single-phase circuits and the actual extent to which it is, depends mainly on the nature of the circuit in which the measurement is being made. It has already been seen that the non-inductive Wattmeter forms the best means of obtaining the *true power* absorbed in a single-phase circuit, but with multiphase circuits usually, though not always, two such instruments are necessary.

The object consequently of the present investigation is not only to state the methods of measuring, but also to prove the truth of them under the several distinctive conditions met with in practice.

The circuit in which the power has to be measured may be of the type shown at (*a*) Fig. 143, which is known as the *star* or *open* form, or of the type shown at (*b*) Fig. 143, known as the *mesh* or *closed* form. (*c*) represents the circuit containing the measuring instruments, which may be connected to either (*a*) or (*b*) arrangements at will, E being the source of polyphase supply.

Now let $A_1\ A_2\ A_3$ and $a_1\ a_2\ a_3$ be the $\sqrt{\text{mean square}}$ values of the currents flowing in the mains and branches respectively for Fig. 143 (*b* and *c*), also $V_1\ V_2\ V_3$ and $v_1\ v_2\ v_3$ the same values of voltages across the mains and branches respectively for Fig. 143 (*b* and *c*); then if the mains are equally loaded we have—

$$A_1 = A_2 = A_3, \text{ and } \therefore\ a_1 = a_2 = a_3, \text{ and } V_1 = V_2 = V_3,$$

whence $A_1 = 2a_2 \sin. 60° = \sqrt{3}a_2$, and $\therefore A = \sqrt{3}a$, since the mains are equally loaded and the *load non-inductive.*

For Fig. 143 (a and c) we have, if $A_1 = A_2 = A_3$ and $E_1 = E_2 = E_3$, that $A_1 = a_1$, $A_2 = a_2$ and $A_3 = a_3$, and since V will now lag 30° in phase behind v, in each main and the corresponding branch circuit, we have $V = 2v \sin. 60° = \sqrt{3}v$, providing the *load is non-inductive.*

Circuits equally loaded and non-inductive.—Here if each main carries the same current A, and if the pressure between

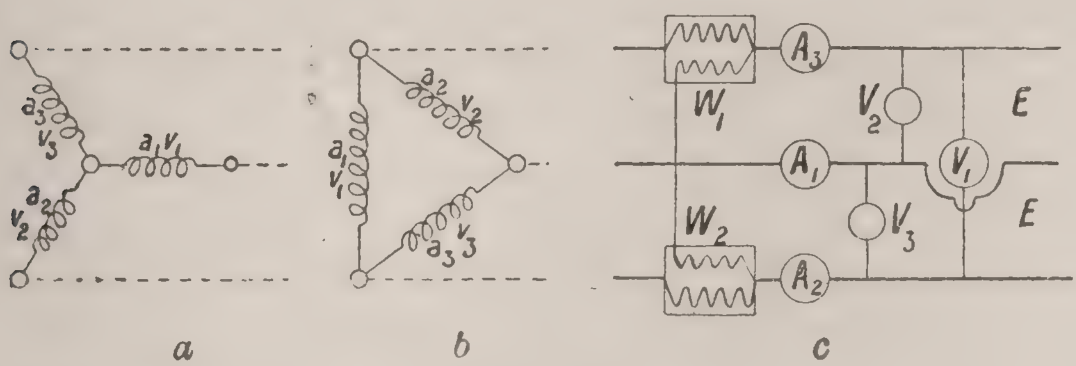


Fig. 143.

each pair of mains $= V$, then the True Power absorbed in a non inductive load, Fig. (b)

$$W = 3av = 3V\frac{A}{\sqrt{3}} = \sqrt{3}A\,V \text{ Watts,}$$

True Power absorbed in a non-inductive load, Fig. (a)

$$W = 3av = 3A\frac{V}{\sqrt{3}} = \sqrt{3}A\,V \text{ Watts.}$$

If, however, the load *is* inductive, then if θ = angle of phase difference between voltage and current, we have, as in the case of single-phase work, that for equal load the True Power absorbed in the inductive load, Figs. (a) or (b)

$$W = \sqrt{3}A\,V \cos. \theta \text{ Watts.}$$

This latter can best be obtained by means of the non-inductive Wattmeter for each of the two following conditions met with in practice.

Circuits equally loaded and inductive—One Wattmeter only needed to obtain the *true power.* Assuming this to be W_1, Fig. 143 ($c\,a$) or ($c\,b$), then with the thick coil in any main (A_3 say, as shown) note the Wattmeter reading (w_1) with its fine coil on

to main A_1, and the reading (w_2) with it on to main A_2 immediately after, then the True Power absorbed in the *equally loaded inductive circuit* $W = w_1 \pm w_2$, where both w_1 and w_2 will vary with load and power factor.

The reason why w_1 or w_2 alone does not give the power of the circuit is because A_3 and V_2 are not in phase, even in a non-inductive circuit, but differ in phase by an angle $= 30° \pm \phi$, for both star and mesh connections. Therefore W_1 will read the product $A_3 V_2$, $\cos 30 = \frac{\sqrt{3}}{2} A_3 V_2$ for unity power factor. Thus we see that $w_1 = A_3 V_2 \cos (30° + \phi)$, and $w_2 = A_2 V_3 \cos (30° - \phi)$, and the sum of these after expansion $= 2A_3 V_2$ ($\cos 30° \cos \phi) = \sqrt{3} A_3 V_2 \cos \phi = w_1 + w_2$, which is the true power in the circuit. If now the load is so highly inductive that ϕ exceeds 60°, *i.e.* the power factor $\cos \phi$ is less than 0·5, then $\cos (30 + \phi)$ becomes $-$ ve, and the wattmeter will reverse for one of its readings w_1 or w_2, which must therefore be considered as $-$ ve since the volt-coil connection must be reversed to get a scale reading.

$\therefore$ the total power $W = w_1 - w_2 = 2A_3 V_2 \sin 30° \sin \phi = 2A_3 V_2 \times \frac{1}{2} \sin \phi = V_2 \times A_3 \sin \phi$, and $\frac{V_2 \times A_3 \sin \phi}{V_2}$ $= A_3$ *sin* $\phi =$ *the wattless or idle line current.*

The above results will be readily understood by a reference to Fig. 144 (corresponding to Fig. 143, *a* and *c*), in which OA, OB, OC, represents the voltages across the respective star stator phase windings in magnitude and phase difference ($= 120°$). AC, CB, BA = voltages V_1, V_2, V_3 between mains in relative magnitude and phase ($= 120°$).

Then obviously $V_1 = AC = \sqrt{3}OA = \sqrt{3}OC$,

also $V_2 = CB = \sqrt{3}OC = \sqrt{3}OB$,

and $V_3 = BA = \sqrt{3}OB = \sqrt{3}OA$.

Now, since the three phase-windings are inductive we can draw three equal lines, Oa, Ob, Oc, to represent the currents in them lagging in phase by equal angles θ behind their respective voltages OA, OB, OC.

Then the current Oa in its phase winding differs in phase from the voltage AB ($= V_3$) by an angle $aDA = \theta - 30°$, and the

current Ob in its phase winding differs in phase from the voltage BC ($= V_2$) by an angle $OcB = \theta + 30°$.

Now, if the current coil of wattmeter w_1 is in the circuit of OC, and hence of main 3, with its volt coil across CB (mains 3 and 1), it will carry the current Oc at a voltage V_2. Similarly the current coil of wattmeter w_2 in the circuit of OA, and hence

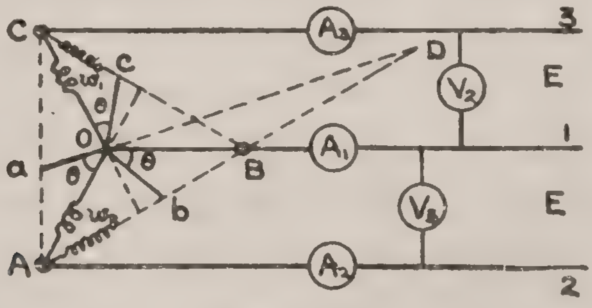


FIG. 114.

of main 2, will carry the current Oa with its volt-coil across AB (mains 2 and 1) at a voltage V_3.

Then $\quad w_1 = A_3 V_2 \cos(\theta + 30°),$

$$= A_3 V_2 (\cos\theta \cos 30° - \sin\theta \sin 30°),$$

and $\quad w_2 = A_2 V_3 \cos(\theta - 30°)$

$$= A_2 V_3 (\cos\theta \cos 30° + \sin\theta \sin 30°).$$

$\therefore$ total power of the circuit $W = w_1 + w_2 = 2AV(\cos\theta \cos 30°) = \sqrt{3} AV \cos\theta$, on the assumption that $A_1 = A_2 = A_3 = A$, and $V_1 = V_2 = V_3 = V$, which should be the case.

By adding and subtracting the values of w_1 and w_2 first given we have

$$\frac{w_1 - w_2}{w_1 + w_2} = \frac{1}{\sqrt{3}} \tan\theta, \quad \text{and putting } \frac{w_2}{w_1} = a$$

we have the power factor

$$\cos\theta = \frac{a + 1}{2\sqrt{a^2 - a + 1}} = \frac{1}{\sqrt{1 + 3\left(\frac{w_1 - w_2}{w_1 + w_2}\right)^2}}$$ (see p. 399),

when $\theta = 0$ the values of w_1 and w_2 are equal, and each $= \frac{1}{2}\sqrt{3} AV \cos\theta$.

As θ increases, w_1 decreases and w_2 increases, when $\theta = 30°$ the value of $w_1 = \frac{1}{2} A_3 V_2$ or $\frac{1}{2} AV$, and of $w_2 = A_2 V_3$ or AV; when $\theta = 60°$ the value of $w_1 = 0$, and of

$$w_2 = \frac{\sqrt{3}}{2} A_2 V_3 \text{ or } \frac{\sqrt{3}}{2} AV.$$

Since in this case the current in the series coil of w_1 differs 90° in phase from that in its pressure coil, any further increase in θ will make w_1 negative and reverse its deflection, so that the connections of one of its coils must be interchanged in order to bring the deflection on to the scale again.

Hence in measuring the power of any inductive 3-phase circuit by either 1 or 2 wattmeters the total power $= w_1 \pm w_2$, *i. e.* if one of the readings reverses, *substract the smaller reading from the larger one* to obtain the total power.

CIRCUITS UNEQUALLY LOADED AND INDUCTIVE—TWO WATTMETERS ONLY *needed* to obtain the *true power*. Assuming the Wattmeters to have their thick coils in any two mains, as shown in Fig. 143 *c* (*a* or *b*), then True Power absorbed $W = W_1 + W_2$.

Hence, when merely the true power in Watts only is required in a three-phase circuit, whether of the *star* or *mesh type*, *one* or *two Wattmeters* are required according to whether the circuits are *equally* or *unequally loaded* respectively. Also when such a three-phase circuit is both equally loaded and non-inductive the true power in Watts is given by the product $\sqrt{3} \times$ amps. in one main × volts. across any pair of mains. (See p. 395 *et seq.*).

Apparatus.—Source of three-phase alternating current (E) and circuit of variable nature to experiment upon (*a* and *b*, Fig. 143).

Two Wattmeters W_1 and W_2; three Siemens dynamometers or Parr direct reading dynamometer ammeters $A_1\, A_2\, A_3$; three electrostatic or hot-wire voltmeters $V_1\, V_2\, V_3$.

Note.—It must be remembered that for any specific measurement, the foregoing rules, and the instruments they entail, can be at once used without reference to the following test, which is devised solely in order to prove these rules.

Observations.—(1) Connect up as in Fig. 143 (*a* and *c*), and adjust the instruments to zero, levelling them if necessary.

(2) With the *load non-inductive* and *the circuits equally loaded*, take the readings of all the instruments for five or six different loads, noting the Wattmeter reading when placing the fine coil of, say, W_1 successively on to A_1 and A_2 mains at each load.

(3) With an inductive load and circuits equally loaded, take the readings of all the instruments for five or six loads, placing the fine coil of, say, W_1 successively on to A_1 and A_2 mains at each load and noting its reading at each. Tabulate your results as follows—

Nature of Circuit.	Loading of Circuit.	Wattmeters.		Voltmeters.			Ammeters.			$W = W_1 + W_2$.	$W = 2 W_1$.	$W = \sqrt{3} A V$.
		W_1.	W_2.	V_1.	V_2.	V_3.	A_1.	A_2.	A_3.			

Inferences.—State very clearly all that can be inferred from your experimental results.

Measurement of Power in Two-Phase Alternating Current Circuits.

Introduction.—Two distinct forms of circuits are met with in the distribution of electrical energy by means of two-phase alternating currents of electricity.

The first entails the use of four wires, forming two circuits completely independent of one another, one to each phase. Since this requires four wires it is usually employed in short distance transmissions.

The second entails the use of only three main wires, and is therefore more economical in first outlay of copper than the above. It will therefore be at once obvious that the measurement of power in two-phase alternating current circuits will be made in more than one way, depending on the form and nature of the circuit in question. We will now deal with such measurements in the case of each possible condition.

Two-Phase Circuits of the 4-wire Form.

Here two cases are possible according to whether the circuits are carrying non-inductive loads, such as incandescent lamps, or inductive loads, such as two-phase motors or transformers, etc.

Non-inductive load.—The product of the amperes and volts in each circuit, obtained in the usual way, when added together gives the true power delivered from the generator; and if the two circuits are equally loaded, *twice* the PRODUCT for one circuit gives the Total True Power.

Inductive load.—Owing to the lag in phase between the current and voltage in each circuit, two non-inductive Wattmeters are necessary, one in each circuit, connected up in the ordinary way

as in single-phase circuits. Then the Total True Power delivered by the generator = sum of the two Wattmeter readings.

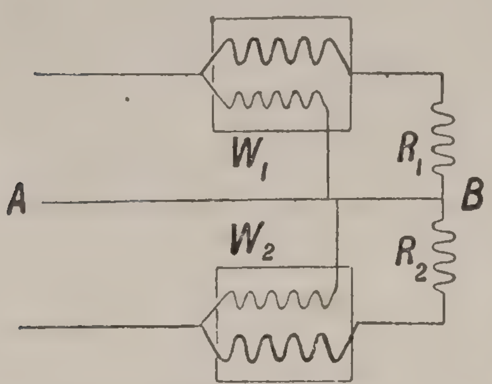


FIG. 145.

If the two circuits are equally loaded, as should be the case when supplying such as two-phase motors, then twice the reading of one Wattmeter gives the Total True Power, and only one such instrument is then necessary.

Two-Phase Circuits of the 3-wire Form.

Here also there are two or three cases depending on whether the circuits are inductive or otherwise.

Equally loaded non-inductive sections.—Total True Power absorbed = twice the product of the current in one outer main and the voltage across the section.

Equally loaded inductive sections.—Total True Power absorbed = twice the reading of a Wattmeter connected with its thick coil in series with either outer main, and its thin coil connected to the centre or larger main which is common to both outers.

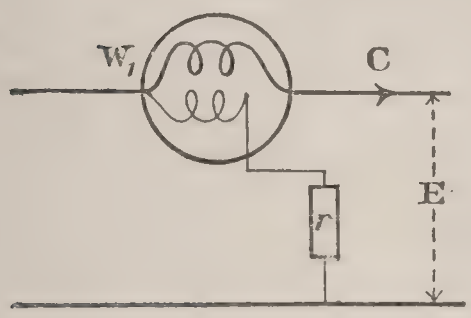
Unequally loaded inductive sections.—Total True Power absorbed = sum of the two readings of the Wattmeters connected with their thick coils in the outers respectively, and their thin coils connected to the common centre wire AB as shown in Fig. 145. This last case would be the one met with when the circuit was partly a lighting and partly a power one, running two-phase motors.

Where the reader may not be quite conversant with the preceding methods of measuring power in two-phase alternating current circuits, a most useful experiment will be to prove the above statements in much the same manner as was set forth in the preceding test on three-phase measurements of power, only three or four ammeters and voltmeters with the two Wattmeters W_1 and W_2 and the variable two-phase rheostat being required.

Measurement of Power in Polyphase Alternating Current Circuits.

As a summary, with some additions, to the methods as given on pages 388–394, the principal arrangements of wattmeters employed for measuring the true power in different kinds of alternating current circuits commonly met with in practice, are given here in diagrammatic form.

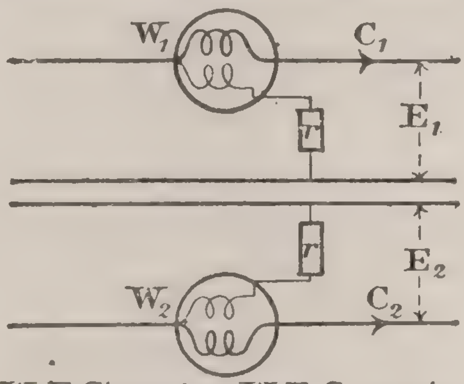
SINGLE PHASE



$W_1 = E\,C\cos\phi$
$W = W_1$

FIG. 146.

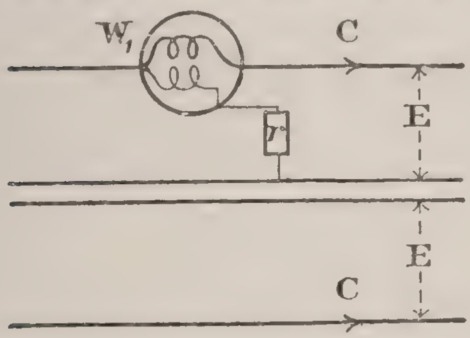
TWO PHASE (UNBALANCED)
INDEPENDENT OR COMMON RETURN



$W_1 = E_1C_1\cos\phi \quad W_2 = E_2C_2\cos\phi$
$W = W_1 + W_2$

FIG. 147.

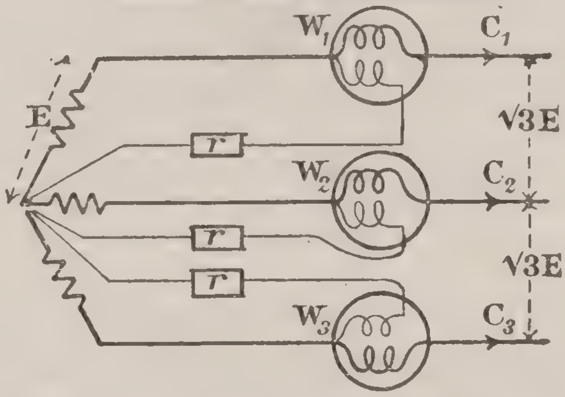
TWO PHASE (BALANCED)
INDEPENDENT OR COMMON RETURN



$W_1 = E\,C\cos\phi$
$W = 2W_1$

FIG. 148.

THREE PHASE (UNBALANCED)
USING THREE INSTRUMENTS



$W_1 = EC_1\cos\phi \quad W_2 = EC_2\cos\phi \quad W_3 = EC_3\cos\phi$
$W = W_1 + W_2 + W_3$

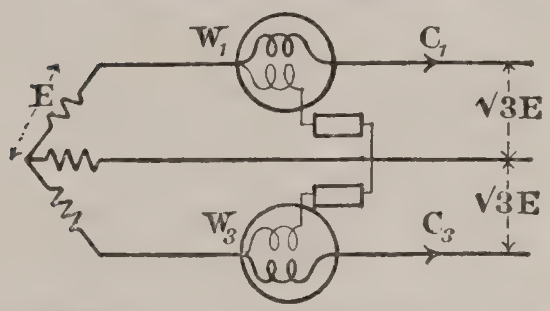
FIG. 149.

W = the total watts in the system.
W_1, W_2, W_3 = „ readings of the various wattmeters.
E, E_1, E_2 = „ voltages „ „ „ sections.
C, C_1, C_2, C_3 = „ currents in „ „ „

r, r_1, r_2 = the non-inductive resistances in series with the fine wire coils of the wattmeters.
$\phi, \phi_1, \phi_2, \phi_3$ = „ angles of phase difference between currents and voltages.

The diagrams and deductions accompanying each explain the principle clearly enough.

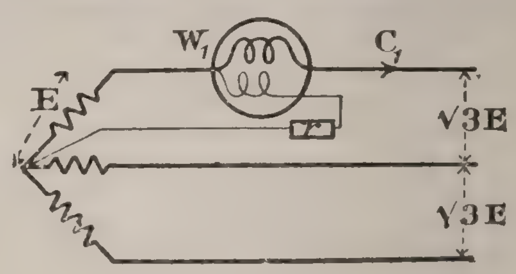
THREE PHASE (UNBALANCED)
USING TWO INSTRUMENTS



$W_1 = \sqrt{3}EC_1\cos(\theta_1 + \phi)$ $W_3 = \sqrt{3}EC_3\cos(\theta_3 - \phi)$
$W = W_1 + W_3$

FIG 150.

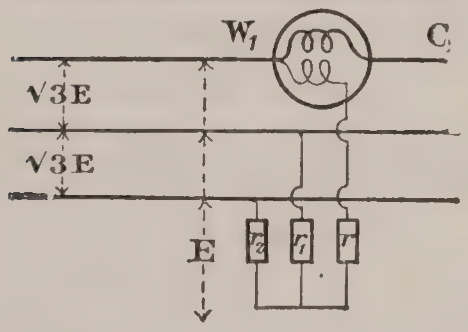
THREE PHASE (BALANCED)
NEUTRAL POINT AVAILABLE



$W_1 = EC_1\cos\phi$
$W = 3W_1$

FIG. 151.

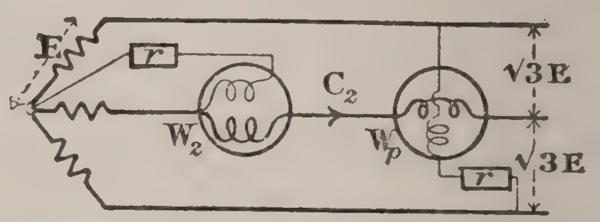
THREE PHASE (BALANCED)
NEUTRAL POINT NOT AVAILABLE



$W_1 = EC\cos\phi$
$W = 3W_1$

FIG. 152.

THREE PHASE (BALANCED)



$W_2 = EC\cos\phi$ $W_p = \sqrt{3}EC_2\cos(90° - \phi)$
$= \sqrt{3}EC_2 \sin\phi$
$W = 3W_2$ $= \sqrt{3} \times$ WATTLESS POWER PER PHASE

FIG. 153.

In Fig. 150, if the power factor of the system is less than 0·5, one of the wattmeters will read negatively and the connections of its fine wire circuit will have to be interchanged in order to obtain deflections on the scale. In this case the *difference of the two wattmeter readings gives the total power.*

In Fig. 152, the resistances $r_1 = r_2 = (r +$ fine wire coil), but an artificial neutral point can be formed by lamps without the expense of the resistances r_1, r_2.

In Fig. 156, unless the resistance of the fixed current coil is

small compared with the resistance of the phase in series with which it is connected, its insertion will throw out the balance of a mesh system and $3W_1$ will not = the total true power. The

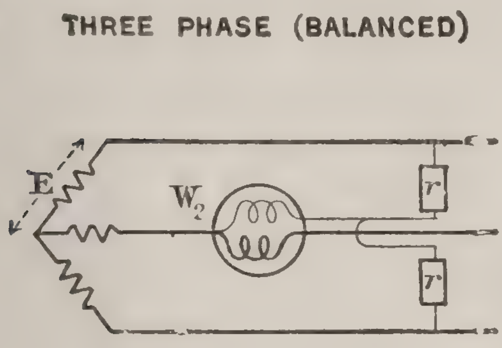


FIG. 154.

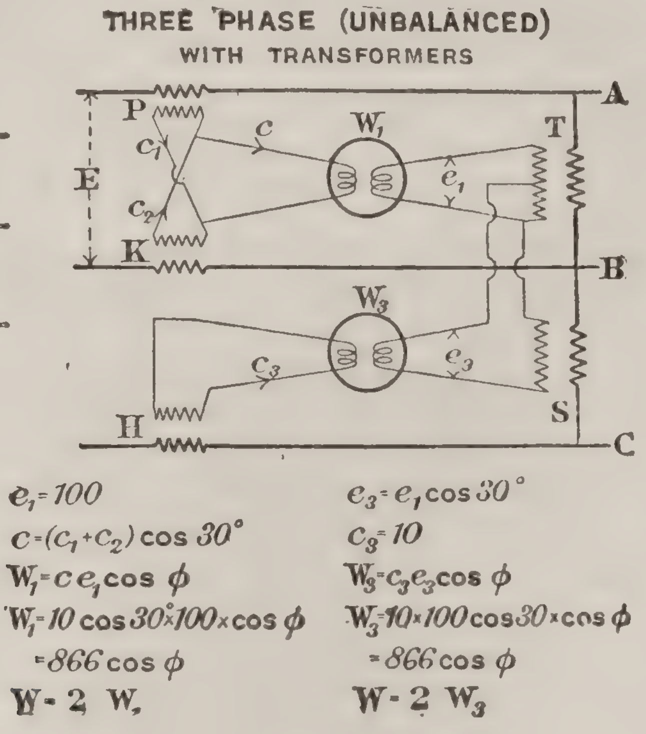


FIG. 155.

arrangement in Fig. 150, or if the system is balanced one wattmeter with a two-way key for connecting one end of the fine

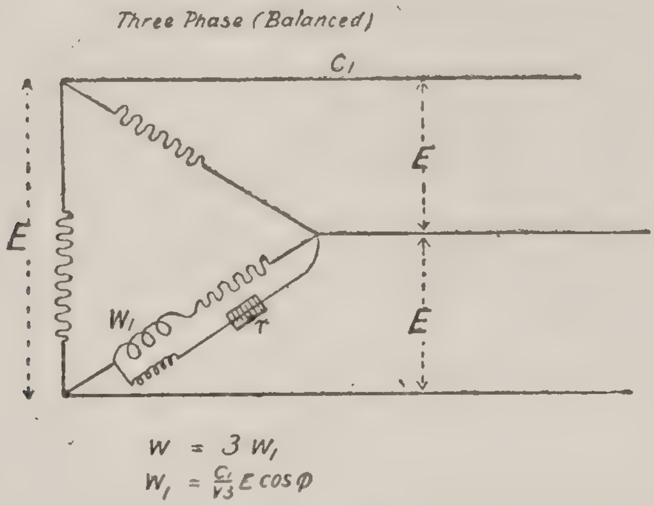


FIG. 156.

wire coil in quick succession to the remaining two mains, is much to be preferred.

Fig. 155 shows a method of connecting two wattmeters in three-phase high tension mains $A\,B\,C$ using current and pressure

transformers. P and K are two 20 to 1 series transformers, giving a secondary current of 5 amps. at full load; while H is a 10 to 1 series transformer, giving a secondary current of 1 amp. at full load. T and S are each 100 to 1 pressure transformers, with 10,000 volts on the primaries. It can be shown

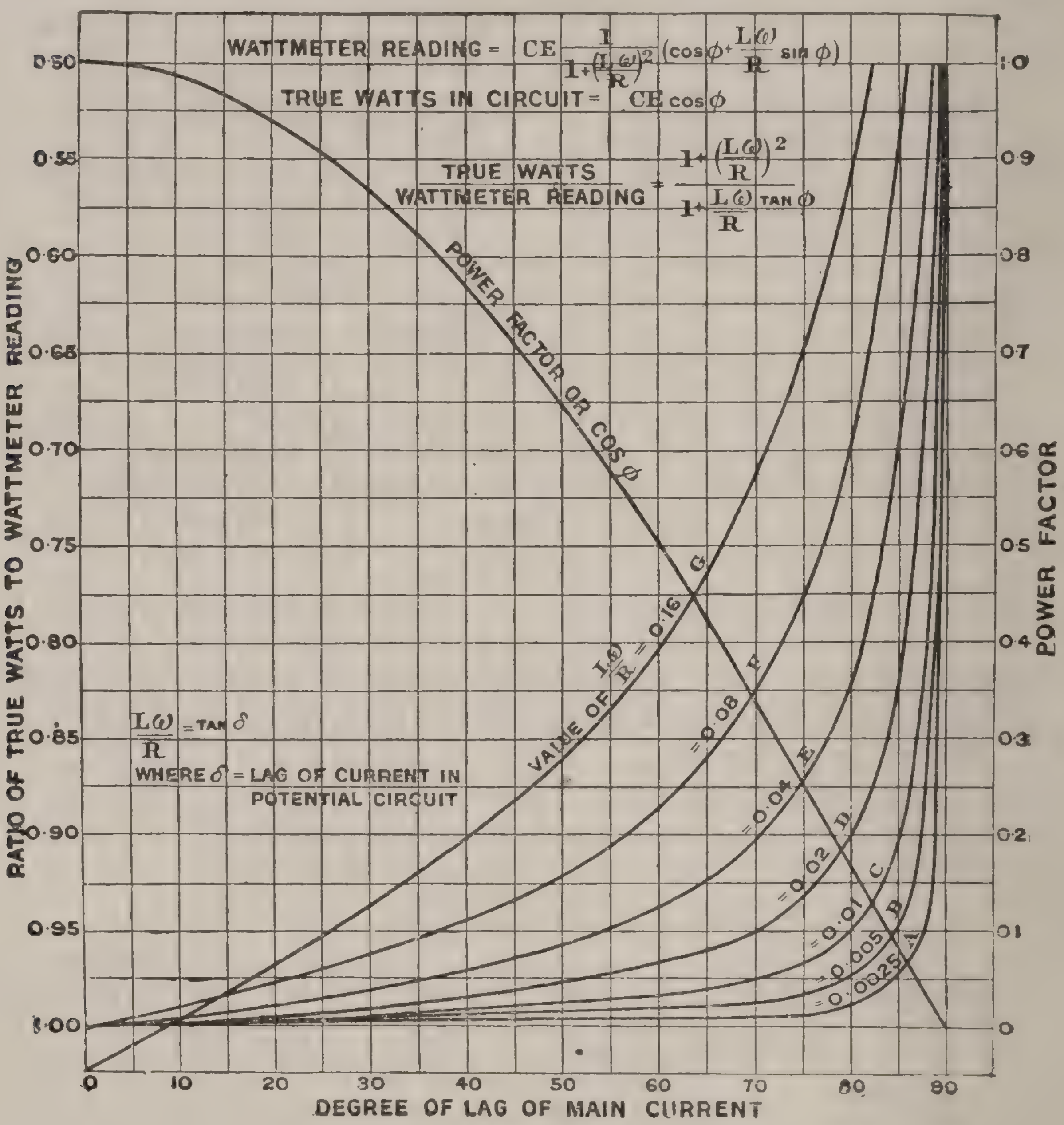


Fig. 157.

that each wattmeter indicates 866 kw. at full load, the total power of the circuit being 1732 kw.

Note.—The above methods are equally applicable to measuring the output of a generator or input into a motor or rheostats.

The measurement of power factor in alternating current circuits can be made by means of power factor indicating

instruments, or by the method described on page 388, in the case of three-phase circuits. Another method is shown in Fig. 153 for three-phase circuits in which a wattmeter W_P connected as shown indicates the wattless power in a phase, or the power factor, if the scale be suitably graduated. The instrument in this case has a central zero and deflects to one side or the other according to whether the current lags or leads with respect to voltage.

As with such an arrangement, a considerable P.D. will exist between the fixed and moving coils, it can only be recommended for the lower voltages.

Another and safer method can be employed with balanced three-phase circuits using one wattmeter in one main and a two-way key for connecting one end of its fine wire circuit in quick succession to the remaining two mains.

If d_1 and d_2 are the two deflections so obtained, then the

$$\text{Power factor of the circuit} = \frac{1}{\sqrt{1 + 3\left(\dfrac{d_1 - d_2}{d_1 + d_2}\right)^2}}$$

Correcting Factor for Wattmeters.—Considering the most common form, namely the electro-dynamometer type, used in practice, it is well known that the current through the moving coil should be exactly in phase with the P.D. at its terminals for the instrument to read *true watts correctly*. It is therefore both interesting and important to know the magnitude of the error introduced into the reading of the wattmeter and the *correcting factor* to be applied to obtain true watts when the current and pressure in the fine wire coil are not in phase due to the coil possessing inductance, which it must necessarily have to some small extent. The following considerations are quite general, and assume that the current and voltage are sine functions—

Let C = the maximum current in the fixed coil,

I = the current (at time t) in the fixed coil,

$E \sin \omega t$ = potential difference between the mains (at time t),

ϕ = angle of lag of the current in the mains,

i = the current (at time t) in the moving coil of self-induction L and total ohmic resistance R, including any resistance in series with it.

$\omega = 2\pi \times$ frequency of the supply.

Then the relations given in Fig. 157 [1] can be shown to hold good.

The relations apply to all types of wattmeters if ϕ, L and ω, the only quantities which vary with the nature of the load and type of wattmeter, are known.

When $\phi = 90°$ the multiplying factor becomes zero, and the reading of the wattmeter is zero, since there is no force between the coils carrying currents which differ in phase by 90°. The curves (Fig. 157) can be used as follows—Suppose we know that $L = 0{\cdot}02$ henry, $R = 628$ ohms, frequency $= 50 \sim$ per sec., and the power factor $= 0{\cdot}5$. Then $\frac{L\,\omega}{R} = 0{\cdot}01$.

Hence curve C is to be used. Now the horizontal line through 0·5 on the power factor scale cuts the power factor curve cos. ϕ at a point, the vertical line through which passes through $\phi = 60°$ and cuts curve C at 0·98, which is the correcting factor of the wattmeter.

Fundamental Considerations Relating to Alternating Current Static Transformers.

General Remarks.—Before considering actual methods of testing *static transformers*, the importance of which, in alternating current systems of distribution of electrical energy, arises from the ease with which a small current at high pressure can be converted to a large current at low pressure or vice versa by such an appliance and with very little loss, some introductory remarks are considered desirable.

There are a great many different forms and ways of building the kind of transformer in question, but they all come under one or other of two main heads, namely—

(*a*) Those with *closed* magnetic circuits in which the magnetic induction or lines of force are contained solely, or nearly so, in iron.

(*b*) Those with *open* magnetic circuits in which the lines of force run partly in the iron core of the transformer, and partly in the air through which they complete their path. This type,

[1] Taken, together with Figs. 104 and 146–155 from a paper on "The Measurement of power in alternating current circuits," by P. Hamilton, *Proc. Inst. C.E.*, vol. cliv, 1902–1903, by kind permission of the Author and Inst. C.E.

however, has now become practically obsolete. In either case (a and b) the iron core is surrounded by or wound with two distinct and separate coils of insulated copper wire termed the *primary* and *secondary*. In all cases the former is the coil connected to the source of supply, while the latter has induced in it an E.M.F. which supplies current to some separate circuit, usually at quite a different E.M.F. to that acting on the primary.

The primary may be either the high tension (pressure) coil or the low, according as to whether the transformer is used as a step-down or step-up appliance respectively. Hence to avoid confusion, the primary will always be that coil which is connected to the source of supply, whether this be high or low tension.

It may now be well to consider certain phrases and quantities met with in static transformers, and which appear in testing work on them. Transformers with "closed" magnetic circuits only need be considered, the "open" magnetic circuit type not having been made for many years. The induced secondary voltage is evaluated as follows—

Let N = total magnetic flux threading the secondary winding of T_s turns,

f = periodicity of the primary supply-current, and hence of this flux,

E_P and E_S = maximum values of E.M.F.s at the terminals of primary and secondary.

Now since in one period of the current wave, the current and hence the flux varies from 0—max., max.—0, then reverses and again varies from 0—max. and then max.—0, the average rate of change in the flux $= 4N$ lines per cycle or period, and the average change $= 4Nf$ lines per sec. Therefore the average E.M.F. induced per turn $= \frac{4Nf}{10^8}$, and therefore the average E.M.F. induced in the secondary winding of T_s turns $= E_S = \frac{4NfT_S}{10^8}$ volts. Since the virtual E.M.F. = average E.M.F. × form factor of the voltage wave, the virtual or R.M.S. E.M.F. $E_S = \frac{4 \times 1{\cdot}11 NfT_S}{10^8} = \frac{4{\cdot}44 NfT_S}{10^8}$ volts, where 1·11 is the value of the form factor of a sinusoidal wave. There will also be an

induced E.M.F. due to self-induction in the primary winding of T_P turns, and since the same flux threads this also, this back E.M.F. of self-induction must $= \frac{4 \cdot 44 N f T_P}{10^8}$ volts. On open secondary circuit the primary supply pressure only exceeds this back E.M.F. by a very small amount, namely, that sufficient to force the energy current through the resistance of the primary winding and provide the necessary magnetizing current for producing the flux in the core. We therefore have the following important relation, namely—

$$\frac{E_P}{E_S} = \frac{4 \cdot 44 N f T_P}{4 \cdot 44 N f T_S} \times \frac{10^8}{10^8} = T_P/T_S$$

very approximately, which is called the *voltage ratio of conversion* or *ratio of transformation.*

If A_P and A_S are the currents flowing in the primary and secondary having resistance R_P and R_S, then the ohmic drop of voltage in each is $A_P R_P$ and $A_S R_S$ respectively, and the core flux is produced by an effective voltage $\overline{E_P - A_P R_P}$, where the bar over the expression indicates that it is a vectorial—and not an algebraical—difference, the primary supply E.M.F., E_P, and energy voltage, $A_P R_P$, not being in phase as indicated in Fig. 158.

The no-load secondary induced voltage will therefore

$$= (\overline{E_P - A_P R_P})\frac{T_S}{T_P} \text{ volts,}$$

and the secondary voltage on load

$$= (\overline{E_P - A_P R_P})\frac{T_S}{T_P} + A_S R_S \text{ volts.}$$

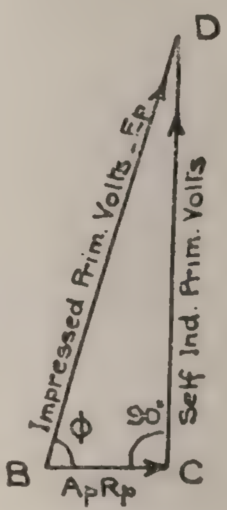


Fig. 158.

When the secondary circuit is *open*, the total loss occurring in the transformer is called the *open-circuit loss*, and the current flowing in the primary is called the *no-load primary current.*

The open-circuit loss is made up of the copper loss due to the no-load current flowing in the primary winding, and which is usually very small compared with the remaining loss due to eddy currents and magnetic hysteresis which are termed the iron core losses.

The no-load primary current, such as would be indicated by an ammeter, consists of two components in quadrature, namely, (*a*) the true magnetizing component, which being an idle or wattless current lags 90° behind the supply voltage, and (*b*) the energy or load component in phase with the supply voltage, and overcoming the above open-circuit losses due to eddy currents, hysteresis, and copper loss.

These three currents can therefore be represented by a right-angled triangle such as Fig. 158, in which BD would be the no-load current, BC the energy component, and CD the magnetizing component. Thus, since $BD = \sqrt{BC^2 + CD^2}$, we see that the no-load current $= \sqrt{(\text{energy current})^2 + (\text{magnetizing current})^2}$ $= \sqrt{A_e^2 + A_m^2}$, and this no-load current would be in quadrature with the supply volts, except for the energy current, which makes the phase difference slightly less than 90°.

The magnetization or core flux, being directly proportional to the supply voltage at constant frequency, is constant at all secondary loads with a constant voltage supply, and hence *the iron losses are constant at all loads.* Further, since we have seen that $\frac{E_P}{E_S} = \frac{T_P}{T_S}$, it follows that $\frac{A_S}{A_P} = \frac{T_P}{T_S} = \frac{E_P}{E_S}$, *i.e.* the primary and secondary currents are inversely $\propto$ to the voltages.

The measurements of current, voltage, and power in tests connected, not only with transformers, but also with alternating currents generally, should be made with instruments possessing practically no self-induction and little or no iron. The best results will be obtained when employing electrostatic, hot-wire, and dynamometer instruments, for such measure the $\sqrt{\text{mean sq.}}$ values of pressure and current and are independent of the variations of frequency. If a circuit supplied with alternating current is *non-inductive*, as for example a bank of electric incandescent lamps run off the secondary of a transformer, then the $\sqrt{\text{mean sq.}}$ values of the amperes × that of the volts = the *true or mean power* in Watts taken up by that circuit or bank of lamps.

If, however, the circuit is inductive this product (amps. × volts) gives what is called the *apparent power* in Watts absorbed, which is in all cases greater than the *true power*. This would be the case if we tried to measure the power given to the primary

of a transformer, which is always very inductive. Recourse must in such cases be had to the so-called *non-inductive Wattmeter*, the fine wire coil of which must have as few a number of fine wire turns as will give the requisite sensibility. Such an instrument will measure the *actual* or *true mean* power given to any circuit, however inductive it is, and no difficulty presents itself in the use of the Wattmeter on a low tension circuit. If, however, the power absorbed in a high tension circuit is required, then a special arrangement of Wattmeter is needed (see p. 42). It is much better, however, to have all measuring instruments on the low tension circuit, and this can be accomplished by employing one of the double conversion methods given in the following pages, a course almost always possible in works and central stations in which two *similar* transformers as regards size and output can generally be obtained.

Another method of measuring the power given to or developed by a transformer is the 3-voltmeter one, and in the case of the primary circuit, a non-inductive resistance of such a *known* value is placed in series with this coil, that the P.D. across its terminals = that across the primary coil, or preferably as nearly so as possible, as this gives maximum accuracy. The method consequently has the somewhat serious disadvantage that the E.M.F. of the supply has to be double that required for the primary alone, which would in the majority of cases preclude its use. Then again a small error in observation may cause a large error in the results.

(136) The Effect on the No-Load Voltage Ratio, Current, and Watts of a Transformer, of Change of Primary Supply Voltage and Frequency. (Magnetization Curve or Open Circuit Characteristic.)

Introduction.—The present investigation is a very important one, in that, amongst other results, it gives the relation between primary terminal voltage ($\propto$ to core flux at constant frequency) and magnetizing current, and which is termed the "open-circuit characteristic" or "magnetization curve" of the transformer.

The voltage used for the relation should, strictly speaking, be that of the back E.M.F. of self-induction, and therefore the vectorial difference $\overline{V_P - AR_P}$; but as both A and R_P are small, their product is negligibly small compared with V_P, and can be neglected. Even with the special low-loss iron now used in transformer cores, these are seldom worked at magnetic induction densities outside the limits, 3500 to 7500 lines per sq. cm., in order to minimize the power (due to the iron-loss or energy-current component) absorbed in magnetization, which is con-

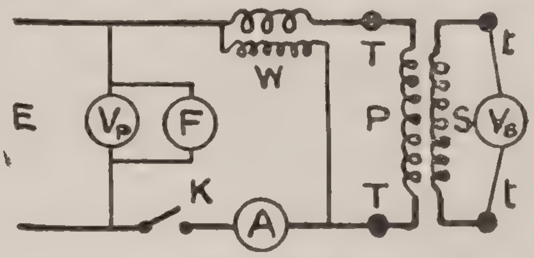


FIG. 159.

verted into heat in the core. For this reason the "knee" of the curve, which corresponds to about 15,000 to 17,000 lines per sq. cm., and is all-important in the design of D.C. apparatus, is never reached in the magnetization curve of a transformer.

Further, since the core loss is obtained in this test and is well known to be practically constant at all loads, it follows that, knowing the resistance of the windings, and hence copper losses (C^2R) in P and S at any load current, the efficiency can be predetermined at all loads.

The test also shows that both the no-load current and watts decrease as the frequency increases, and hence that higher frequencies reduce the size of core and cost of manufacture for a given output.

Apparatus.—Transformer under test, of which P is the primary and S the secondary; low-reading wattmeter W; voltmeters $V_P V_S$; switch K; frequency meter F; low-reading ammeter A; source of supply E, preferably a motor-driven alternator, the speed and excitation of which is variable over a wide range.

Observations.—*With Variable Voltage Supply at Constant Frequency.*

(1) Connect up as in Fig. 159, levelling and adjusting such instruments to zero as need it, the terminals *tt* of the high

tension winding used as the secondary being open-circuited as shown.

(2) With the frequency adjusted to the normal value for the transformer, and the field regulator of the alternator full in, close K and take simultaneous reading of V_P, V_S, F, W and A at each of some ten different values of V_P, rising by about equal increments from the lowest readable values to not exceeding 20 % above normal, by adjustment of field regulation or otherwise, and at *constant normal frequency.*

(3) *With Variable Frequency Supply at Constant Voltage.*

With the voltage adjusted to the normal value for the transformer, take simultaneous readings of V_P, V_S, F, W and A at each of some ten different values of F, rising by about equal increments between the lowest and highest values convenient, *at constant voltage* V_P.

(4) Measure the ohmic resistances of the primary and secondary windings P and S; that of P by either (a) the ammeter-voltmeter method (p. 86), using Ohm's law and a direct current supply for E, taking care to connect a suitable main current variable rheostat in circuit between E and P; or (b) the comparative deflection method (p. 84). The resistance of S may be obtained by either (c) method (a) above mentioned, or (d) a Wheatstone bridge. In the case of the ammeter-voltmeter method, the voltmeter must be connected to the actual terminals TT or tt of the windings.

Tabulate all your results as follows—

Transformer: No. . . . Type . . . Maker . . .
Normal output = . . . *KVA.* Frequency = $\sim$ per sec. Voltage ratio = . . .
Secondary: volts = . . . Amps. = . . . Resistance R_S = . . . ohms @ . . . °C.
Turns T_S = . . .
Primary: volts = . . . Amps. = . . . Resistance R_P = . . . ohms @ . . . °C.
Turns T_P = . . .

Frequency f.	Voltages		Voltage Ratio V_S/V_P.	Amps A.	Apparent Watts V_PA.	True Watts W.	Power Factor $\cos\phi = W/V_PA$.	Angle of Lag $\phi°$.	Energy Current Component $A_E = W/V_P$.	Idle or Magnetizing Current Components $A_m = \sqrt{A^2 - A_E^2}$.	Core Flux $N = \frac{10^8 E_P}{4{\cdot}44 f T_P}$.
	Primary V_P.	Secondary V_S.									

(5) From obs. 2 plot the "open-circuit characteristic" (otherwise known as the "magnetization curve") of the transformer having values of V_P as ordinates, with magnetizing current A_m as abscissæ.

Also curves having the same scale values of V_P as ordinates, with (*a*) total no-load current A; (*b*) energy current component A_E; (*c*) no-load watts W (practically all iron-core losses); (*d*) voltage ratio V_S/V_P, as abscissæ, in each case. From obs. 3 plot on another curve-sheet curves having values of frequency f as ordinates, with A, A_E, A_m, W and V_S/V_P, respectively, as abscissæ.

Inferences.—From a study of the table of results and shape of curves state clearly all that can be deduced.

(137) Measurement of Copper Losses in a Transformer (by the Short Circuit Test).

Introduction.—The total internal loss W in any static transformer is made up of the iron loss W_I due to eddy currents and magnetic hysteresis in the iron core, together with the copper loss W_C due to the currents A_P and A_S in the primary and secondary windings of resistances R_P and R_S.

Then $$W_C = A_P^2 R_P + A_S^2 R_S \text{ and } W = W_I + W_C.$$

Knowing R_P and R_S, the copper loss W_C can be calculated for any or a series of measured load currents, but the value so found may differ considerably from the actual working or effective value, owing to the eddy current and "skin effect" present with the larger sizes of conductor, when carrying alternating current, causing an apparent increase in the resistances R_P and R_S. The present test, comprising the direct measurement of the total copper loss, would therefore appear to be a means of obtaining it under working conditions, and hence more accurately than by calculation.

Another source of error may, however, now creep in, for the wattmeter necessary for measuring the loss must obviously be a low-reading one, and have a current capacity equal to that of full load for the winding chosen as primary, while its pressure coil will be subject to a small fraction of what would probably be its normal pressure (a condition introducing an error in its

indication) unless, of course, the wattmeter is a specially designed one for low pressure. The small applied voltage necessary for keeping the short-circuit current within safe limits, will produce a very small induction, and therefore loss due to magnetization of the core. This last named may be negligibly small, when the wattmeter will indicate the copper loss only. If the iron loss is not so small, the wattmeter will give a reading at the applied voltage of short circuit when the secondary is open-circuited, and this reading must be subtracted from all of its indications on short-circuited secondary. Further, care must be taken that the wattmeter reading does not include any losses in connecting or short-circuiting cable. If it does, the loss in such must be separately calculated from their measured resistance and each current, and deducted from the reading.

If T_P and T_S = the number of primary and secondary turns respectively, and V_P = a small supply voltage applied to the primary in order to send full-load current A_P through it with secondary short-circuited, then the total resistance "drop"

$$= A_P\left(R_P + R_S\left(\frac{T_P}{T_S}\right)^2\right) \text{ volts.}$$

From the values of this "drop" and V_P the *characteristic triangle* of the transformer can be drawn and the *leakage drop* determined.[1]

Apparatus.—That for test No. 136, excepting that W and V_P must now both be low-reading instruments, while V_S is replaced by a low-resistance ammeter A_S for short-circuiting the terminals *tt* of the secondary winding S, the range being large enough to indicate at least full-load current of that winding.

Observations.—(1) Connect up as in Fig. 159, with the ammeter A_S across (*tt*) and the pressure circuit of W across TT, in order to eliminate errors due to including in the reading of W any copper loss in the primary connecting cables. Level and adjust to zero any instruments which need it.

(2) With an *efficient short circuit* of S the primary P will practically constitute a metallic resistance and require, by Ohm's law, probably only two or three volts or so to be applied to it in order to obtain full-load current through it.

[1] *Vide* "The Testing of Transformers," by Morris and Lister (*Journal I.E.E.*, vol. 37, p. 264, 1906).

This low voltage required at the normal frequency of the transformer, and from whatever source obtainable, must be adjusted by a suitable variable resistance, in series with P, so as to give eight or ten currents through P, varying by about equal amounts from its full-load value to the lowest readable, the readings of V_P, W, A and A_S being noted at each.

(3) Disconnect the secondary short circuit and note the iron-loss reading, w_I on W, for the *same value of* V_P as used in obs. 2 (if any is readable).

(4) Measure the resistance r_S of the short circuit (namely, A_S and its two short connecting leads) and tabulate your results as follows—

Transformer: No. = . . . Made by . . . Type . . . Voltage Ratio = . . .
Full load: Output = *KVA.* Amps. = . . . Volts = . . . Frequency = . . .
Resistances: Primary (R_P) = . . . ohms @ . . . °*C* Secondary (R_S) = . . . ohms @ . . . °*C.* Secondary short-circuit (r_s) = . . . ohms.

Frequency F.	Volts V_P.	Primary Amps (A).	Secondary short-circuit Amps. A_S.	Total Watts W.	Power Factor $\cos\phi = \frac{W}{AV_P}$.	Angle of Lag ϕ.	Ohmic Drop $\frac{W}{A}$.	Core Watts (if any) W_I.	$A_S^2 r_s$.	Actual Copper loss $W_C = W - w_I - A_S^2 r_s$.

(5) Plot a curve having values of copper loss W_C as ordinates, with A as abscissæ.

Note.—The impedance voltage V_P is entirely spent in overcoming the equivalent impedance of the windings with short-circuited secondary, being partly spent in overcoming resistance and partly in reactance.

The reactance voltage = $\sqrt{V_P^2 - (W/A)^2}$.

Deduction of the Regulation of a Transformer for any Load and Power Factor from the "Open" and "Short Circuit" Tests.

From the curves obtained in the preceding open and short-circuit tests, the drop in volts in a transformer on non-inductive or inductive secondary load can be predetermined. To obtain this drop is needed the "open-circuit" volts and the triangle of voltages relating impedance voltage, ohmic drop or resistance voltage, and the reactance voltage as obtained from the "short-circuit" test.

The voltage drop in the transformer for any load and power factor can thus be obtained from an exactly similar construction to that given on p. 182 for an alternator, and which will not, therefore, be repeated here.

(138) Determination of the Regulation of a Static Transformer. (Differential Method.)

Introduction.—The meaning of the term "regulation," as applied to a transformer, was explained and defined in test No. 139, p. 412, and its measurement in a single transformer there given. When, however, two similar transformers $T_1 T_2$ are available, the present method of measurement is both simple, convenient, and *direct reading*, whereas that of test No. 139 necessitates taking the difference between two voltages, and is less accurate. It is applicable to any pair of high-tension or low-tension transformers, but the secondary circuit should preferably be the L.T. side, on account of the greater safety in handling instruments at low tension.

Apparatus.—Source of A.C. supply E, whether low or high tension; two transformers $T_1 T_2$, similar in all respects; voltmeter V_S; switches $S_P S_S$; load resistance R; ammeter A_S; and (if available—for interest, but not as a necessity) two ammeters A_{P1} and A_{P2}.

Observations.—(1) Connect up as in Fig. 160, levelling and adjusting to zero such instruments as need it.

(2) First connect for V_S a voltmeter capable of reading (or glow lamps capable of absorbing) the *sum* of the normal voltages S_1S_2. Then with S_S and S_P open, and E giving the normal voltage and frequency of T_1 or T_2, close S_P. If V_S shows a fairly large voltage, S_1 and S_2 are in *helping* series, and the connections of one of them must be interchanged to bring their voltages into *opposing* series, when V_S will show very little.

(3) Now replace V_S by a *low-reading voltmeter*, and, with S_P closed (S_S still being open), note the readings of A_{P1}, A_{P2} and V_S (if any). If T_1T_2 are either exactly *similar*, or *unloaded*, or both, V_S should now read 0.

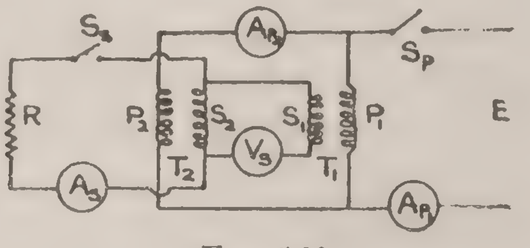


FIG. 160.

(4) With the supply voltage constant, and R *non-inductive* and full in, close S_S, taking the readings of all the instruments for each of a series of six or eight load currents A_S, rising by about equal increments from 0 to the full-load current of T_1 or T_2.

Note.— V_S gives the difference of the voltages between the terminals of the loaded (T_2) and unloaded (T_1) transformer, which is the required "drop."

The secondary output of transformers is usually expressed in kilo-volt-amperes (K.V.A.), irrespective of the power factor of the secondary circuit, and not in true K.W. at unity or some lower P.F. The values of V_S may, however, be obtained if desired on inductive loads by repeating obs. 4 with a variable choker (C) (not shown), connected in series with the non-inductive resistance R (preferably a bank of lamps) and a voltmeter with key to measure the volts V_R across R and V_C across the choker at the same current, when the power factor of the circuit will be

$$\cos\theta = \frac{R}{\text{impedance}} = \frac{V_R}{V_C}.$$

Tabulate your results as shown—

Currents for reference.		Secondary Load Current A_S.	Voltage Drop V_S.
A_{P_1}.	A_{P_2}.		

(5) Plot curves having values of V_S, as ordinates, with values of A_S as abscissæ.

(139) Determination of the Efficiency and Regulation of Transformers. (Single Conversion Method.)

Introduction.—This method is one of the simplest, though not the most accurate, and entails using only the one transformer to be tested. In all cases by the primary of the transformer is meant that winding connected to the supply mains whether these are at high or low tension.

The efficiency of any transformer, supplied at constant voltage and frequency, is the ratio of the secondary output to the primary input, or W_2/W_1.

The regulation of a transformer is the amount by which the secondary terminal voltage at any secondary load differs from that on open secondary circuit, *i.e.* it is the "drop" in voltage under load, and is due to both the resistance and reactance of the winding.

The regulation curve therefore relates secondary terminal voltage as ordinates and secondary load current as abscissæ. The ordinate intercept between this curve and a horizontal straight line through the "open circuit" voltage point at any secondary load gives the "drop" of voltage at that load.

Caution.—In the case of transformers which have to be run off a high tension alternator and are tested by this method, the one operating the high tension instruments must not only wear a pair of carefully selected and good india-rubber gloves, but must also stand on an india-rubber mat, and to guard against the possibility of accidents even in the case of manipulating the low tension instruments, the one operating these must either wear the pair of india-rubber gloves provided or stand on an india-

rubber mat. Before switching on, the "danger boards" provided must be placed close to the high tension wires.

Note.—Great care must be taken that the india-rubber gloves are not scratched, cut, or pierced in any way, as this would tend to render them useless for the purposes of insulation.

Apparatus.—Alternating current ammeters $A_1 A_2$ (Fig. 251), and voltmeters $V_1 V_2$; non-inductive Wattmeters $W_1 W_2$ (with their separate anti-inductive resistances $r_1 r_2$, if any); load absorbing resistance R, preferably non-inductive (p. 598); switches S_1 S_2; source of alternating current supply and transformer T to be tested.

Note.—V_1 and V_2 should be either hot-wire or electrostatic instruments, of which V_2 may preferably be of the latter type. If R is strictly non-inductive, then W_2 could be dispensed with; it may, however, as well be used if available.

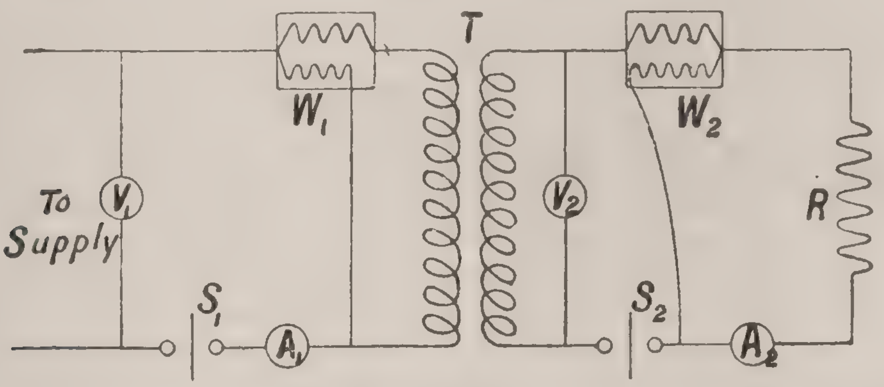


FIG. 161.

Tests.—(1) Measure the ohmic resistances R_1 of primary and R_2 of secondary coils in a suitable manner.

(2) Connect up the apparatus as indicated in Fig. 161, carefully levelling such instruments as need it, and seeing that their pointers are at o. Adjust the voltage and frequency (if possible) of the supply to the normal value required for the transformer.

(3) With S_2 *open*, close S_1 and note the readings of A_1, V_1 and W_1 simultaneously. The "open circuit losses" occurring in the transformer will thus be obtained.

(4) Make R large, and close S_2 as well as S_1. Then note simultaneously the readings of all the instruments for about ten different secondary currents from 0 to full load or to 15% over full load, rising by about = increments.

In all cases the *frequency* and primary voltage must be kept constant.

(5) Repeat 4 with a higher and lower frequency than the normal.

(6) Repeat 4 and 5 on an inductive load of constant power factor, or otherwise obtain the readings, as detailed on p. 182, necessary for plotting the regulation curves between secondary volts and current, each at different but constant power factors.

(7) Find, experimentally, the copper losses in primary and secondary by passing direct currents of various strengths, between 0 and full load, through the coils, and noting the losses by means of a Wattmeter.

NAME . . . DATE . . .

Transformer: No. . . . Made by . . . Transformation Ratio . . .
Normal output = . . . Kilowatts. Frequency = . . . ∼ per Sec. Type . . .
Primary Resistance R_1 = . . . ohms at . . . °C. Secondary Resistance R_1 = . . . ohms at . . . °C.

Primary.				Secondary.									Copper Losses in Watts.				
		Watts.				Watts.											
Volts V_1.	Amps. A_1.	Apparent A_1V_1.	True W_1.	Volts V_2.	Amps. A_2.	Apparent A_2V_2.	True W_2.	Voltage Ratio V_1/V_2.	Regulation $\left(\frac{V_1 - V_2}{V_1} \times 100\right)\%$.	Total Loss in Transformer $W_1 - W_2$.	Power Factor $\frac{W_1}{A_1V_1}$.	Primary $A_1^2R_1$.	Secondary $A_2^2R_2$.	Total L_c.	Iron Loss in Watts $W_1 - W_2 - L_c$.	Efficiency % $100\frac{W_2}{W_1}$.

Frequency used = . . . ∼ per sec. Total Secondary drop = . . . Volts.

(8) Plot the following curves having values of—(*a*) Total copper losses; (*b*) total iron losses; (*c*) secondary voltage; (*d*) primary power factor; (*e*) efficiency; (*f*) voltage ratio, respectively as ordinates and secondary load currents as abscissæ in each case.

Inferences.—State concisely all the inferences which you can draw from the results of your experiments.

(140) Determination of the Efficiency of Transformers. (Double Conversion Method.)

Introduction.—This method can be used when two *similar* transformers are at hand, and particularly when only low tension measuring instruments are available.

Caution.—To guard against the possibility of accidents, even in the case of manipulating the low tension instruments, the pair

of india-rubber gloves provided must be worn by the one manipulating the tertiary circuit instruments, and the india-rubber mat must be used by the one reading those on the primary circuit. *On no account must any part of the secondary (high tension) circuit be touched while "alive,"* and before switching on the primary current, the "danger boards" provided must be placed close to the high tension leads.

Note.—Great care must be taken that the india-rubber gloves are *not scratched*, *cut*, or *pierced* in any way, as this would tend to render them useless for the purposes of insulation.

Apparatus.—Alternating current ammeters $A_1 A_3$ (Fig. 251), and voltmeters V_1 V_2 V_3, of which V_2 is not absolutely essential to the

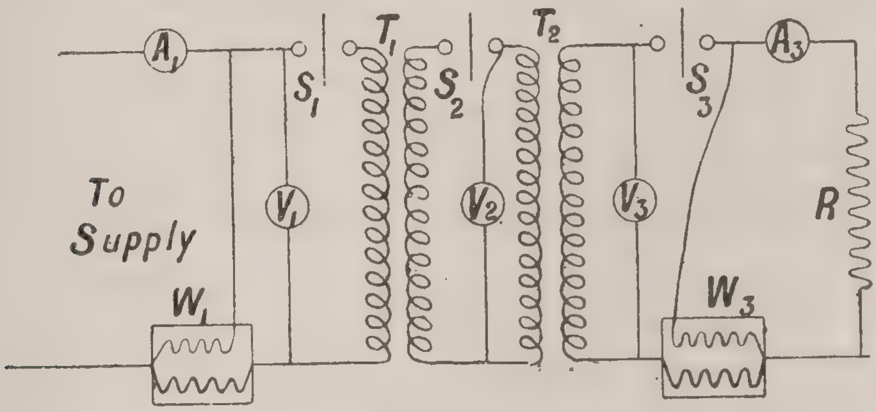


FIG. 162.

test; non-inductive Wattmeters W_1 and W_3 (with their separate anti-inductive resistances r_1, r_3, if any); switches S_1 S_2 S_3; load absorbing resistance R, preferably non-inductive (p. 598); source of alternating current supply and the two transformers T_1 T_2 to be tested.

Note.—Both V_3 and the high tension voltmeter V_2 should, if possible, be of the electrostatic type. If R is strictly non-inductive, then W_3 can be dispensed with; it may, however, as well be used if available.

Tests.—(1) Measure the ohmic resistance of each of the coils of the transformers T_1 and T_2 in a suitable manner.

(2) Connect up the above apparatus as indicated, carefully levelling such instruments as need it, and seeing that their pointers are at zero. Adjust the voltage and frequency (if possible) of the supply to the normal value required for the transformers.

(3) With S_2 and S_3 *open*, close S_1, and note simultaneously the readings of A_1, V_1, W_1. The "open circuit losses" occurring in transformer T_1 will thus be obtained.

(4) Make R large and close all the switches. Then note simultaneously the readings of all the instruments for about ten different tertiary currents from 0 to full load, rising by about = increments.

(5) Interchange T_1 and T_2 so that the latter now becomes the "step-up," and repeat exp. 3 and 4, tabulating your results in two tables similar to that shown.

Note.—In all cases the frequency and secondary voltage must be kept constant.

(5A) Repeat 3—5 for a higher and lower frequency than the normal.

(6) Find, experimentally, the copper losses in each of the coils by passing direct currents of various strengths between 0 and full load through them, and noting the losses by means of a Wattmeter.

NAME . . . DATE . . .

Transformer T_1 { No. . . . Type . . . Made by . . . Used as . . .
T_2 { ,, . . . ,, . . . ,, . . . ,, . .
,, T_1 { Normal Output = . . . / = . . . Kilowatts. Frequency = . . . / = . . . ∼ per sec.
,, T_2 {
Change ratio = . . . / = . . .

Primary.				Power Factor Cos. $\theta = \frac{W_1}{A_1V_1}$.	Angle of Lag θ.	Volts V_2.	Tertiary.				Total Loss in Two Transf. $W_1 - W_3$.	Copper Losses in Watts.				Iron Loss in Watts $W_1 - W_3 - L_c$.	Efficiency of	
		Watts.							Watts.									
V_1.	A_1.	A_1V_1.	True W_1.				V_3.	A_3.	A_3V_3.	True W_3.		$A_1^2R_1$.	$A_2^2R_2$.	$A_3^2R_3$.	Total L_c.		Combination $\frac{W_3}{W_1}$.	Each Transformer $\sqrt{\frac{W_3}{W_1}}$.

Resistances: Primary = . . . ohms. Total Secondary = . . . ohms. Tertiary = . . . ohms. at °C.
Frequency used = . . . ∼ per sec. Total Secondary drop $\frac{T_1}{T_2}$ = . . . volts.

(7) Plot the following curves having values of—(*a*) Total copper losses; (*b*) total iron losses; (*c*) tertiary voltage; (*d*) power factor; (*e*) efficiency respectively as ordinates and tertiary load currents as abscissæ in each case.

Inferences.—State clearly all the inferences which you can draw from your experimental results.

(141) Efficiency of High Tension Transformers. (Sumpner's Differential Method.)

Introduction.—A neat and convenient method of measuring the efficiency of high tension transformers, and which is susceptible of greater accuracy than most methods, is that due to Dr. W. E. Sumpner, and detailed as follows:—A small auxiliary transformer (C), the output of which need not be greater than the waste of power occurring in the two larger transformers A and B, to be tested, at full load, is required for the purpose of furnishing a small extra voltage (say 5 to 12 volts) necessary for driving the full load or any other currents through A and B. Its efficiency, goodness, or badness is a matter of indifference, and all we need in connection with it, is the output (w_1) of its secondary in Watts as measured by the Wattmeter (W_1).

The particulars as regards this output can be deduced as follows:—Suppose that two 2250 Watt transformers have to be tested each converting from 100 to 2000 volts or *vice versâ*. Then their probable efficiency would be about 94% (say), and hence each would absorb $22{\cdot}5 \times 6 = 135$ Watts at full load. Consequently the output of the auxiliary transformer C need not exceed 2×135 or 270 Watts.

Hence if used on low-pressure 100 volt mains the primary should take about 2·7 amps. at 100 volts, and the secondary give out 22·5 amps. at 12 volts. In the ordinary test of efficiency of high tension transformers, in which two similar ones are used—one as a step-up from the low-pressure primary mains, and the other as a step-down to the tertiary mains, the efficiency is deduced from measurements of primary imput and tertiary output which are of nearly equal magnitude. Consequently the percentage error in measuring these two quantities re-appears as the same percentage error in the efficiency so obtained.

The present method, which is much to be preferred of the two, consists in *actually measuring* the *losses* (w) occurring in the two transformers *directly*, and comparing these with the input to obtain the efficiency.

The method is economical in cost of energy used, especially when testing large transformers; with the methods used in tests

139 and 140 it would be a serious consideration, while the supply of full-load current would make a serious demand on a public supply, or necessitate a large testing alternator. The present method is accurate because the total loss (w) in the two transformers is obtained by *adding* together two quantities, and not by subtracting them, and is most convenient for finding the temperature rise after a run of a prescribed number of hours on full load.

The principle of the present method is strikingly analogous to Dr. Hopkinson's combined efficiency test of a pair of dynamos, the distinguishing feature of which is to couple two similar machines together both mechanically and electrically, one to run as a dynamo and the other as a motor. Energy is supplied to one by which it is transferred to the other, this latter returning it again to the source; the balance of energy supplied actually by the source is therefore equal to the waste which occurs in the double transformation and corresponds with the loss (w) above mentioned. This then is what takes place in the present case, for energy is taken from the mains by the "step-up" (A or B, whichever is used as such), then transferred to the "step-down" transformer, and finally back to the mains again.

Thus, while both transformers can be loaded to any extent by controlling the current circulating between them, the power taken from the supply is only some 4 to 20 % of the full-load K.W. capacity of either, depending on their efficiency—being only that necessary to make up the total internal losses in the two transformers together. Whether the L.T. or H.T. windings are connected to the supply is merely a matter of convenience depending on which supply is available, but usually the L.T. sides are connected to an L.T. supply for safety in handling the more commonly available low-tension instruments, etc. Calling whichever are connected to the supply the primaries, the *secondaries* must be so connected in series that their *E.M.F.s oppose* each other. If the primary E.M.F.s are equal, so also will be the secondary E.M.F.s, and no current will flow in the secondary windings. By making the small auxiliary transformer in series with one of the primaries provide a $+^{ve}$ or $-^{ve}$ boosting E.M.F., the out-of-balance primary E.M.F.s so produced will cause out-of-balance secondary E.M.F.s and a circulating secondary

current, the strength of this current depending on the difference between the E.M.F.s. Should the connections be such that the secondary E.M.F.s are in helping series instead of opposing series, as they should be, a short circuit will result. To avoid this, and to ensure the connections of the secondaries being correct, close S and S_1, when B will induce a voltage in the L.T. winding of A equal to that of the supply, but opposite in phase if the connections are correct. Hence, if either a voltmeter or lamps, each having a voltage range equalling twice that of the supply, are connected across the open switch S_2 and neither show any voltage, the connections are correct for the two L.T. windings, and therefore also the two H.T. windings are then in opposition. If otherwise, the voltmeter or lamps will show *twice the voltage of the supply.* In this event the connections of one of the H.T. secondaries must be reversed. It should be noted that (a) will indicate the load current, while an ammeter (a_1) in series with S_1 will give the magnetizing current.

If W = load in Watts supplied to the primary of the "step-up," and $w_1\, w_2$ = the Watts at this load as measured by w_1 and w_2, and λ = loss of power in the connecting leads, current meter a and the current coil of W_1, then the total loss in the two transformers

$$w = w_1 + w_2 - \lambda.$$

Hence the *efficiency* of *double conversion* $= 1 - \dfrac{w}{W}$

and the efficiency of either transformer $= \sqrt{1 - \dfrac{w}{W}}$

As the ratio of $\dfrac{w}{W}$ is small—not greater than $\frac{1}{20}$ with a transformer of 95% efficiency, the efficiency of each transformer is given quite accurately enough by the relation

$$\Sigma = 1 - \tfrac{1}{2}\frac{w}{W} - \tfrac{1}{8}\frac{w^2}{W^2}.$$

An error of 10% in estimating $\dfrac{w}{W}$ only affects the combined efficiency to 1%, and that of either transformers to $\frac{1}{2}$% only. Hence can be seen the superiority of the present method over the preceding one. The quantity W can be obtained with quite

sufficient accuracy by the product of the current A and the P.D. in volts supplied to the primary of the "step-up."

If (A) is the "step-up," then $W = 100 \times$ current in low tension coil of A, whereas if B is the "step-up," the power returned to the mains by $A =$ the above quantity, and hence the input of the primary of B is

$$W = 100 \times \text{ current of } A + (w_1 + w_2 - \lambda).$$

That transformer will be acting as "step-up" which has the higher P.D. of the two (A and B) on its low tension coil. If, say, 12 volts are supplied by C to the primary of B, the P.D. at the terminals of B will be either 112 or 88 volts according as the 12 volts from the auxiliary and the 100 of the mains are in phase or opposite phase. If R was very inductive, the above voltages would be out of phase, and B's P.D. might be anything between 88 and 112 volts.

Apparatus.—The two high-tension transformers A and B to be tested of say 2000/100 volts; an auxiliary Boosting one C, the primary of which is in series with a variable non-inductive resistance R of sufficient range to produce only a few secondary volts; two non-inductive Wattmeters W_1 W_2; Siemens electro-dynamometer or direct reading alternating current ammeter (a); switches S_1 S_2 S_3 and S; voltmeter V for maintaining the mains at 100 volts; alternator D, or some other source of alternating current.

Caution.—On no account whatever is the high tension circuit of either A or B to be touched while "alive." The india-rubber gloves and mat must be used by the operators to ensure immunity from accidental shock, or break-down of the insulation between primary and secondary of A and B.

Observations.—(1) Connect up as in Fig. 163, and adjust the instruments to zero where necessary. See that all lubricating arrangements are working properly, also that the gloves are in good order and the mat suitably placed.

(2) Measure the losses due to the resistance of the leads and instruments employing alternating currents, which can be done without altering any of the connections thus—with S and S_1 *open, short circuit* the primaries of A and B and close S_2 S_3.

(3) Vary R so as to obtain about six different currents through (a) from 0 to the full load current of A or B by causing the secondary voltage of C to vary suitably. Note the corresponding

readings on W_1 (the power given out by the secondary of C), which therefore at once gives the required losses in Watts in leads and instruments for each particular current passing through them.

(4) Measure the copper losses in the coils of the two transformers A and B by opening S_1 and closing S, S_2 and S_3 and observing the readings of W_1 for some six different currents from 0 to the maximum of A and B, as read off on a.

(5) Measure the iron or core losses in the two transformers A and B by closing *all* the switches and observing the reading of W_2.

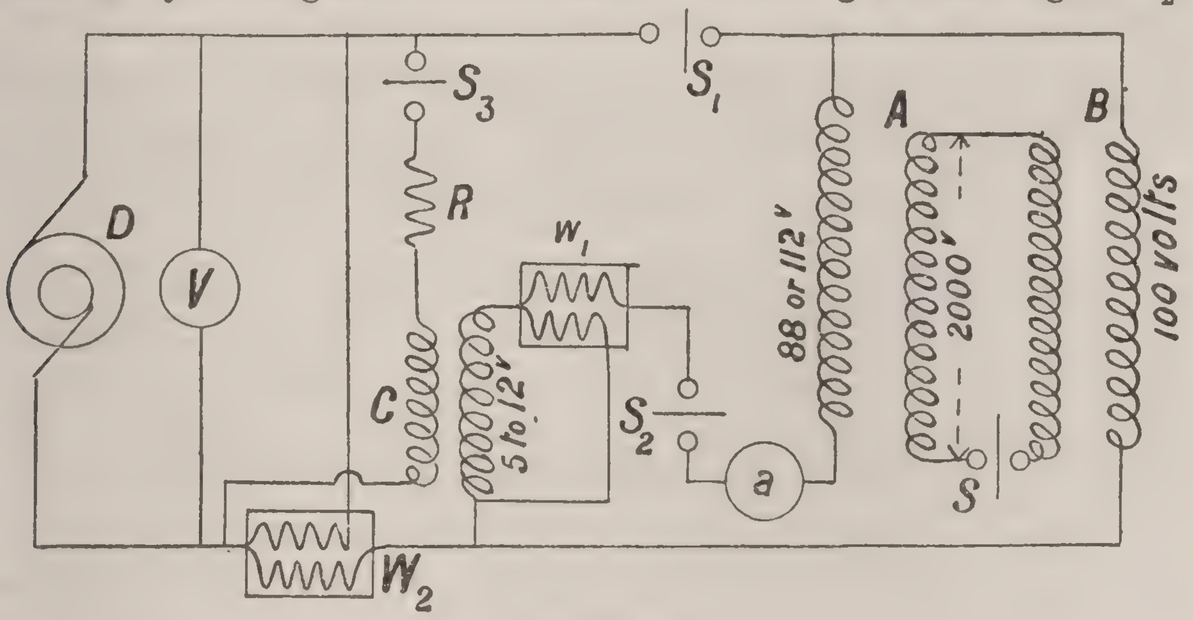


FIG. 163

(6) With S_2 and S_3 open, close S_1 and S and note the readings of a_1, a_2 and W_2. Then a_1 will indicate twice the no-load current of either transformers A or B, and W_2 twice the no-load losses in either.

Tabulate your results as follows—

NAME . . . DATE . . .

Transformer used as Step-up: No. . . . Type . . . Make . . . : Resist. $R_P = \ldots$ $R_S = \ldots$ Ohms.

" " Step-down: No. . . . " . . . " . . . : " $R_P = \ldots$ $R_S = \ldots$ "

" No. . . . Output: Volts = . . . Amps. = . . . Constant of Wattmeter $W_1 = \ldots$ "

" " . . . " : " = . . . " = . . . " " $W_2 = \ldots$ "

Speed in Revs. per min. N.	Frequency in $\sim$ per sec. $n = \frac{KN}{60}$.	Volts V.	Current.		Total Losses in				Total Input into Primary of Step-up $W = 100\,a + w$.	Efficiency	
			Reading on a.	Amps. a.	Coils, Leads and Instruments (W_1 in 4) w_1.	Iron Cores (W_2 in 5) w_2.	Leads and Instruments W_1 in 3 = λ.	the two Transformers $w = w_1 + w_2 - \lambda$.		of Combination in % $100\left(1 - \frac{w}{W}\right)$.	of either transformer $100\sqrt{1 - \frac{w}{W}}$.

(7) Plot the following curves on the same curve sheet having currents (a) as abscissæ, and for the ordinates the following—(i) Losses in leads and instruments; (ii) iron core losses; (iii) a^2R losses in the coils of A and B; (iv) total a^2R losses in A and B. Also with efficiency as ordinates, and load W in Watts as abscissæ.

(8) Reverse the positions of A and B, and repeat the above tests.

Inferences.—State very clearly all that you can infer from your experimental results.

(142) Measurement of the Efficiency of ordinary Single-Phase Transformers by Blakesley's 3-dynamometer Method.

Introduction.—This method necessitates the use of two ordinary Siemens electro-dynamometers, in which of course the moving coil is in series with and carries the same current as the fixed coil, whence the angle of torsion is proportional to the $\sqrt{\text{mean square}}$ value of the alternating current, and in addition the use of a third Siemens electro-dynamometer, arranged so that the moving coil has its own separate terminals, and is not in series with the fixed coil.

If then two alternating currents of equal period, from either the same or different sources, flow through the two independent coils, the periodic time of oscillation of the moving coil being very large compared with that of the current, the angle of torsion is proportional to the *mean* product of the simultaneous instantaneous values of current throughout the period, and is called the "*split dynamometer*" reading.

If A_o and A_o^1 = maximum values, and A, A^1 the mean values of two simple periodic alternating currents, one of which lags behind the other by an angle α, then the ordinary dynamometer will give $A = \frac{1}{2}A_o^2$ and $A^1 = \frac{1}{2}(A_o^1)^2$. On passing these currents through the split dynamometer its reading ϕ would be $= \frac{1}{2}A_oA_o^1$ cos. α, and hence cos. $\alpha = \dfrac{\phi}{\sqrt{AA^1}}$.

The following method is quite general, and does not assume that the current is a simple sine function of the time, but does

assume that there is no magnetic leakage, *i. e.* that the number of lines cutting the primary and secondary are the same. This is not true in all types of transformers on full load, but is nearly so in closed magnetic circuit types.

Since the split dynamometer gives no reading on open secondary circuit, this method is useless for determining the "open circuit" losses.

Apparatus. — Two ordinary Siemens electro-dynamometers $A_1 A_2$, and one split dynamometer (A); transformer T to be tested; non-inductive resistance L (such as a bank of lamps to take up the secondary load) (p. 598); alternator D; switches $S_1\,S_2$; voltmeters $V_1\,V_2$; non-inductive Wattmeter W, inserted merely for the purposes of comparison.

Observations.—(1) Connect up as in Fig. 164, and adjust the instruments to zero where necessary. See that all lubricating cups in use feed slowly and properly, then start D.

(2) S_2 being open, close S_1, and adjust the speed and excitation

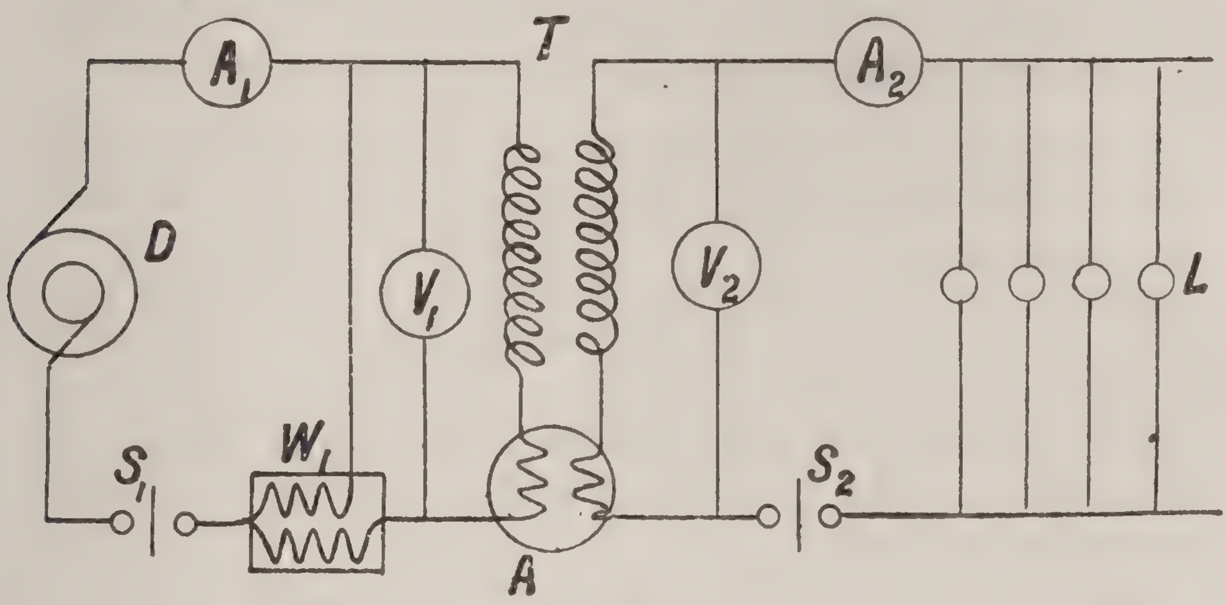


Fig. 164.

so that V_1 reads the normal voltage required for the primary at the normal frequency of the transformer. Note the readings of A_1, V_1 and W.

(3) Close S_2 and adjust L so that A_2 reads about $\frac{1}{10}$ of the maximum secondary current, the voltage V_1 being kept normal by varying the excitation. Now note the readings of A, A_1, A_2, V_1, V_2 and W.

(4) Repeat 3 for about 10 secondary load currents, rising by

about equal increments to the maximum allowable, and tabulate as follows—

NAME . . . DATE . . .

Transformer tested : No. . . . Type . . . Make . . .
Primary turns $N_1 = . . .$ Resistance $R_1 = . . .$ ohms. at . . . °C.
Secondary „ $N_2 = . . .$ „ $R_2 = . . .$ „ . . . °C.
Normal : Volts . . . Amps. . . . Transformation Ratio = . . .

Speed Revs. per min. N.	Frequency $\sim$ per sec.	V_1.	Dynamo-meter Reading.			Amps.			Wattmeter dW_1.	True Watts W_1.	Apparent Primary Watts A_1V_1.	Primary Input $W_P = (R_1A_1 + \frac{N_1}{N_2}R_2A)$.	Secondary Output $W_S = A_2V_2$.	Cosine Angle of Lag $\frac{A}{\sqrt{A_1A_2}}$.	Angle $\alpha°$.	Power Factor $\frac{W_1}{A_1V_1}$.	Efficiency $\frac{A_2V_2}{W_P}$.
			dA_1.	dA.	dA_2.	A_1.	A.	A_2.									

(5) Plot curves having values of W_S as abscissæ, with efficiencies and V_2 as ordinates.

(143) Measurement of the Efficiency of Multi-phase Alternating Current Transformers.

Introduction.—The determination of the efficiency of ordinary single-phase transformers has already been fully considered in the preceding pages.

The present test does not differ materially in principle from those in question, and practically the only difference is in the method of measuring the power absorbed and developed by the multiphase transformer, and which possesses some characteristic differences from that used in the case of the ordinary single-phase transformer.

Most of the preceding methods are equally applicable in the present case whether the transformer is of the two or the three phase type. The reader should refer to p. 388 for the method of measuring electrical power in two and three phase alternating current circuits, where a more detailed description of them will be found. If in the present instance, as in fact with any others, the rheostats or circuits in which the load is to be absorbed are strictly non-inductive, *i. e.* are of the nature of incandescent lamps or water rheostats, then providing such load-absorbing devices operate equally on each of the sections of the circuit, thus main-

taining a balanced system, the output can quite accurately enough be obtained from the ammeter and voltmeter readings in the manner set forth on pp. 388 *et seq.*

For the present test we will assume that the efficiency of a 3-phase transformer is required by, say, the single conversion method.

Apparatus.—The 3-phase transformer to be tested, of which P is the primary winding and S the secondary shown in Fig. 165, with the star or open winding; two non-inductive Wattmeters W_1 and W_2; three Parr's direct-reading dynamometer ammeters A, A_1 and A_2 (p. 572); three voltmeters V, V_1 and V_2; 3-phase variable rheostat R (non-inductive) capable of operating equally on each line (p. 608); source of 3-phase current E; two 3-throw switches $S\,S\,S$ and $S_1\,S_1\,S_1$.

Note.—If the 3-phase rheostat R is not non-inductive, then two additional Wattmeters will be necessary in the secondary

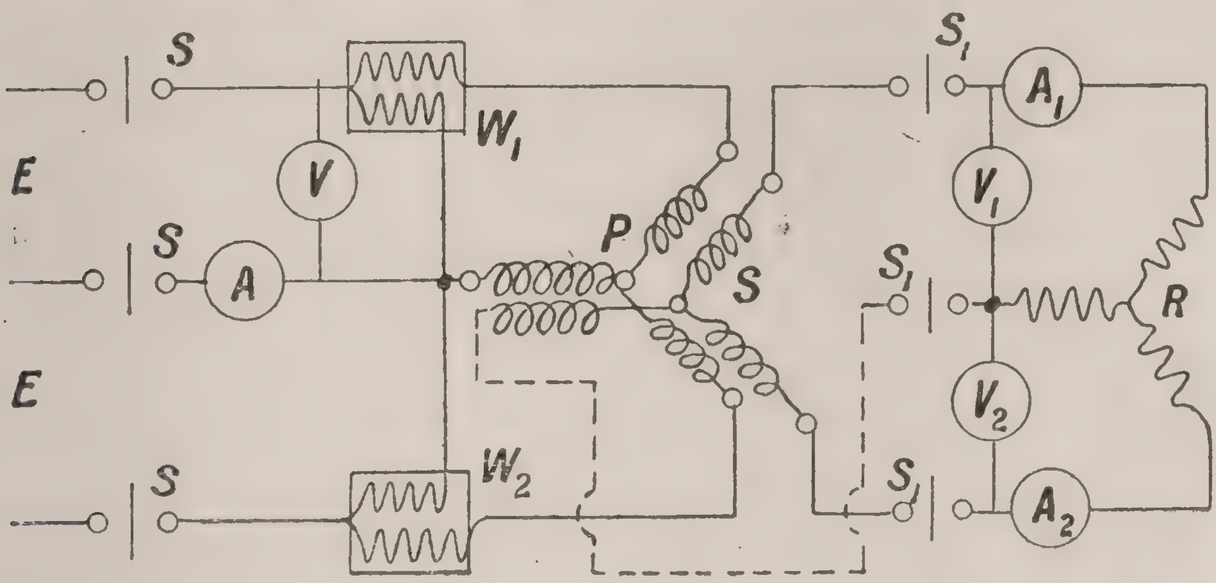


FIG. 165.

circuits connected up in precisely the same way as those shown in the primary circuit, the secondary output being then given by the sum of their readings at any particular load.

Observations.—(1) Connect up as in Fig. 165, and adjust all the instruments to zero, levelling such as require it. See that all lubricating cups in use feed slowly and properly if the source of 3-phase current supply E is controllable.

(2) With $S_1\,S_1\,S_1$ open, close $S\,S\,S$, and adjust the speed of the generator so as to give the proper periodicity for the transformer

and then the excitation, so as to have the desired voltage, shown by V across the primary.

Note the respective Wattmeter readings W_1 and W_2, and if possible that of A in addition to V. Then $(W_1 + W_2)$ = the *no-load primary input* = the magnetizing losses.

(3) With R at its maximum, close S_1 S_1 S_1 and note the readings of all the instruments for some ten or twelve secondary load-currents from the smallest to the maximum permissible, rising by about equal increments at a time for constant secondary voltage.

(4) Calculate the secondary loads (W_S) from the relation—

$$W_S = \sqrt{3}\, A_1 V_1 = \sqrt{3}\, A_2 V_2, \text{ etc.,}$$

and tabulate as follows—

NAME . . . DATE . . .

Transformer tested: No. . . . Type . . . Maker . . .
Normal: Volts . . . Amps. . . . Periodicity . . . Watts . . . Constant of Wattmeter. $\left.\begin{matrix} W_1 = \ldots \\ W_2 = \ldots \end{matrix}\right.$
Resistance: Each Primary coil R_1 = . . . ohms. @ . . . ° C.
" : " Seconding coil R_2 = . . . " @ . . . ° C.

Primary Circuit.				Power Factor $\frac{W_1 + W_2}{\sqrt{3}\, AV}$.	Angle of Lag $\phi°$.	Secondary Circuit.			Total Loss in			Efficiency $\eta = 100 \frac{W_S}{W_P}$ %.
Volts V.	Amps. A.	Apparent Watts $\sqrt{3}\, AV$.	True Watts $W_P = W_1 + W_2$.			Volts V_1 or V_2.	Amps. A_1 or A_2.	Watts $W_S = \sqrt{3}\, A_1 V_1$ or $\sqrt{3}\, A_2 V_2$.	Transformer $W_P - W_S$.	Copper Coils W_C.	Iron Core $W_P - W_S - W_C$.	

(5) Measure the resistance of the transformer coils by means of either the Wheatstone Bridge or Potential Difference method.

(6) Plot the following curves between—

Efficiencies η as ordinates and secondary loads ($\sqrt{3}\, A_1 V_1$) as abscissæ.

Power Factor as ordinates and secondary loads ($\sqrt{3}\, A_1 V_1$) as abscissæ.

Inferences.—State clearly all that can be inferred from your experimental results.

(144) Efficiency of a Nodon Valve Electrolytic Rectifier.

Introduction.—The necessity of obtaining continuous current for certain purposes, such as electrolytic work and the charging of secondary cells, where, frequently, the only available public supply is alternating current, has led to the introduction of rectifiers for rectifying alternating into continuous or unidirectional current.

Of such appliances, there are now several commercially successful forms; that known as the nodon valve consists of as many pairs of cells grouped according to the Leo Gratz method (Fig. 166) as there are phases of current or distributing mains, in order to obtain a single rectified current. Each cell consists of plates formed of an alloy, mainly composed of aluminium, acting as cathode, immersed in a solution of borate or phosphate of ammonium or other salt formed from tartaric, acetic, oxalic or gallic acids. The solution is capable of rapidly altering the condition of the polarizing film formed by an alternating current on the aluminium. The containing cell is made of lead and constitutes the anode. The electrolytic action taking place is as follows—

In one half period of the alternating current, current tends to flow from *aluminium to lead*, but cannot, owing to an insulating film of very high resistances being formed over the aluminium (cathode) plate. In the next half (reversed) period, the current actually is able to flow from *lead to aluminium* owing to the instantaneous de-polarization or reduction of the film on the aluminium plate. The principle on which both semi-waves of the period of an a.c. supply are utilized, is that proposed by Leo Gratz, and shown in Fig. 166, for a single phase alternating—to direct—current transformation. A and L are the aluminium and lead plates respectively of the four cells I, I, and II, II. The continuous arrows represent the direction of current in the valve during one half of a period when the current of the alternating supply flows from P to R. The dotted arrows show the direction of current in the valve in the next half period when the supply current flows in the reverse direction from

R to P. Thus for the first half period it is blocked in cells II, II, and in the second half period it is blocked in cells I, I. A unidirectional current therefore always flows from D to C through any load (r) whether motor, secondary cells or resistance, etc. To obtain greater constancy or uniformity of d.c. voltage a suitable condenser can be connected across D and C. A single valve will stand a.c. pressures up to 140 volts between Q and R, that between D and C being about 90% of this. For higher a.c. pressures two or more valves may be combined, or an "economy coil" type of transformer connected between valve and a.c. supply. The pressure between D and C may be varied to any extent by a corresponding variation of that of the supply between Q and R.

The temperature of the electrolyte must not be allowed to rise much above 50° C., and in large valves, forced air draught around the cells is resorted to in order to keep down the temperature. The valve may be used on any periodicity employed in practice up to 100 ~ per sec. or more. A starting resistance (S) must be employed with the valve when this has been out of use for a few hours, in order to reform the insulating pellicule on the aluminium plate. This only takes a few seconds and prevents a sudden heavy rush of current through the valve. The resistance or inductance of S is cut out entirely afterwards.

Evaporation of the solution is made up by adding distilled water and the solution need only be renewed at long intervals.

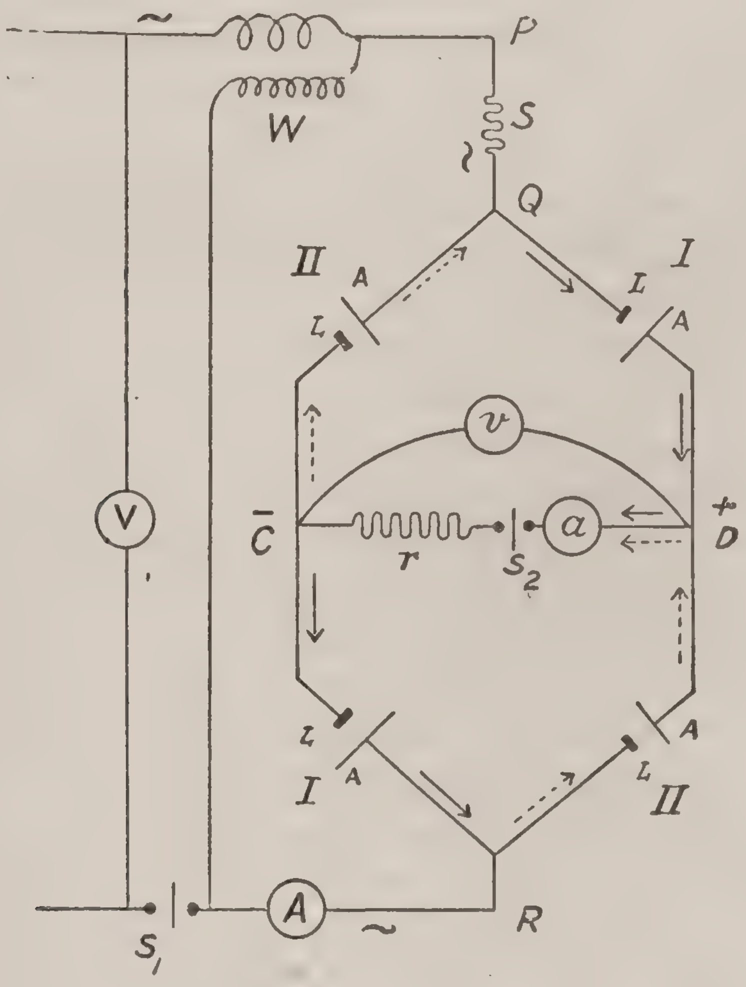
Apparatus.—The nodon valve complete; starting resistance (S); alternating current ammeter (A), voltmeter (V), wattmeter (W); direct current ammeter (a), voltmeter (v); load or variable resistance (r); thermometer; source of a.c. supply; economy coil or transformer if a.c. supply exceeds 140 volts; switches S_1, S_2.

Observations.—(1) Connect up as in Fig. 166, and adjust all the instruments to zero, levelling such as require it. Q and R are the terminals marked **ALT** on the valve, and are to be connected to the a.c. supply: D and C are the terminals marked + and −. If machinery is being run for supplying the valve, see that all oil cups feed very slowly and properly.

(2) With S_1 and S_2 open and **S full in**, adjust the a.c. supply so that V reads about 140 volts, the periodicity being kept constant at normal value. Now close S_1, and note the readings of all the instruments.

(3) With S_2 still open, gradually cut out S to short circuit and again note all instrumental readings and the temperature of the electrolyte.

(4) Re-insert S and with (r) full in, close S_2 and gradually cut out (S) to short circuit. Next adjust (r) so that (a) reads about $\frac{1}{10}$th full output current and note the readings of all the instruments.



Three Phase (Balanced)

FIG. 166.

(5) Re-adjust (r) so as to obtain some ten different load currents on (a) rising by about equal increments to the maximum for which the value is intended, and note the temperature and readings of all instruments at each.

(6) Repeat (5) for a widely different but constant periodicity (if available) above and below normal at the same voltage if possible.

(7) Repeat (5) for a constant supply voltage, say 50% less than before, at normal periodicity.

(8) Open (S_2) and at constant normal periodicity, note the readings of all the instruments for ten different voltages between 0 and 140 volts.

(9) With a convenient constant supply voltage and S_2 open, take readings of all the instruments for ten different periodicities, ranging from the maximum obtainable downwards.

(10) Repeat both (8 and 9) for S_2 closed, constant full load being maintained on (a) by varying (r), and tabulate all results as follows—

NAME . . . DATE . . .
Nodon Valve: No. . . . No. of cells . . .
Area of Anode . . . sq. in. Area of Cathode . . . sq. in.
Maximum output Amperes = . . .

Value of S.	Tempr. of Solu-tion.	Periods per Sec.	Primary.						Secondary.			Voltage Ratio of conver-sion $\frac{v}{V}$	Effici-ency $\frac{av}{W}$
			Volts (V).	Amps. (A).	Watt-meter Read-ing DW.	True Watts W.	Ap-parent Watts AV.	Power Factor $\frac{W}{AV}$	Volts (v).	Amps. (a).	Watts (av).		

Note.—If the valve is cooled by forced air draught, the power absorbed in producing the draught must be added to the true watts (W), or watts (av), according to whether it is supplied by the primary or secondary circuit respectively.

(11) Plot curves on the same sheet, having values of—power factor; volts (v); efficiency; and voltage ratio as ordinates, with secondary load (av) as abscissæ; also between efficiency as ordinates and temperature as abscissæ at constant secondary load.

(12) Plot curves (for Exp. 8 and 10) with voltage as abscissæ and the other quantities as ordinates; also (for Exp. 9 and 10) with periodicity as abscissæ and the other quantities as ordinates.

Inferences.—State clearly all the inferences deducible from experimental results.

(145) Efficiency of a Rotary Rectifier.

Introduction.—Rotary rectifiers are employed for the purpose of rectifying single-phase alternating current into unidirectional or continuous current, and comprise a special form of commutator driven at synchronous speed by a suitable single-phase synchronous a.c. motor.

The now well-known Ferranti rotary rectifier comprises, in addition, a constant-current static transformer, which, when supplied with varying a.c. at constant pressure, automatically delivers constant direct current at varying pressure for supplying arc lamps in series. Since the motor is driven in this case from a separate secondary coil on the static transformer, the ratio of the d.c. power output to the a.c. power intake by the primaries of the transformer gives the overall net efficiency which may be over 91% at a full load of 40 H.P. with a power factor of 0·90.

In the Morton and Wright rotary rectifier there is merely the special commutator and synchronous motor, the rectification being from varying current at constant a.c. voltage to varying current at constant voltage on the d.c. side.

Apparatus.—The rectifier complete comprising—motor M, commutator B, diphaser P and starting switch S; ammeters A_m, A_A and A_D; voltmeters V and V_D; switches S_1, S_m, S_A and S_D, and two-way voltmeter key K; wattmeter W; load absorbing device R_D; and non-inductive regulating resistance r_m (if necessary).

Observations.—(1) Connect up as in Fig. 167. The terminals marked (m) Fig. 167, are for the motor circuit, and those marked DC and AC are the terminals for the direct and alternating currents sides of the rectifying commutator B. Adjust all the instruments to zero, levelling such as require it, and see that the bearings of the rectifier, and those of any other machine in use, are properly lubricated before starting. See also that the two brushes, which rub on the central-sectioned part of the rectifying commutator, are adjusted to touch on the thin strips simultaneously.

(2) With the *switch* (S) *on the stud marked Start*, close S_1 and S_m only, and move the brush rocker on the motor itself, until the machine emits a constant hum and runs quite sparklessly, when it will then be in synchronism with the supply. Now switch S to the right-hand contact and if necessary re-adjust the position of the rocker to obtain sparkless commutation.

(3) With K on V_m, note the readings of W, A_m and V.

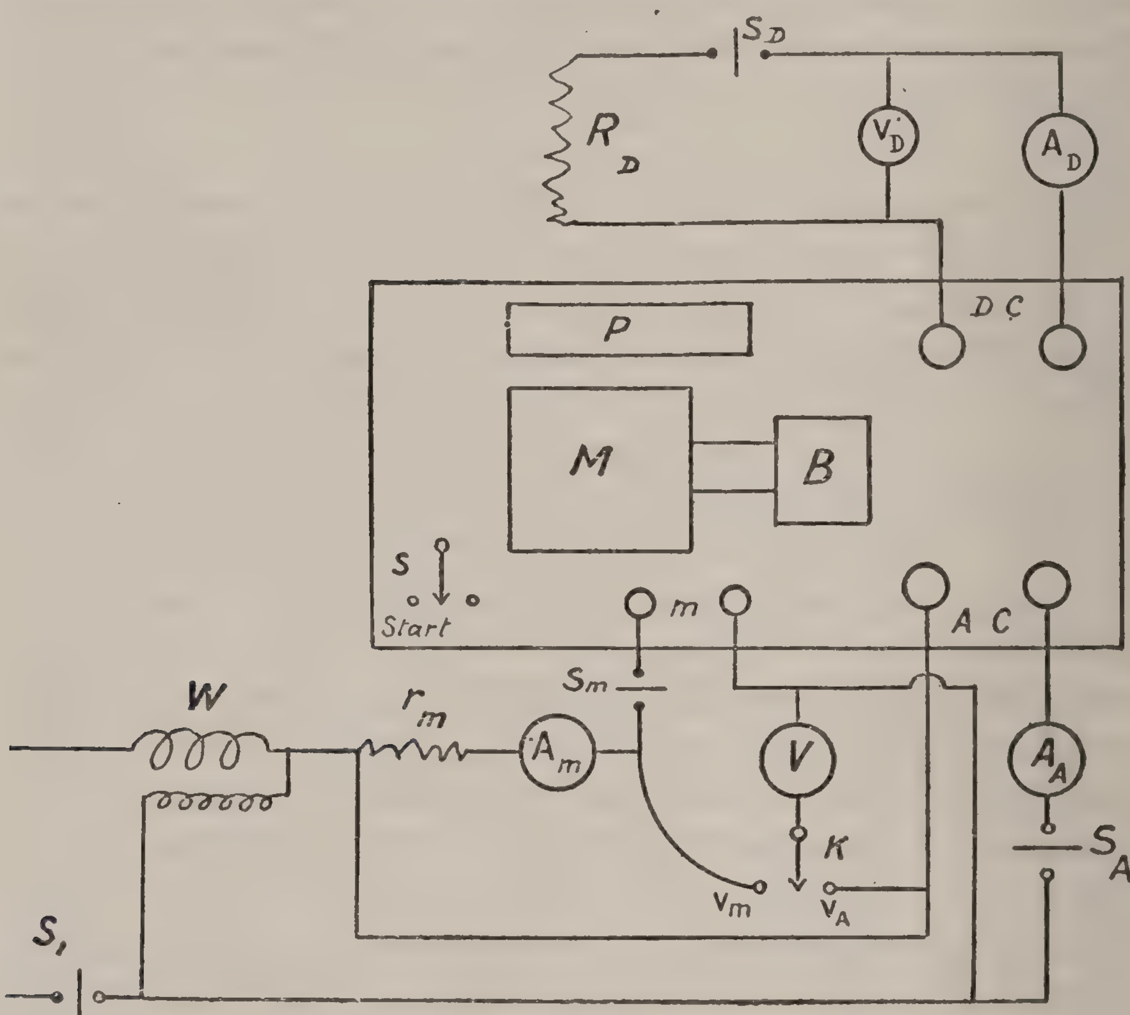


FIG. 167.

(4) With R_D full in and S_D open close S_A and again note W, A_m, V, A_A and V_D, K now being on stud V_A and V_m in quick succession.

(5) Close S_D and note the readings of all the instruments for about ten different currents on A_D, from 0 to full load, by varying R_D; adjusting the brush rocker of the rectifying commutator to get sparkless rectification at all loads.

Tabulate all your results as follows—

Name . . . Date . . .
Rectifier: No. . . . Type . . . Maker . . .
Full-load Output = ... Watts . . . . Volts . . . . Amps.
Value of non-inductive resistance r_m = . . . ohms.

Wattmeter Reading D_W.	True Watts W.	Motor				Volts V_A.	Amps. A_A.	A_AV_A.	$\frac{W-W_m}{A_AV_A}$	$\frac{W}{A_AV_A+A_mV_m}$.	Volts V_D.	Amps. A_D.	Watts W_D.	Overall Efficiency $\frac{A_DV_D}{W-A_m^2r_m}$.	Voltage Ratio of Conversion $\frac{V_D}{V_A}$.
		Amps. A_m.	Volts V_m.	Apparent Watts A_mV_m.	Power Factor $\frac{W_m}{A_mV_m}$.										

(6) Plot the following curves :—between output W_D as abscissæ and volts V_D; watts W; $\frac{W-W_m}{A_AV_A}$; efficiency; and voltage ratio of conversion, as ordinates in each case on the same curve sheet.

Inferences.—State concisely all the inferences which may be deduced from the results of the above tests.

(146) Efficiency and Characteristic of Alternating Current Rotatory Converters. (Run from the Direct Current Side.)

Introduction.—The rapid development of multiphase alternating current machinery, but perhaps more especially of that particular class of the same, known now commonly by the name of the *Rotatory Converter*, marks one important epoch in the history of this all-important and ever-increasing branch of industry—Electrical Engineering. There are several different types of transformers, but all come under one or other of two main heads.

(1) Static transformers or converters with no moving parts.

(2) Rotatory transformers or converters having moving parts, and on which latter their very existence depends. The former of course include the ordinary every-day transformer which we are so accustomed to see.

The type 1 transforms electrical energy of one species at a particular pressure into the same species but at a different pressure, while type 2 transforms electrical energy of one species

into that of another. To this class belong the various forms of multiphase rotatory converters. Those converting from multiphase alternating currents to continuous currents or *vice versâ*, are usually multipolar machines, having any number of pairs of poles up to about 16 or more, with a periodicity ranging from 20 to something like 60 $\sim$ per sec. Owing, however, to the conditions imposed by the relations between voltage, speed and size, they usually operate best at the lower periodicities.

The rotatory converter to be tested consists of an ordinary direct-current machine, with the usual armature winding and its commutator at one end and three slip rings at the other, connected to three points on the armature winding—0, $\frac{2}{3}$ and $\frac{4}{3}$ of the polar pitch apart, *i.e.* in a two-pole machine at 120° apart. The machine when driven as a motor by direct currents taken in at the ordinary commutator end develops a 3-phase alternating current at the slip rings. It is this type of converter which is beginning to be used now on a large scale, only with more than one pair of poles, in long distance transmission of power, as follows—Polyphase alternating currents being transmitted at high pressure from the distant generating station, are reduced to, say, 100 to 300 volts by static transformers at the near end and then converted by the rotatory converter into direct currents, which may be employed for tramway, lighting, electrolytic purposes, or for charging storage cells. In any converting appliance, and therefore in any converter, the (*total energy put in*) – (*total energy given out*) = (*total internal losses*). These are made up of mechanical frictions at journals, brushes, and due to wind or air churning, magnetic hysteresis, eddy currents and copper losses.

Owing to the armature reactions of the dynamo and motor currents practically balancing one another, *no lead* of the brushes in either direction is needed for sparkless running.

A rotary converter is usually run from the A.C. side in practice, but when "inverted," *i.e.* run from the D.C. side, as in the present instance, the nature of the external circuit will have the same effect on its field as it has on that of an A.C. generator, but with additional effects.

Thus on inductive load or power factor less than unity, a leading current will cause an armature reaction which will strengthen the field and hence reduce the speed, while a lagging

current will cause a reaction that will weaken the field and hence increase the speed, in either cause producing a change of frequency.

In fact, the increase of speed may become excessive from either small lagging power factors or short circuit causing excessive weakening of the field, which can only be counteracted by an *increase* in the natural excitation of the field proportional to the effect causing the increase of speed.

Apparatus.—Multiphase converter C to be tested in the present case assumed to be for 3-phase currents; source of direct-current supply E; direct-current ammeters A and a and voltmeter V; alternating current ammeters A_1 A_2 A_3, and voltmeters V_1 V_2 V_3; non-inductive 3-phase rheostat R_1 R_2 R_3, and ordinary ones R

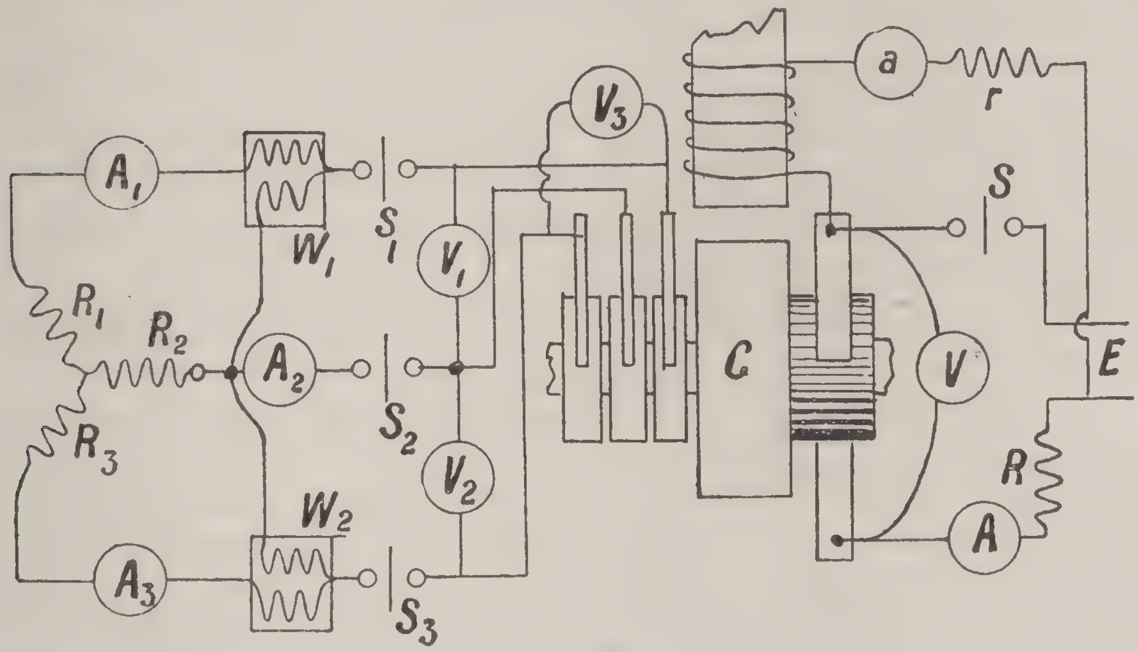


FIG. 168.

(p. 606) and r (p. 599); triple-pole switch S_1 S_2 S_3 and S; non-inductive Wattmeters W_1 and W_2; tachometer.

Note.—Certain pieces of the above apparatus are not absolutely necessary to the test, but when available may preferably be inserted and used so as to clearly show what is actually taking place. Thus if the three resistances R_1 R_2 R_3 constitute a proper 3-phase rheostat (water or otherwise) (p. 607), which *operates equally* on each of the mains, then in addition to apparatus as before we need only have—one single 3-phase rheostat R_1 R_2 R_3; one ammeter A_1 and voltmeter V_1 instead of the three; one 3-throw switch S_1 S_2 S_3, to close the three circuits simultaneously, and one Wattmeter.

The reason for such an alteration is fully described on p. 389, in connection with power measurements in multiphase circuits that are symmetrically loaded. The method or rule for deducing the power absorbed in such cases will be found there, and must then be used. In the present case we will assume that the circuits are not symmetrically loaded.

For a more detailed description of power measurements in multiphase circuits, see p. 388 *et seq.*

Observations.—(1) Connect up as in Fig. 168, and adjust all the instruments to zero, levelling such as require it. See that all lubricating arrangements in use act properly. Increase R_1 R_2 R_3 to a maximum and r to a minimum. See that all the switches are *open* and that the brushes are fixed in the neutral position.

(2) Start C like an ordinary D.C. motor, the speed and volts V being adjusted to, and kept constant at, the *normal* values. Take readings of all the instruments with S_1 S_2 S_3 open, and again with S_1 S_2 S_3 closed, for about ten different load currents on A_1 A_2 A_3 rising by about equal amounts to the maximum permissible by varying R_1 R_2 R_3.

(3) The excitation (a) and volts (V) being now kept constant at normal value, repeat the readings in 2.

(4) The speed and excitation being next kept constant at normal value, repeat 2.

Name . . . **Date . .**

Rotatory Converter: No. . . . Type . . . Maker . . .

" " : Normal output: Volts = . . . Amps. = . . . Speed = . .

" " : Resistances: Shunt Coils r_s = . . . ohms at . . . °C.

Armature ", r_a = . . .

Normal ratio of conversion = . . .

Periods per revol^n. K = . . .

Speed Revs. per min. N.	Frequency (n) ∼ per sec. $\frac{KN}{60}$.	Amps. (a).	Amps. A.	Volts V.	Volts.			Amps.			True Watts.		Total H.P.		Total Losses		Conversion Ratio $100\frac{V_1}{V}\%$.	Efficiency of Converter $\Sigma = 100\frac{H_2}{H_1}$.	Power Factor Cos. ϕ $= \frac{W_1+W_2}{\sqrt{3}\,A_1V_1}$.
					V_1.	V_2.	V_3.	A_1.	A_2.	A_3.	W_1.	W_2.	put in $H_1 = \frac{AV + a^2r_s}{746}$.	given out $H_2 = \frac{W_1+W_2}{746}$.	in Converter $H_1 - H_2$.	used as D.-C. Dynamo.			

(5) Repeat 2–4 for a highly inductive circuit R_1 R_2 R_3.

(6) From observations 2–5 plot the efficiency curve having H_2 as abscissæ and Σ as ordinates.

The external or *a-c* characteristic with V_1 as ordinates and A_1 as abscissæ.

The current characteristic with A as ordinates and A_1 as abscissæ.

The voltage ratio, input, and speed as ordinates and H_2 as abscissæ.

From the current characteristic indicate how the efficiency of the converter could be calculated at any load and also work out the ratio of current transformation.

Inferences.—State very clearly all that can be deduced from your experimental results.

Note.—If n = number of armature windings per radian,

e = maximum E.M.F. per turn of winding,

and if we *assume* the flux in the interpolar space to be sinusoidally distributed, and that the E.M.F. is a sine function of the time, then the voltage ratio of conversion with 3-phase connections

$$= \frac{\sqrt{\frac{3}{2}}\,ne}{2\,ne} = \tfrac{1}{2}\sqrt{\tfrac{3}{2}}$$

and the virtual voltage across any pair of slip collector rings will $= 61{\cdot}23$ when that impressed on the direct current side $= 100$ volts.

In other words, the voltage ratio of conversion *with sine distribution* of flux in the interpolar space

$$= 61{\cdot}23\%.$$

As, however, the flux is never so distributed, and, moreover, as the voltage ratio depends to a large extent on the polar arc, pole, shape and position of the brushes, the above ratio is only roughly about what may be expected.

The C^2R total loss and temperature rise will be less in the machine used as a converter than when used as a dynamo.

(147) No-Load (open circuit) Characteristic or Magnetization Curve of Continuous-Alternating Current Rotary Converters. (Run from the Continuous Current Side.)

The No-Load Characteristic or curve of magnetization of a converter from which its magnetic properties and most suitable excitation is seen can be obtained in one of the two following ways—

(1) By driving the rotary at *constant speed* from a direct coupled motor, or by belting, and noting the readings of the voltmeters across the d.c. and a c. sides respectively for each of some ten values of exciting current obtained from some outside d.c. supply, and differing by about equal amounts from 0 to say 25% above normal excitation and taking a similar descending set of readings.

(2) By connecting up exactly as in Fig. 168 and driving the rotary as a motor from its d.c. side.

With the field current (a) adjusted to say 20 % above its normal value (if possible) start the rotary up in the usual way to maximum speed obtainable with this excitation, and R cut out. Note the readings of speed (to insure constancy throughout) and both a.c. and d.c. voltages for this maximum excitation, and for each of about ten smaller values obtained by increasing r and differing by about equal amounts down to the minimum practicable, the speed being kept at the same constant value by increasing R, and tabulate as on page 436. Plot the magnetization curve, having exciting currents as abscissæ, and the a.c. and d.c. voltages as ordinates respectively.

Deduce a third curve by joining the points obtained on deducting the armature drop (= current $\times$ its resistance) from each of the d.c. voltage ordinates.

Plot also a curve having a.c. volts as ordinates with d.c. volts as abscissæ and deduce the voltage ratio of conversion. Compare this ratio with the theoretical value and explain the reason for any difference.

(148) Effect of Variation of (1) Excitation (2) Speed on the Voltage Ratio of a Rotary Converter (Run from the Direct Current Side.)

Observations.—(1) With exactly the same connections as in Fig. 168, and with S_1, S_2, S_3 open, note the readings of all the instruments for constant speed throughout, for about eight different values of exciting current (a) differing by about equal amounts between the minimum and maximum values possible.

(2) Repeat (1) with S_1, S_2, S_3 closed, and at $\frac{1}{4}$, $\frac{1}{2}$ and full load respectively, the load being kept constant by varying R_1, R_2, R_3.

(3) Repeat 1 and 2 for a similar variation of speed at constant normal excitation and tabulate as on p. 436.

Plot curves for Tests 1 and 2 at each load having exciting currents (a) as abscissæ, and (1) a.c. volts, (2) d.c. amps, (3) voltage ratio of conversion, as ordinates in each case, and for tests (3) curves having speeds as abscissæ with voltage ratio; a.c. volts and d.c. amps. as ordinates.

Inferences.—State concisely all that can be deduced from the results of your investigations in the present test.

(149) Efficiency and Characteristics of Alternating-Continuous Current Rotary Converters. (Run from the Alternating Current Side.)

Introduction.—The investigations to be made in the present case are all the more important and instructive because the usual application of this kind of rotary, commercially, is to convert a.c. to d.c., the machine being supplied with a.c. and running as a synchronous a.c. motor. The speed at which it runs will therefore solely depend on the number of poles in the field, and on the periodicity of the a.c. supply, and if this latter is constant the speed will be also, irrespective of the load developed at the d.c. side, or of the excitation. As the load increases, for constant power factor, a.c. voltage, and frequency, "armature reaction" causes an increasing drop of d.c. voltage, and, further, a decrease of excitation is necessary to maintain constant power factor, but

an increased variation of d.c. voltage results. On the other hand, a constant excitation increases the intake current, but decreases the variation of d.c. voltage. By varying the excitation to maintain constant d.c. voltage, this latter, and also the efficiency, is increased. In order to minimize the variation of d.c. voltage, and maintain a constant voltage ratio as the load changes, rotaries are usually compound wound, the series coils, just as in the case of an ordinary compound dynamo, producing an increase of excitation proportional to the load, and simultaneously, the necessary change of power factor. A rotary can be over-compounded so as to give an increasing d.c. voltage with load to make up for loss of voltage in transmission, in which case unit power factor is obtained by field regulation at some fraction of full load, thereby giving leading currents at full load. On a light constant load, a given variation in the excitation causes a much greater change in the current intake and power factor than it would do on a heavy constant load. Since the lost armature volts in a rotary run from its a.c. side = (current × armature impedance), while when run from its d.c. side this quantity = current × armature resistance, and also owing to the power factor not being unity; to the wave-forms of a.c. supply, and of the rotary (run from the d.c. side) being different; and to armature reaction, the voltage ratio of conversion will be different when run from the d.c. and a.c. sides. Further, when the power factor of the circuit is low and the current comparatively large and lagging, the supply and lost armature volts will be more nearly opposite in phase, and hence a larger armature drop results at the small excitations. A rotary has unit power factor at a particular excitation, also too low an excitation causes the current to lag, while too high an excitation causes it to lead as shown by the V curves between excitation and power factor obtained in the above investigation. Constant d.c. voltage at all loads can be maintained by adjusting the excitation to give unit power factor at full load.

Apparatus.—Precisely that prescribed for Test 146, except that the a.c. supply is substituted for, and takes the place of, R_1, R_2, R_3, and that some form of synchronizer is needed. If the phases are equally balanced, or if all the instruments on the a.c. side are unavailable, then any two ammeters such as A_1 and A_3 and any two voltmeters such as V_1, V_8 may be omitted. Also one

of the two wattmeters shown might be omitted, means being provided by a two-way key for connecting one end of the fine wire coil of the wattmeter used to the remaining two supply mains in quick succession. It will be noticed that a three-phase rotary is assumed for the test, but the same considerations and investigations would apply to single- and two-phase rotaries.

Connections.—To be as shown in Fig. 168, unless modified by a reduction in the number of a.c. instruments as mentioned above. The simplest and most convenient form of synchronizer to employ consists of two ordinary glow lamps supported in two ordinary bayonet holders connected in series and carried on a base board with two terminals. The sum of the voltages marked on the lamp bulbs *must not be less than* the sum of the supply and converter a.c. voltages. The two terminals of this "lamp synchronizer" may be connected so as to short circuit S_1, say a piece of thin wire short circuiting, say, S_2.

The Process of Synchronizing the rotary with the a.c. supply is usually most conveniently accomplished as follows—

(*a*) Adjust all instruments which need it, and with S_1, S_2, S_3 open, start the rotary up as a d.c. motor from a d.c. supply E, by closing S, and operating the starter (not shown), with which the machine is provided, in the usual way.

N.B.—If there is no starter, the adjustable load resistance R may be used for starting up.

(*b*) Adjust R so as to obtain the *same voltage* on V_2 as that of the main a.c. supply, and then adjust r to give such a speed that the lamps go out. The a.c. supply and a.c. voltage of the rotary are now *equal* to, and *opposing*, one another, and of the same frequency, and hence in synchronism. The switch $S_1S_2S_3$ must now be closed, and S at once opened, when the rotary will continue to run, now as a self-exciting three-phase synchronous a.c. motor.

It may be mentioned that the a.c. voltages of the main supply and of the rotary are in assisting series when the lamps show steady luminosity, while between this and the "quite out" condition they pulsate in brightness due to the current pulses of the two E.M.F.s trying to catch up to one another.

Of course, a separate motor, or driving source, if available, might be used to run the rotary up to synchronous speed instead

of the d.c. supply above named. If the d.c. supply used for starting-up purposes is unsuitable for giving the necessary a.c. voltage on the rotary at the required speed, the rotary may be run up to a much higher speed than that of synchronism, S then being opened, and afterwards $S_1 S_2 S_3$ closed, at the moment when the speed falls to such a value that the lamps go out.

Note.—If an outside d.c. supply is used in synchronizing, then directly the rotary is running synchronously on the a.c. supply and S is opened, *at once disconnect the mains E from the auxiliary d.c. supply,* and connect them together to avoid the possibility of a future mishap by forgetting to do this at the time. Several investigations on the operation of the converter under different conditions can now be undertaken.

Effect of variation of Direct Current Load on the operation of the Converter at Constant Direct Current Voltage and Excitation, and Alternate Current Frequency.

Observations.—(1) Adjust the exciting current (a) by means of (r) until the intake a.c is a minimum, then with S still open, note the readings of all the instruments and the speeds of the rotary and generator (giving the main a.c. supply) respectively for normal frequency.

(2) Close S and note the readings of all the instruments and the speed, for about eight different d.c. loads rising by about equal amounts between 0 and full load, by varying R; keeping the current (a), the supply frequency, and the d.c. voltage (V) (by varying the excitation of the main generator), constant throughout.

(3) Repeat (1 and 2) above for a *lower* and also for a *higher constant excitation* than that previously found in Test 1 above, and tabulate your results as shown in the table.

(4) Plot curves between d.c. output in amps. A as abscissæ, and (a) efficiency Σ, (b) power factor, (c) mean intake a.c. amperes, (d) mean a.c. volts, as ordinates in each case.

(150) Effect of variation of Direct Current Load on the operation of the Converter at Constant Alternate Current Voltage and Frequency, and Direct Current Excitation.

Observations.—(1) With constant normal a.c. voltage and frequency maintained throughout, repeat (1—4) Test 149 above, plotting for (4*d*) above the d.c. volts instead of mean a.c. volts as ordinates. The curve between *V* and *A* is called the d.c. characteristic of the rotary.

(151) Effect of variation of Excitation on the operation of a Converter at Constant Alternate Current Voltage and Frequency, and Direct Current Load ("V" curves).

Observations.—(1) With *S* open, and the converter running at constant normal a.c. voltage and frequency, note the readings of all the instruments, and the speeds of the main generator and rotary for about eight different exciting currents (*a*) between the lowest and highest permissible by altering (*r*).

(2) Close (*S*) and repeat (1) for constant loads of about $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and full d.c. load and tabulate as indicated.

(3) Plot curves between d.c. amperes of excitation as abscissæ and (*a*) power factor, (*b*) intake alternating current, (*c*) voltage ratio of conversion, (*d*) intake a.c. Watts.

(152) Variation of Excitation to Maintain Maximum Power Factor for Varying Direct Current Load at Constant Alternate Current Voltage and Frequency.

Observations.—(1) With *S* open, and the rotary running at constant normal a.c. voltage and frequency, adjust the excitation (*a*) so as to obtain minimum intake current, and note the readings

of all the instruments and the speeds of main generator and rotary.

(2) Close S and note the value of the exciting current (a) necessary to give *minimum intake current* at about 8 different d.c. loads between 0 and the maximum (by altering R), the a.c. voltage and frequency being the same at each load. Note all the other instrumental readings and tabulate as indicated.

(3) Plot curves between d.c. load in watts as abscissæ and (a) d.c. volts, (b) exciting current, (c) a.c. amperes, (d) power factor, (e) voltage ratio of conversion, (f) efficiency.

Name . . . Date . . .

Rotary: No. . . . Type . . . Maker . .

d.c. Side: Volts = . . . Amps. = . . . Speed = . . .

a.c. Side: Volts = . . . Amps. = . . . Wattmeter Constants K_1 = . . . ; K_2 = . . .

Resistances:—Armature r_a = . . . Shunt r_{sh} = . . . Series r_{se} = . . .

Speed of		Main Alternating Current Supply.								
			Currents.			Voltages.			Wattmeters.	
Generator.	Rotary.	Frequency.	A_1.	A_2.	A_3.	V_1.	V_2.	V_3.	W_1.	W_2.

Total Watt Input.		Power Factor	Direct Current Output.					Nett useful Efficiency	Mean Voltage Ratio of Conversion	Gross Efficiency
			Amps in							
True $K_1W_1+K_2W_2 = W$.	Apparent $\sqrt{3}\,A_1V_1$.	$\cos.\theta = \frac{W}{\sqrt{3}\,A_1V_1}$.	Shunt a.	Main A.	Volts V.	Watts $w = AV$.	Lost Excitation Watts W_F.	$\frac{AV}{W} = \Sigma$.	$\frac{V_1+V_2+V_3}{3V}$.	$\frac{w+W_F}{W}$.

Inferences.—Very carefully consider and state all the inferences which can be deduced from the results of your investigations.

(153) Efficiency and Output of a "Booster" or of a "Motor Generator Set."

Introduction.—It frequently happens in practice that either electrical power in one form at a certain pressure is required in the same form but at a different pressure, or that electrical power of one nature is required in quite a different nature at the same pressure or otherwise. The electrical appliance by means of which such transformations can be effected is variously termed a

"Motor Generator," "Booster," "Rotatory Converter," "Continuous Current Transformer," etc.

In all these appliances the desired effect is produced by machinery in motion, and only so long as it is in motion. The motor-driven Booster at the present day is essentially a device for transforming direct-current energy from one pressure to another. Speaking in general terms a Booster is a machine for adding a small percentage of E.M.F. to a large generator and is much used in storage battery systems. The Motor Generator and Converter very frequently constitute a device for transforming electrical energy in the form of direct currents into that of the

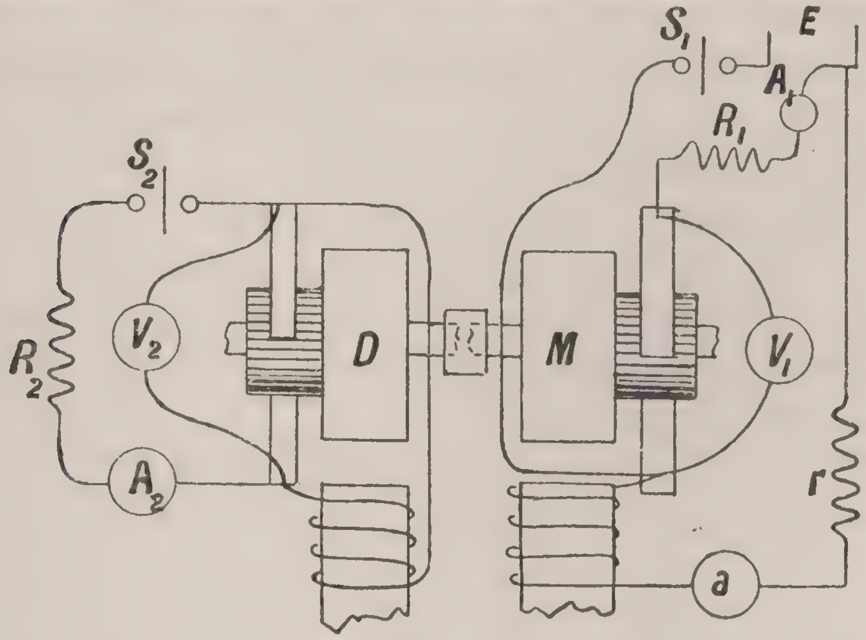


FIG. 169.

form of single and multiphase alternating currents or *vice versâ*. The motor generator frequently takes the form of two separate machines, on the same bed plate, with their shafts in alignment and coupled mechanically. One machine M constitutes an electromotor, fed from an external source of electric supply, the other is a generator D which is driven direct by the motor and develops electrical power.

As machines of this nature are used to some considerable extent as "regulators" to "feeder mains," and also with some slight constructional difference as "equalizers" on the various systems of parallel distribution, the determination of their efficiency and output becomes of considerable importance.

Apparatus.—Motor generator MD; source of electrical energy E; ammeters A_1 A_2 and a; voltmeters V_1 and V_2; rheostats R_1 R_2 (p. 606) and r (p. 599); switches S_1 and S_2; tachometer.

Note.—Prior to starting, all lubricators must be seen to feed properly.

Observations.—(1) Connect up as in Fig. 169, and adjust the pointers of all the instruments to zero. Increase R_1 R_2 to their maximum and r to a minimum. See that S_1 S_2 are open and both sets of brushes down.

(2) Close S_1 and adjust R_1 so that the normal speed N for the particular "set" tested is obtained. Note simultaneously the readings of A_1, a, V_1, V_2 and N.

(3) Close S_2 and adjust R_1 R_2 so that A_2 reads about $\frac{1}{10}$ full load current in amperes, N being the same as before. Again read all the instruments.

N.B.—It may be found necessary to vary the excitation of M by means of the resistance r in order to keep the speed constant throughout any one set of readings.

(4) Repeat 3 for about ten different load currents A_2 up to the maximum, rising by about equal increments at a time.

(5) Repeat 3 and 4 for speeds 20% above and 50% below normal, and tabulate your results as follows—

NAME . . . DATE . . .

Maker of Motor Generator . . . Resistance of Shunt Coil of M (r_s) = . . . ohms.
Number of " . . . Type . . . Normal Output = . . . Watts. at . . . revs. per min.

Speed N.	Shunt Current a.	Watts used in Shunt $a^2 r_s$.	Current A_1 Amps.	Voltage V_1.	Watts A_1V_1.	Total Power absorbed by Motor $W_1 = A_1V_1 + a^2r_s$.	Current A_2 Amps.	Voltage V_2.	Nett Output of D in Watts $W_2 = A_2V_2$.	Efficiency of Motor-generator $\Sigma = \frac{W_2}{W_1} \times 100\%$.

(6) Plot an efficiency curve for each speed having W_2 as abscissæ and Σ as ordinates. Also curves having W_1 as ordinates.

Inferences.—State clearly all the inferences which you can draw from the results of your experiment. Could the combined efficiency be increased by any structural alterations in the "set" tested?

(154) Determination of the Periodic E.M.F. and Current Curves of an Alternator.

Introduction.—It may sometimes be desirable to determine the periodic curve or wave of E.M.F. and current in an alter-

nating current circuit, for the shape of such curves has an important influence on the losses occurring in the iron cores of any appliances in the circuit. In fact, the more peaked the E.M.F. curve, or the more nearly it approximates to a sine or even a triangular curve, the less will be the losses occurring in such appliances and the greater will be what is called the "*form factor*."

Two cases may arise in which it is desired to obtain the periodic curves, namely (1) when the alternator supplying the circuit is at a long distance away, and consequently inaccessible in a sense, (2) when the test can be applied close to the alternator, if necessary. In either case some convenient form of rotating contact maker must be used to close the circuit of a suitable measuring arrangement for an instant once every revolution, at any definite point in the period of alternation, corresponding to the position of the brush or contact arm. Hence by moving this contact arm into various angular positions, the periodic curve of instantaneous values of varying E.M.F. and current at different instants can be obtained throughout the whole period or wave. Such a contact maker is illustrated and described in the Appendix, p. 619, and in case 1 above it is fitted to and driven by a synchronous alternating current electromotor run off the supply of which the E.M.F. or current curve is desired. Such a motor always runs at a speed bearing a definite and fixed ratio to the periodicity of the supply current. In case 2, the one we shall here consider, the contact maker is fixed to, and is driven by, the rotating portion of the alternator itself.

Knowing then the periodic curves of E.M.F. and current in a circuit together with their *phase difference*, at once seen from the relative positions of the two curves, the true instantaneous power developed in that alternating current circuit can at once be deduced.

The following method of obtaining such curves consists in charging a condenser to a certain E.M.F. by periodically connecting it, by means of the contact breaker, to the alternating circuit to be tested, and measuring this E.M.F. by an electrostatic voltmeter. This will therefore be the instantaneous value of the potential at the point of the period of alternation corresponding to the position of contact of the brush on the contact maker.

The function of the condenser is to maintain the instantaneous voltage at a uniform value and so insure a steady reading on the voltmeter, notwithstanding the leakage usually occurring from it, in the interval between successive contacts.

If a Kelvin multicellular voltmeter (preferably dead beat type) is used for the test it will be necessary to obtain a false zero of about 30 on its scale, as the one volt graduations only commence from this point upwards. For this purpose an auxiliary voltage of about 30, which can be supplied by a battery of about 15 small secondary cells, may be used. As the circuit is closed by the contact maker for a small fraction of a second only, the condenser should have a small capacity, otherwise it may not be fully charged. Even though V and C are well insulated a certain amount of leakage may occur, depending on the rate of contact—*i.e.* on the speed of D. A known steady P.D. should therefore be applied in place of the alternator P.D., and the speed adjusted to the value above, then if there is leakage V will read differently when the contact is stationary and when periodic; the ratio of these two $\times V_1$ gives the instantaneous P.D. In the determination of the current curve a low non-inductive resistance should be used, so that its introduction may not affect the existing conditions of the circuit to any appreciable extent. The P.D.s in this case will be small, and can be measured by the ordinary "Null deflection," or balance method on a potentiometer. The amperes per scale division can be found by passing a "known steady direct current" through (r) from a supply in place of D, the rate of contact being as above; then known current $\div$ scale reading = amperes per div.

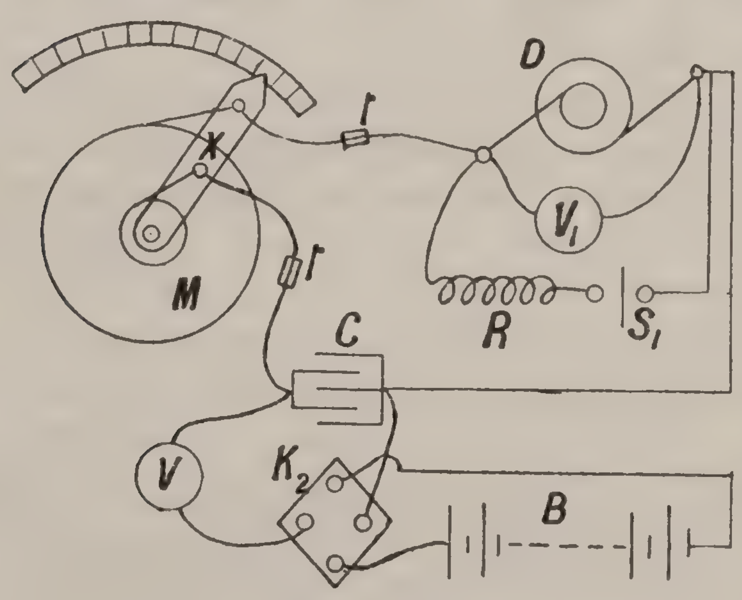


Fig. 170.

Apparatus.—Alternator D with its exciting circuit; contact maker M (p. 619); well-insulated electrostatic voltmeter V (p. 563), and also $\frac{1}{4}$ to $\frac{1}{3}$ *m.f.d.* condenser C; well-insulated battery B; A.C. voltmeter V_1; A.C.

ammeter A; switches S and S_1; load resistance R, in which the current wave is to be found; non-inductive resistance r, of such a size as will carry the current without sensible heating; sensitive H.R. galvanometer (p. 569); potentiometer PQ (Fig. 171), with its slider key K_1; E.M.F.—E (2 volts), and reversing key K_2 (p. 585).

Note.—All lubricating arrangements must be seen to feed properly when the machines are started.

E.M.F. CURVE.—*Experiments*—

(1) Connect up as in Fig. 170. Set X to 0 on its scale and measure the auxiliary voltage (v) on V by short-circuiting C by a wire and afterward removing this.

(2) Start D and adjust its speed to the normal value, say, also its excitation so as to give about normal voltage on the voltmeter V_1.

(3) With the speed and terminal P.D. constant, note the *steady* reading on V, which therefore is a measure of the instantaneous E.M.F. of D for this position of X.

(4) Repeat 3 every 20° from 0 throughout the period of alternation (360°) by moving X, noting the points when K_2 is used to reverse the E.M.F. of B.

(5) Repeat 2–4 when D is giving a convenient constant current through a suitable *inductive* circuit R.

CURRENT CURVE.—*Experiments*—

(1) Connect up as in Fig. 171. Set X to 0 and repeat 2 above.

(2) With R at its maximum, close S and adjust the current to the same value as mentioned in 5 above, by means of the exciting current.

(3) With the speed normal and terminal P.D. constant, obtain balance with K_1 so that, on pressing it, no deflection occurs on G. Note the scale reading (d) of K_1, which

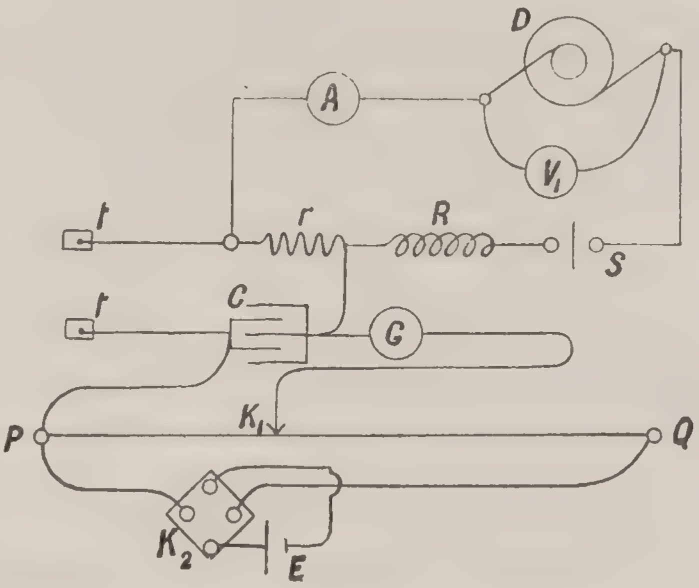


FIG. 171.

therefore is a measure of the instantaneous P.D. at the resistance r.

(4) Repeat 4 above, and tabulate your results as follows—

NAME . . . DATE . . .

Speed of D = . . . Revs. per min. Frequency = . . . ∼ per sec.
Capacity of C = . . . *m.f.d.s.* Aux. E.M.F. v = . . . volts.
Non-ind. Resistance r = . . . ohms.
Reading of voltmeter V_1 = . . volts Volts per div. of P.Q. (α) = . .
,, ammeter A = . . . amps. Amps. ,, ,, (β) = . . .

Reading of X = Phase Angle $\theta°$.	Total E.M.F. V.	Nett Inst. E.M.F. $V_I = V - v$.	Reading of P.Q. (d).	Actual Inst. E.M.F. at r. (αd).	Actual Inst. Current through r. $A_I = \frac{\alpha d}{r} = \beta d$.	Angle of Phase between V_I and A_I.

Note.—2 v must either be added or subtracted, *on reversing the battery switch*, according to whether (v) was previously in helping or opposing series. Half the difference between the readings = v.

It will be obvious that the instantaneous terminal voltage of D can be obtained in addition to the current, if in Fig. 171 E has a value at least = the maximum value of that voltage ; PQ has a high resistance, and a two-way key be used in place of the permanent connecting wire between C and r, so that it will connect C successively to (r) as shown, and to the junction of R and S, which latter gives the terminal instantaneous voltage.

(5) Plot the E.M.F. and current curves on the same curve sheet, having $\theta°$ as abscissæ in each case and V_1 and A_1 as ordinates respectively, and calculate out from them the $\sqrt{\text{mean square}}$ of the instantaneous values of V and A.

In determining the periodic curves of E.M.F. in a high tension circuit a potentiometer arrangement should be used ; the wires going to D (Fig. 170) being connected to the ends of a suitable known fraction of a known high resistance placed across the mains, and which can carry an appreciable current, say $\frac{1}{4}$ to $\frac{1}{2}$ an ampere, thus illuminating any error arising from the capacity current of the voltmeter.

(155) Determination of the Periodic E.M.F. and Current Curves of an Alternating Current Circuit. (Ballistically.)

Introduction.—Should an electrostatic voltmeter not be available as in the preceding method, a reflecting ballistic galvanometer

G may be used, and it should preferably be of the moving coil D'Arsonval type, so as not to be affected by the stray magnetic fields invariably met with in a dynamo-room. In all other respects the present method is precisely similar to the last. K_3 is a reversing key (Fig. 255) for obtaining deflections each side of zero. When the two-way key K (Fig. 256) is put to stud 1 the condenser C is charged to an E.M.F. corresponding to that for the position of the contact arm X in the period of alternation. On putting K to 2, C is discharged through G, and since the resulting throw is $\propto$ the quantity flowing out of C, which in turn is $\propto C \times V$, where C = capacity of the condenser and V the E.M.F. to which it is charged, we see that the resulting momentary throw on $G \propto V$, since the capacity is a constant.

This should be larger the smaller the charging E.M.F. to be measured The current curve is obtained in a precisely similar manner to that indicated in Fig. 171.

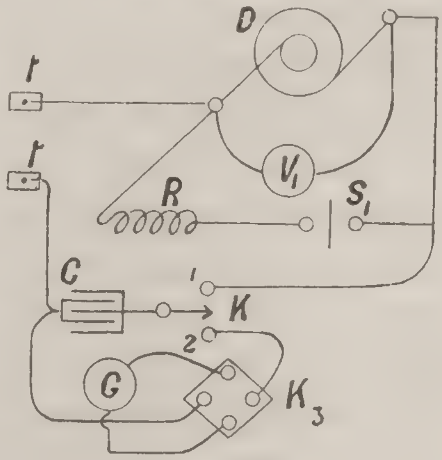


FIG. 172.

(156) Delineation of Wave-Forms by means of the Duddell Oscillograph.

Introduction.—The shape of wave-forms in general, but perhaps more especially those of current and pressure in alternating current circuits, are of the utmost importance to electrical engineers. For instance the efficiency of transformers and a.c. motors, and even the working of the latter, is in some cases seriously affected by the wave-form of the supply. On the other hand, the optical efficiency of the a.c. arc has been found to be 44% higher with a flat-topped E.M.F. curve than with a peaked curve, while transformers work most efficiently on peaked curves. Again the wave-form reveals the presence or otherwise of higher harmonics due to accidental though often avoidable resonance effects, so dangerous in causing the breakdown of the insulation of high-tension electric cables.

The oscillograph itself consists of a highly specialized reflecting d'Arsonval galvanometer having an extremely small periodic

time, when undamped, of from $\frac{1}{1800}$th to $\frac{1}{10000}$th of a second, depending on the type of instrument.

For all ordinary frequencies the oscillograph is perfectly dead-beat, absolutely free from hysteretic errors, and has practically no self-induction or capacity.

It is therefore an accurate instantaneous ammeter or voltmeter capable of giving a deflection which is at any moment accurately proportional to the instantaneous value of the variable even with frequencies of 300 or more periods per second on any wave-form whether periodic irregular, or non-periodic and whether continuous or alternating.

Thus in addition to recording E.M.F. and current waves, such an instrument will indicate the charge and discharge curves of condensers, the changes of P.D. and current on breaking an inductive circuit, the P.D. and current changes in the armature coils of a dynamo or motor, or in the primary of an induction coil or even the very rapid changes when the d.c. arc hisses. Doubtless it will be employed for a vast number of other determinations as time and necessity arise, but the preceding merely serve to indicate some of the uses to which this highly important instrument has already been put.

Construction of the Duddell Oscillographs.

The apparatus consists of the galvanometer, combined either with a rotating or vibrating mirror, moving photographic film, or falling photographic plate.

Fig. 173 is a diagrammatic view of the galvanometer part of the instrument showing the principle on which it works. In the narrow gap between the poles N, S of a powerful magnet are stretched two parallel conductors s, s formed by bending a strip of phosphor-bronze back on itself over the pulley P which is attached to a light spring balance. At the bottom ends the strips are clamped on a block, K, while at the top they are held in position by the bridge piece L. By altering the tension on the spring stretching the phosphor-bronze loop, the periodicity of the instrument can be varied at will. Each strip or leg of the loop passes through a separate gap (not shown) in the magnetic circuit. The clearance between the sides of the

gaps and the moving strip is but 0·38 mm., and these gaps are filled with a viscous oil, over which is placed a small lens, which is held in position entirely by the surface tension of the oil, and serves in its turn to keep the oil in place. The object of the oil is to damp the movements of the strips. A small mirror

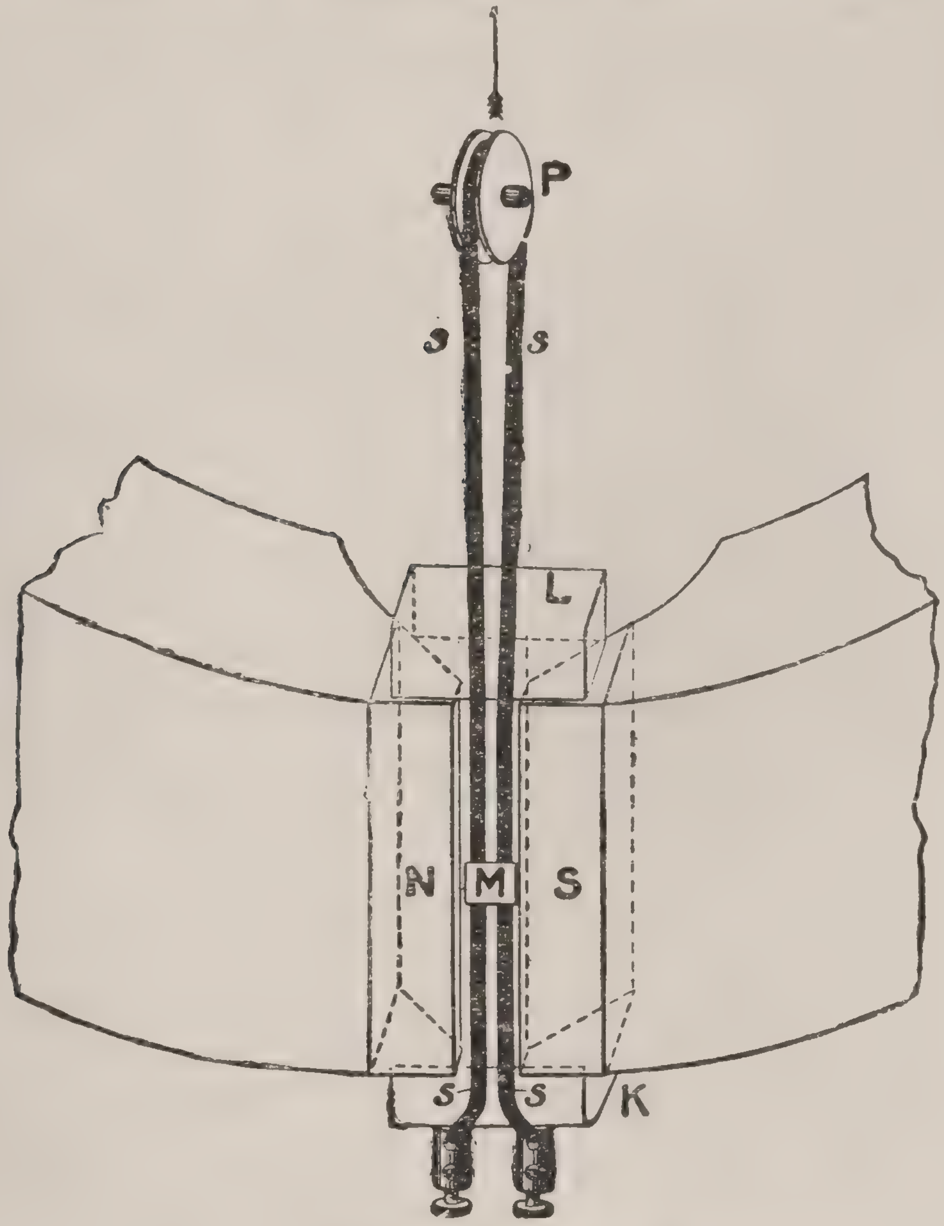


Fig. 173.—Essentials of the Vibrating System.

marked M is attached to the loop, as shown. The effect of passing a current through one of these loops is to cause one leg of it to advance whilst the other recedes, and the mirror is thus turned about a vertical axis. In the high frequency instrument the natural period of vibration of the loop is $\frac{1}{10000}$th of a second, and the clearances being, as stated, extremely small, the damping effect of the oil is so great, that the instrument can be relied

upon to give accurate results even when the periodicity of the current to be tested is over 300 periods per second. Small fuses below the loops protect these from injury in case of accidental excessive current. The fuses consist of very fine wires enclosed in glass tubes, which are held in position by spring clamps.

The beam of light reflected from the mirror M is received on a screen or photographic plate, the instantaneous value of the current being proportional to the linear displacement of the spot of light so formed. With alternating currents the spot of light oscillates to and fro as the current varies and would thus trace a straight line. Hence to obtain an image of the wave-form, it is necessary to traverse the photographic plate or film in a direction at right angles to the direction of movement of the spot of light. A second mirror can be interposed in the path of the beam of light, and this mirror caused to vibrate or rotate so as to impart to the beam of light a uniform motion proportional to time in a plane at right angles to the plane of vibration of the beam due to the current. The spot of light will now trace out on a stationary screen or plate the time curve of the variation of the P.D. or current as the case may be. If the variations are periodic, as in alternating currents, then the second mirror can be synchronized and the spot of light caused to trace out the wave-form over and over again.

The various methods of examining and recording the wave-forms will be described later.

The Oscillograph is provided with an adjustment for slightly increasing the periodic time and sensibility. This may be done by altering the tension of the strips. This is not advisable, however, as it is liable to spoil the definition of the spots.

Four standard types of Duddell Oscillograph are made and are in common use, namely—

Type I. **The Double High Frequency Oscillograph**, which is the most sensitive type and has a powerful electro magnetic field. The magnetizing coils are wound in eight sections, and by suitably connecting them the field can be excited direct from any voltage between 25 and 100 volts or from 200 volts with an 8-c.p. lamp in series. The magnetic circuit being saturated, a change of 4% up or down in exciting current only changes the sensibility about 1%. To introduce damping oil, the vibrator cover should be removed, the lens lifted up, the vibrator being held horizontal and a drop

or two of the special oil placed on the gaps over the mirrors. The lens is then lowered into place, the lower edge first, care being taken not to imprison any air bells. The temperature of this oil is measured by inserting the bulb of a thermometer at the back of the vibrator.

II. The Single Permanent magnet, and III. the Double Permanent magnet, Oscillographs are similar to one another. In each the electro magnet of Type I. is replaced by a permanent magnet, and the damping oil, which is introduced by means of a small cup at the back of the instrument, is adjusted to give correct damping at 15° C., and practically correct damping betwen 10° C. and 20° C., so that no thermometer is needed. These two types are portable and can be easily insulated for research on 10,000 volt circuits by placing it on an ebonite table.

IV. The Double Projection Oscillograph is somewhat similar to Type I. and is most suitable for teaching and lecture work. Wave-forms having a total amplitude of 1 metre can be thrown on to a screen.

The above types are *single* or *double* according as they possess *one* or *two strips* respectively. The double or two-strip pattern is practically two single instruments built compactly in one magnetic field and capable of recording simultaneously any two distinct wave-forms. A fixed mirror is fitted in addition to give the datum or zero line. Types II. and III. can be arranged in portable form.

Three Methods are generally employed for Observing and Recording the Movements of the Spots.

1. Visual observation. A rotating mirror is placed with its axis horizontal in such a position that the reflection in it of the moving spot on the screen can be examined; when, owing to persistence of vision, the moving spot will appear drawn out into a bright time curve of the variations which it is required to observe, and this curve can be sketched if required for future reference.

2. Recording by photography. A photographic plate or film

is caused to move rapidly at right angles to the plane of vibration of the beam of light, so that the moving spot traces out on it the required variations.

The photographic method is very expeditious, and gives permanent records which are free from all errors of a personal nature ; it is the only satisfactory method of recording irregular non-periodic variations of P.D. and current.

3. Tracing. In this method, which is only applicable to periodic variations, the beam of light from the Oscillograph is reflected by an additional mirror with its axis horizontal before

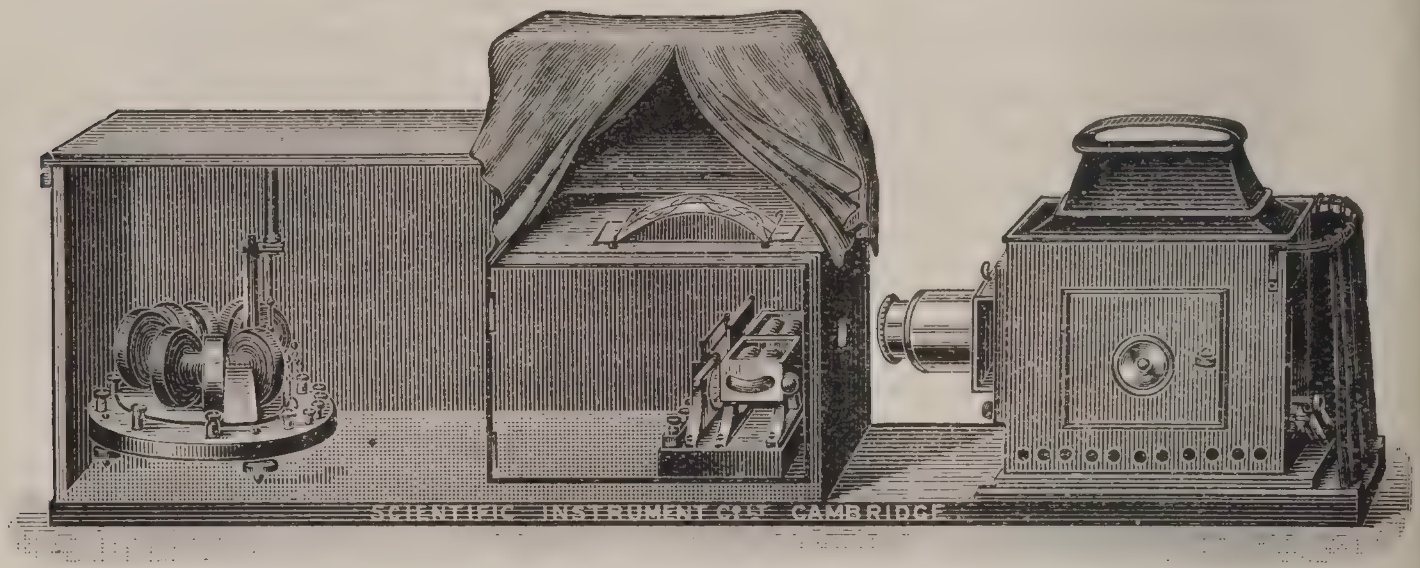


Fig. 174.—Type I. Double Oscillograph with Synchronous Motor and Tracing Desk.

reaching the screen, and this latter mirror is caused to move uniformly and synchronously with the period of the variations to be recorded. This combination of the two motions at right angles, the one proportional to the instantaneous value of the current through the Oscillograph, and the other to the time, causes the spot to travel continuously along the time curve of variation of the current, which curve, if the frequency is sufficiently high, will appear as a stationary bright line of light. This curve may be recorded by tracing or photography. In the Projection Oscillograph, this stationary line of light can be thrown either on the screen or on the tracing desk.

The arrangement employed for tracing a.c. wave-forms which remain fairly constant in shape and frequency and thus obviating the necessity for using photography is shown in Fig. 174. In

it the light from the Oscillograph mirrors is reflected vertically by a small mirror which is made to vibrate synchronously by means of a specially designed alternate current motor. The light is thrown on to a curved screen, on which tracing paper is held by means of a clip and on which the wave-forms appear as

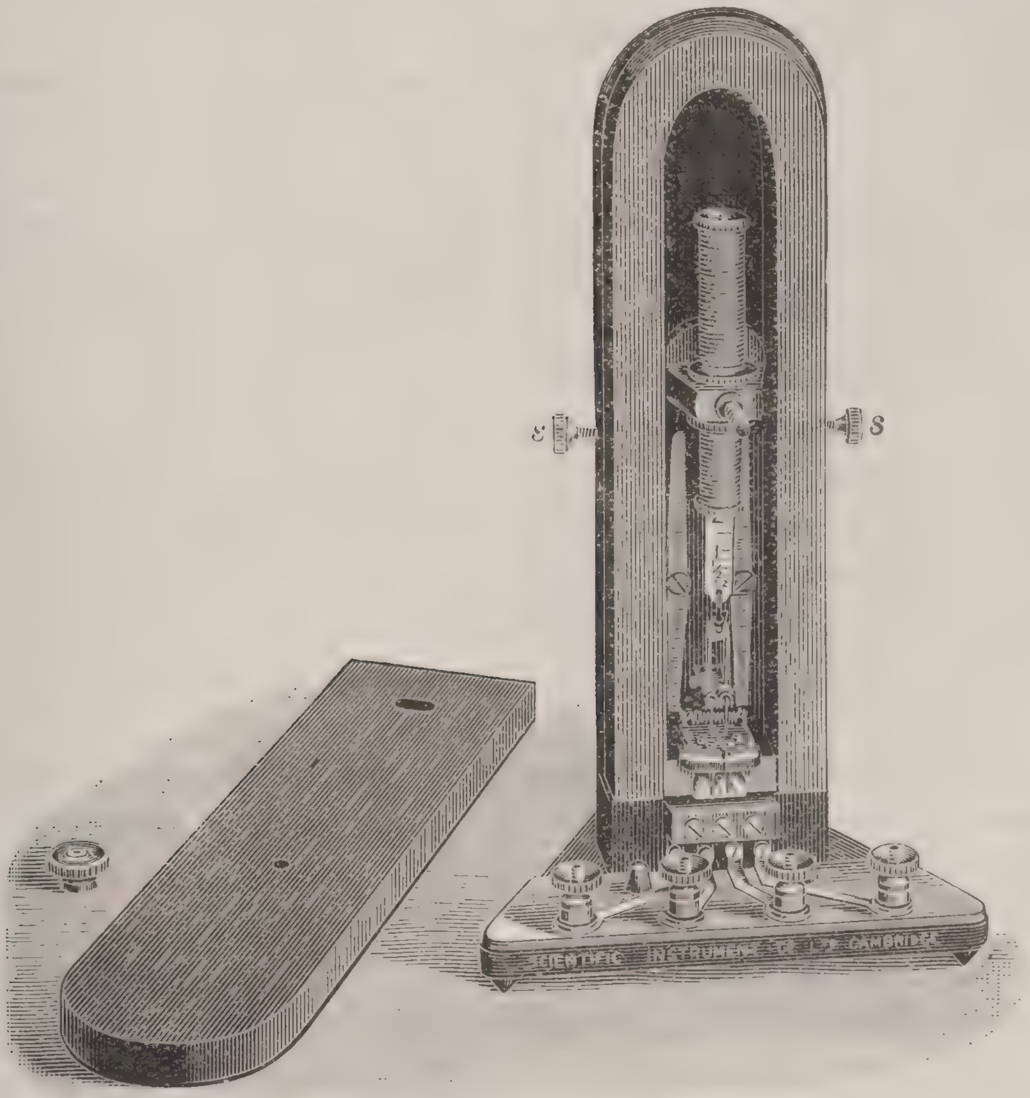


FIG. 175.—Type III. Double Permanent Magnet Oscillograph.

stationary curves of light, and may be traced by hand. The small mirror is vibrated by means of a cam attached to the motor shaft. The cam is so arranged that the mirror moves uniformly for about $1\frac{1}{2}$ complete periods during which the wave-form is observed; it then returns rapidly to its starting point during the remaining $\frac{1}{2}$ period. During the half period of return motion, the light is cut off from the Oscillograph by means of a sector fixed to the motor.

The Oscillograph shown in Fig. 175 can of course be substituted for that shown in Fig. 174, the tangent screw heads *s*, *s*, Fig. 175, being for the purpose of bringing the spots of the Oscillograph to zero on the screen.

The Synchronous Motor.—Seeing that this must run dead synchronously with the wave-forms to be recorded, it should be supplied from the same circuit.

To start the latest type of motor, connect the three terminals on it to the phase-splitting board by means of the three-way flexible lead attached to the latter. Care must be taken to connect the wires to the terminals correspondingly marked, the remaining two terminals on the phase-splitting board being connected to the a.c. supply having a P.D. of *100 to 120 volts* with any frequency between 30 and 100 ~ per sec.

See that the armature of the motor is quite free by turning the milled head, and that the bearings are well oiled, then after pushing the movable core of the choking coil in as far as it will go, close the switch. Now give the armature a start by sharply twisting the milled head on its spindle at the vibrating mirror end in the clockwise direction, when it should continue to run and increase in speed up to synchronism.

If the motor does not attain synchronism (indicated by the "beats" in the sound emitted), draw out the core of the choker little by little until a constant rhythmic hum is given out.

The position of the core for synchronism depends on the wave-form and frequency, being further out the higher the latter. If the core is too far in, the motor will not attain synchronism, and if too far out the motor will take too much current and get hot.

It might be necessary to remove the load from the motor at starting by pressing back the mirror so as to lift the "follower" of the cam.

The wave-forms will have more than one complete period, and will move to and fro on the tracing disc or screen if the motor is running below synchronous speed.

Method of Connection of Oscillographs to the Circuits to be Investigated.

The P.D. required to work the Oscillographs when fuses are used in series with the strips is only 1 to 1·5 volts. For current curves a non-inductive shunt R_3 should be placed in the main circuit and connected as shown in Figs. 176 and 177. The low resistance R_2 serves to adjust the sensibility to a round number of amperes per millimetre. An Ayrton-Mather shunt giving six sensibilities for currents from 1·5 to 60 amperes is made for this purpose. When the sensibility is adjusted by altering R_2 to a round number of amperes per millimetre for any one of the

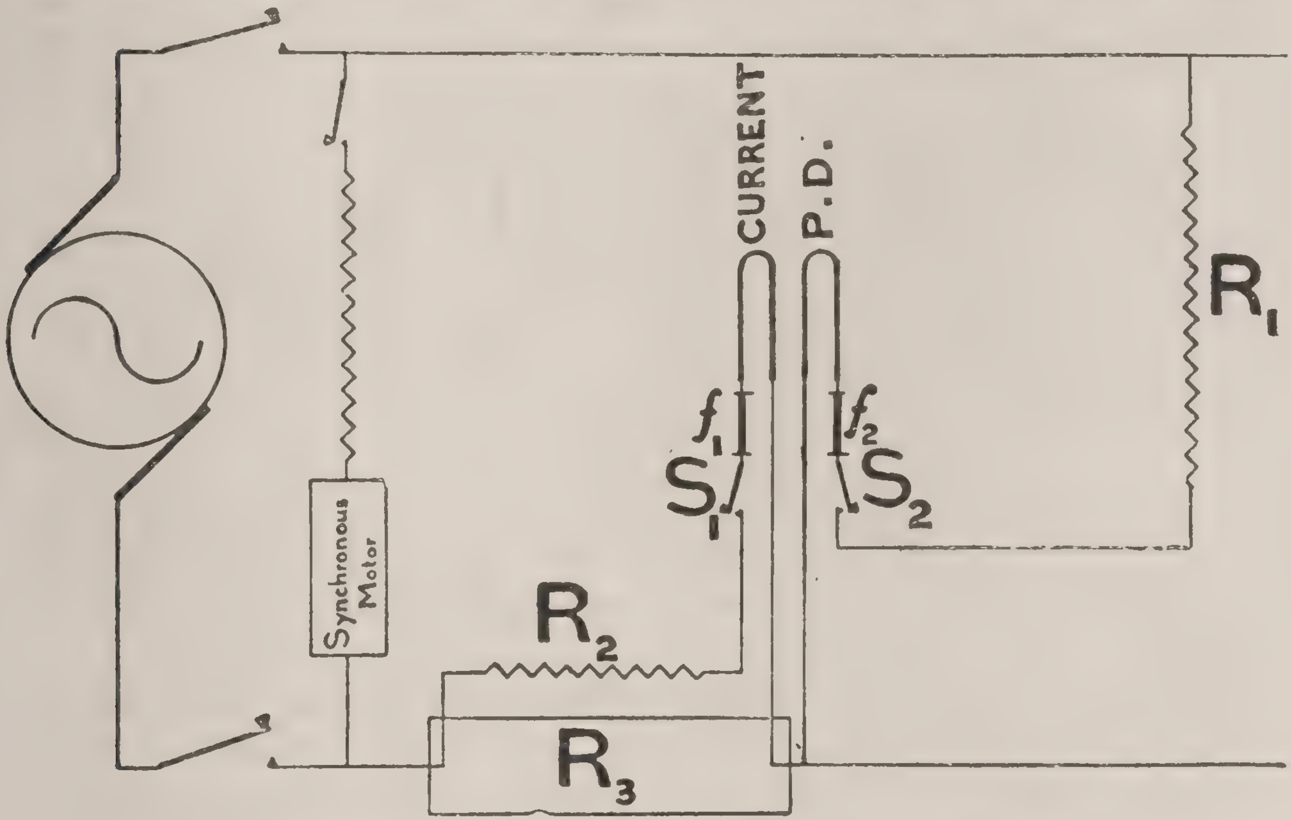


FIG. 176.—Diagram of Connection of Oscillograph to Low Tension Circuit.

sensibilities, all the other positions of shunt are simple multiples of the same; thus with Oscillographs Types I., II. and III. the sensibilities may be 0·05, 0·1, 0·2, 0·5, 1·0, 2·0 amperes per millimetre. Standard Potentiometer shunts constructed for a drop of 1 to 1·5 volts can also be used in place of R_3.

For P.D. measurements up to 250 volts a non-inductive resistance R_1 is placed in series with the strips, Fig. 176, which is adjusted to give a round number of volts per millimetre deflection. The switches S_1, S_2 and fuses f_1, f_2 should be arranged as

shown in the diagram so that no P.D. can exist between the P.D. and current strips due to their action.

For P.D.s up to 15,000 volts the arrangement shown in Fig. 177 is much safer. R_1 consists of several specially wound 10,000 ohm resistance frames joined in series and giving about 7 to 10 ohms for each volt, so that on a 10,000 volt circuit R_1 would be 70,000 to 100,000 ohms; R_5 is a 100 or 200 ohm coil forming with R_1 a potential divider; R_4 is a resistance to adjust the sensibility

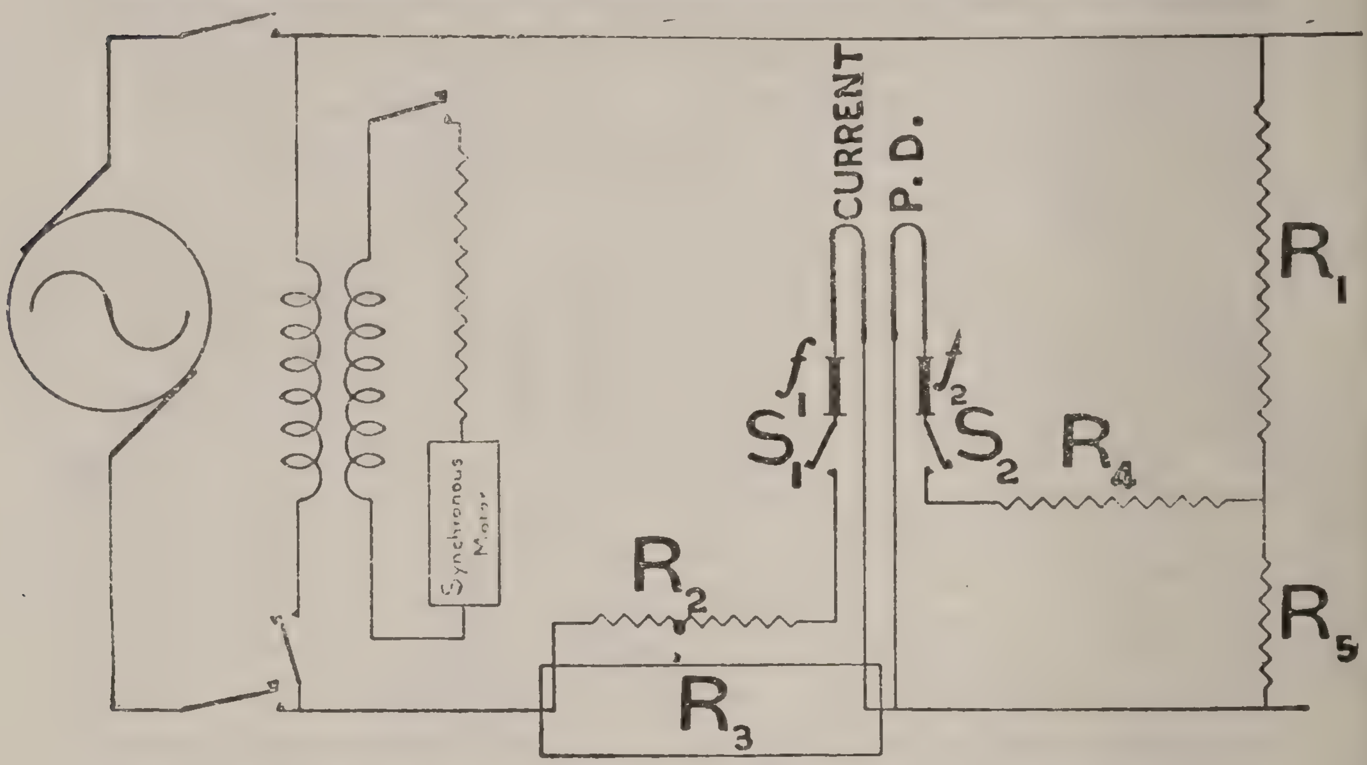


FIG. 177.—Diagram of Connection of Oscillograph to High Tension Circuit.

to a round number of volts per millimetre deflection. The same resistance box as is used for R_1, in Fig. 176, is suitable. When P.D. curves only are being recorded then R_5 should be connected at that point in R_1 which has the least potential above earth. All the resistances R_1, R_2, R_3, R_4 and R_5 must be insulated so as to stand four or five times the working voltage to earth. Unless the side of the circuit containing R_3 is *permanently connected to earth it is better not to use any switches or fuses in the Oscillograph circuit as shown, and permanent magnet Oscillographs Types Nos. II. or III. should be used.* It is also advisable to connect one terminal of the strips to the frame so as to screen it from electrostatic effects. On high voltage circuits the synchronous motor if used is best supplied through a suitable transformer.

When using the double Oscillographs the two pairs of strips should be so connected to the circuit that it is impossible under any circumstances that a higher potential difference than 50 *volts should exist between one pair of strips and the other, or between either pair and the frame.*

The Oscillographs should be calibrated with continuous currents and the resistances R_1, R_2, R_4 adjusted so that one mm. deflection corresponds to a convenient number of amperes or volts, as the case may be.

The following table gives some useful approximate data for the various forms of Duddell Oscillographs.

TABLE VII.

	Double High Frequency Oscillograph.	Single Permanent Magnet and Portable Oscillograph.	Double Permanent Magnet and Portable Oscillograph.	Double Projection Oscillograph.
Resistance of Field Coils in series at 30° C. . . .	360 ohms	—	—	650 to 700 ohms
Normal Exciting Current with the Coils in series .	0·28 ampere	—	—	0·27 ampere
Working Temperature for correct damping with the oil supplied . .	25° C. to 35° C.	10° C. to 20° C.	10° C. to 20° C.	30° C. to 35 C.
Normal Tension on strips . . .	8 ozs.	$1\frac{1}{4}$ ozs.	$2\frac{1}{2}$ ozs.	6 lbs.
Periodic Time (undamped) with the above tension .	$\frac{1}{8000}$ to $\frac{1}{10000}$ sec.	$\frac{1}{4000}$ to $\frac{1}{5000}$ sec.	$\frac{1}{4000}$ to $\frac{1}{5000}$ sec.	$\frac{1}{1800}$ to $\frac{1}{2000}$ sec.
Normal Scale distance . . .	50 cms.	50 cms. (25 cms. when used as Portable Oscillographs)	50 cms.	300 cms.
Sensibility with the above tension, normal exciting current and scale distance with damping oil in instrument . .	290 mm. per ampere	320 mm. per ampere	290 mm. per ampere	50 cms. per ampere
Normal Working Current in strips for alternate current wave-forms .	0·05 to 0·10 ampere	0·05 to 0·10 ampere	0·05 to 0·10 ampere	0·5 ampere
Resistance of strips without fuse and connections . .	about 5 ohms	about 5 ohms	about 5 ohms	about 1 ohm
Do. do. with one fuse and connections . . .	about 10 ohms	no fuses	no fuses	about 1·5 ohms

(157) Determination of the Efficiency of an Electro-Motor-Fan Set.

Introduction.—The very extensive use to which the electrically driven fan is put, coupled with the large number of combinations of different types of motors and fans in use at the present day, makes it desirable to obtain some measure or gauge of the efficiency of such an electric fan as an air circulating device.

From a commercial point of view, the utility of a fan can best be judged by knowing (1) the quantity of air, reckoned say in cubic feet, passed by the fan in unit time, say 1 minute; (2) the power required to drive the fan to give this. Also, from a mechanical point of view, the speed of the fan under these conditions, and, as a matter of scientific interest, the pressure of the air thus passed.

In order to carry out the test, it will be necessary to provide the fan with an air conduit for the purpose of restricting the current of air to a definite path. This may very simply and conveniently consist of a tubular casing of either circular or square cross section, open at both ends, and from three to four diameters long. The fan must just fit into one end, and any crevices due to loose fitting between the sides of the conduit and fan must be filled in air-tight, so that the only path for the air is through the fan itself.

The determination of the pressure of the draught through the conduit *CD*, but which, however, is not essential to the test, and merely of interest, can be determined by a special form of water or spirit gauge *G*.

Since the pressure of the draught is very small, an ordinary vertical U pressure tube would hardly indicate it, and not sufficiently accurately for any practical use. To measure such small pressures, the gauge, in the present instance, may conveniently take the form of a slanting U glass tube about $\frac{1}{8}''$ to

$\frac{1}{4}''$ bore, one limb L of which is extended and bent into the form shown at LO, the end O being bent so as to face the incoming draught of air from the end C of the conduit.

If now this U tube contains coloured alcohol, any slight pressure at O will cause a difference in level of the liquid in the two limbs, the *vertical* distance between the ends of the columns being a measure of the pressure at O.. Thus even though this is very small, the end of the liquid column in either limb may have moved through a considerable distance, which increases as the angle of slope (θ) gets smaller. Hence if the tube is provided with a scale and calibrated, it can be made to read small fractions of an inch pressure of water, etc.

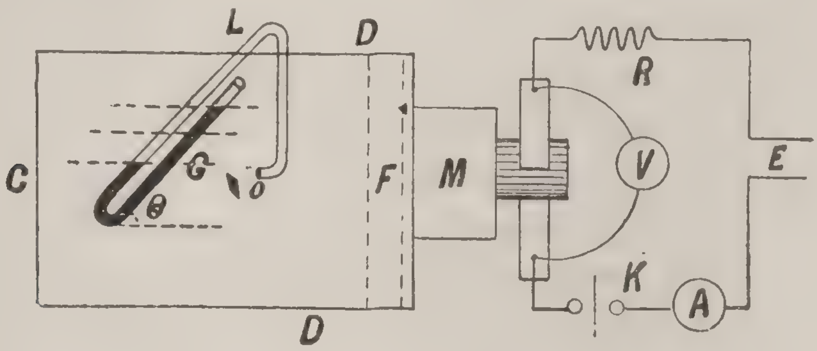


FIG. 178.

A little cotton wool placed in each end of the tube helps to damp the motions of the liquid column caused by rapid fluctuations of air pressure.

In order to obtain the volumetric discharge of air for any particular speed of the motor M, the velocity of the propelled air must be obtained by means of a rotatory anemometer placed in different positions in the cross section of the conduit some little distance from the end C, the instrument being supported at the end of a stout wire or rod so as not to alter the flow of air. From the various readings a mean may be obtained for the whole section.

Now let v = mean velocity of the air in feet per minute as given by the anemometer, and let s = area of cross section of the conduit in square feet, then the volume of air discharged per unit time, *i.e.* number of cubic feet of air per minute $= vs$; this will vary with the speed of the motor.

Let w = weight in lbs. of 1 cubic foot of air at the barometric pressure and temperature of the room in which the test is made (see table, p. 650), then $W = wvs$ = the weight in lbs. of air discharged per minute; but the kinetic energy of a mass (m) lbs, moving with a velocity v feet per second $= \frac{1}{2}mv^2$ foot-poundals, and this is a measure of the work done.

Hence the work done on the air $= \frac{1}{2}mv^2 \div g$ foot lbs.

$$\therefore \text{ H.P. developed} = \frac{\frac{1}{2}Wv^2}{32{\cdot}2 \times 33000} = \frac{wvs}{2 \times 32{\cdot}2} \times \left(\frac{v}{60}\right)^2 \times \frac{1}{33000}.$$

Apparatus.—Motor M with its attached fan F to be tested; suitable conduit CD; anemometer; air pressure gauge G; voltmeter V; ammeter A; switch K; rheostat R (p. 606) and source of electrical supply E; speed indicator for obtaining the speed of the motor.

Observations.—(1) Connect up as shown in Fig. 178, and adjust the instruments to zero if they require it. See that all lubricating cups feed slowly and properly.

(2) With R at its maximum, close K, and adjust the speed of the motor M to the lowest convenient amount, then note the readings of V, A, speed, anemometer, and gauge G.

(3) Take the anemometer reading at different positions in the cross section of the conduit at this speed of M, and take the mean.

(4) Repeat 2 and 3 for twelve or fifteen different speeds of the motor, rising by about equal increments to the maximum permissible, by varying the rheostat R, noting simultaneously the readings of all the instruments at each speed. Tabulate your results as follows—

NAME . . DATE . . .

Electro Motor: No. . . . Type . . . Maker . . . Normal: Volts . . . Amps. . . . Speed . . .
Fan: No. . . . ,, . . . ,, . . . No. of blades . . .
Sectional Area of Conduit s = . . . sq. ft. Weight of 1 cubic foot of air at time of test = . . . lbs.

Mean anemometer reading (v) ft. per min.	Pressure of Draught in inches.	Discharge in cub. ft. per min. (vs)	Speed of Motor revs. per min.	Volts V.	Amps. A.	Horse Power: absorbed $\frac{AV}{746} = H_1$.	Horse Power: developed $\frac{w.v^3.s}{765072 \times 10^4} = H_2$.	Efficiency of Fan % $= 100\frac{H_2}{H_1}$.

(5) Plot the following curves on the same curve sheet having (*a*) the speed in revolutions per minute of the motor, (*b*) H.P. absorbed by the motor as abscissæ, with the volume of air discharged per minute as ordinates in each case; also between (*a*) as abscissæ with (*c*) the mean velocity of the air draught as given by the anemometer, (*d*) the H.P. absorbed in the transference of air as ordinates in each case; lastly, between efficiency as ordinates and speed as abscissæ.

(158) Determination of the Commercial Efficiency of a Gas Engine-Dynamo Generating Set.

Introduction.—It is most desirable that any generating unit such as the above should be tested at various loads or electrical outputs in order that the best running conditions may be discovered and the performance generally of the *unit* observed.

This is done by "*indicating*" the engine at the various electrical load outputs desired, and so finding the relation between the total power exerted on the piston of the engine, commonly termed its indicated horse-power (I.H.P.), and the corresponding power utilized or developed by the generator in the external circuit, which may be reckoned in $\frac{\text{Watts}}{746}$ or electrical horse-power (E.H.P.).

In order that the cost of running at any given electrical output per hour (say) may be determined, it will be necessary to measure the *volume of gas used* in the engine, which can usually be done easily enough, as nearly all gas engines are provided with separate gas-meters on the inlet pipe of the engine. Should, however, this not be the case, the test must be made when all gas-jets are out and the readings on the main meter recorded.

The *jacket water*, or volume of cooling water passed through the water jacket of the cylinder of the engine, should be measured, and this can best be done by a water-meter inserted in the inlet water-pipe. If such a meter is not available, a fairly large tank can be used (the volume of which can be calculated) to supply the

water jacket, then the time taken to empty the calculated known volume of water will enable us to get what is required. If the temperature of the inlet and outlet water of the jacket is taken, the heat removed from the cylinder in thermal units can at once be deduced, knowing the volume of water passed in a given time.

The engine must be provided with a stop-cock in communication with the interior of the cylinder, and into the outer end of which the nipple of the indicator is screwed. This indicator may be of the Richards' type, and, if the gas engine is running at a high speed, the indicator should be a high speed one. If the diagrams taken by this indicator in such cases are not shortened, a stronger drum-spring will be needed to get over the effects of inertia in the drum which carries the card.

If the engine has an ordinary double (Otto) cycle and gets the maximum number of explosions possible, which would occur when it is running at or near full load, then the speed in revolutions per min. ÷ 2 will give the number of explosions per min. If, however, the gas engine is running on light loads it will "miss" an explosion frequently, in which case, since the number of such per min. is a factor of the I.H.P., they must be counted separately, either mentally or automatically by an attachment or counter actuated by the inlet gas-valve lever.

Furthermore, in order to obtain the total cost of electrical energy delivered at the switch-board, we must know the amount of oil, cotton waste, wages, and interest on depreciation and first cost of the plant for the period over which the run extends; these items, however, pertain merely to what we may term the economic efficiency of the plant.

The electrical power developed by the dynamo can be taken up either in the apparatus on the circuit to be supplied, or in suitably designed water rheostats having ample plate area to avoid variations in the output. This may take the form of a rectangular water-tight wooden trough, having a fixed zinc, iron, or copper plate at one end, connected to one terminal of the dynamo, and a similar movable plate, capable of being moved to a considerable distance from the fixed plate, thus enabling the current output to be varied; this plate is perforce connected to the other terminal of the generator.

The mean effective pressure P in lbs. per sq. in. during the

explosion, which is required in the calculations, is calculated, from the indicator diagrams taken, in the manner described on pp. 471 and 530.

Note.—The effective area of each plate of the water rheostat should be something like 4—9 sq. ft. for currents of about 500 amperes.

Apparatus.—Gas engine-dynamo set D to be tested, of which the gas engine is not shown; *indicator* (p. 531) and reciprocating lever gear for rotating the card drum; tachometer; voltmeter V; ammeter A; trough rheostat T for absorbing the electrical output from D.

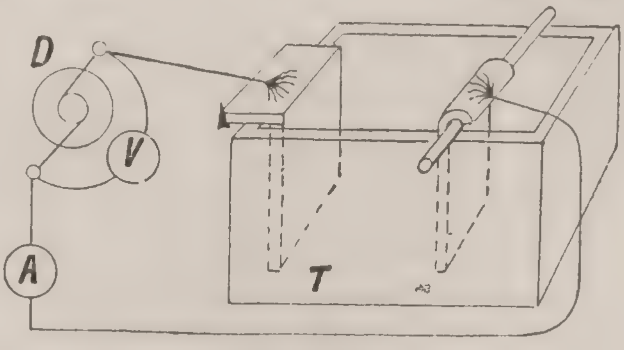


Fig. 179

In addition a "cut-out" and switch are desirable in the main circuit, and a water-meter with necessary separate gas-meter for the engine.

Observations.—(1) Connect up the electrical circuit as in Fig. 179, and adjust all instruments to zero. Fill T with water in which a few handfuls of washing soda have been dissolved and set the plates at the extremities of the rheostat.

N.B.—The plates may be provided with massive terminals for connection to the main leads, otherwise the ends of these latter should be spread out, fan-wise, and soldered to the plates.

(2) Measure approximately how much *oil* is required to *fill all lubricators* in use, which must be set to feed properly just before starting the trial.

Insert the most suitable spring in the indicator and a card on the drum, then screw the indicator to the cylinder cock and connect to the reciprocating gear.

(3) Start the "set" up to its normal speed and take an indicator diagram from the engine with D on open circuit.

(4) At a noted instant simultaneously read all the instruments and meters, then quickly switch on and adjust A to full load and take an "engine card" again.

Note.—At least four observers will be required for the trial.

(5) Simultaneously read all the appliances every twenty minutes

throughout the trial, which should last at least three hours, taking a "card" at each.

(6) Repeat 2—5 for $\frac{1}{2}$ full load on D if possible, and tabulate your results as follows—

NAME . . . DATE . . .

Gas Engine: No. . . . Makers . . . Type . . . Normal speed = . . . revs. per min.
Piston: Area (a) = . . . sq. inches. Stroke (L) = . . . feet.
Dynamo: No. . . . Makers . . . Type . . . Normal: Volts = . . . Amps. = . . . Speed = . . .
Indicator: No. . . . Type . . . Scale of Spring used . . . Mean $\left\{\frac{\text{E.H.P.} = \ldots}{\text{I.H.P.} = \ldots}\right\}$ during trial.

Time		Output from Dynamo.			Gas Engine.											Efficiency of "set" = mean $\frac{\text{E.H.P.}}{\text{I.H.P.}}$
					Gas.					Water.					I.H.P. calculated from cards $\frac{PLna}{33000}$.	
of observations.	in Hours from Start.	Volts V.	Amps. A.	E.H.P. = $\frac{AV}{746}$.	Explosions per min. (n).	Meter reading.	Total Vol. used in trial.	Cubic feet per mean. I.H.P.	Cubic feet per mean. E.H.P.	Meter reading.	Total Vol. used in trial.	Gallons per mean. I.H.P.	Gallons per mean. E.H.P.			

Inferences.—What inferences can be deduced from the results of the trial?

Calculate the total cost per E.H.P. hour delivered at the switch-boards, taking the approximate average costs of the various factors.

(159) Determination of the Commercial Efficiency of a Steam Engine-Dynamo Generating Set.

Introduction.—We have already described in detail the usual methods of determining the efficiency of both direct and alternating current generators without reference to any prime mover such as a steam, gas, or oil engine. The common practice, however, at the present day, of employing "*direct coupled sets*" in central stations, consisting of the generator placed on, and fixed to, the same bed-plate as the engine and coupled direct to it, makes the test of the performance of such a combined generating set one of extreme importance.

This practice has resulted from the endeavours of central

station engineers to curtail the amount of floor area required for a given station and to avoid the loss of power and trouble inseparable from driving by belts and ropes. In order to determine the combined efficiency of a generating unit, whether direct coupled or otherwise, we require to measure the useful or nett electrical output, which can be done with the aid of an ammeter, voltmeter, and one or more suitable rheostats in the manner described in the earlier pages of this book.

In addition we must know the gross or total H.P. exerted by the piston of the engine, usually termed the Indicated H.P. This can be determined by aid of the *engine indicator*, for the detailed theory of which the reader is referred to special "works" dealing with such *indicators* almost entirely. There are, however, some important details connected with the use in general of the various forms of these instruments, which may with advantage be mentioned here, but otherwise it will be assumed that the theory, construction and action of the engine indicator is understood.

Relation between length of indicator diagram and engine speed.—It must be carefully remembered that as the paper drum of the indicator is rotated by a reciprocating motion from the piston or cross-head of the engine, its inertia at the higher speeds may introduce errors in the diagram; in other words, its motion may continue, from this cause, beyond what actually represents the true driving motion. To minimize such an error the angular motion of the drum must be reduced, and consequently a shorter diagram obtained as the speed increases, in order to insure a true and properly proportioned diagram.

In this connection it may be noted that for *speeds up to* 200 *revs. per min.* the driving of the drum should be so arranged that its angular motion gives a diagram about $4\frac{1}{2}''$ long, and this should be made to diminish almost inversely proportional to the increase of speed. Since in addition increase of speed will require a stronger piston spring, the height and length of diagram will decrease in about the same proportion for increase of speed, thus giving a properly proportioned and accurate diagram.

Gearing of Engine Indicators.—In order that an accurate diagram may be obtained it is all-important that the reducing-gear for driving the drum should reduce the piston motion in

exactly the same proportion at *any* and *every* part of the stroke. For such gears the reader is referred to works dealing with the subject of engine indicators. In the use of the indicator great care should be taken to keep it free from all dust and well oiled with watch oil, as the least friction in the cylinder or multiplying levers may cause a distortion of the diagram, the boiler pressure and engine speed will determine what strength of spring is to be used in the cylinder, and the lighter this is, the higher the diagram and the more accurate the measurement, providing inertia effects are absent.

The engine piston must of course never be allowed to close the outlet pipe between the engine and indicator cylinders, hence the latter should be screwed on to a cock at the end of the engine cylinder. For accurate work diagrams must be taken at both ends of the cylinder. Fig. 180 represents the approximate shape of a diagram which would be taken from an ordinary non-condensing engine. The indicator is first made to draw the straight line $O'A'$, done by putting both sides of its piston in communication with the atmosphere by opening the stop-cock so as to cut off all steam from the cylinder of the engine and allow the air to enter underneath the indicator-piston. The pressure both sides of this latter will thus be the same and equal to that of the atmosphere, consequently if the pencil be lightly pressed against the moving card, the horizontal straight line $O'A'$, termed the *atmospheric line*, will be described. OA is the *absolute zero line*, parallel to $O'A'$, and drawn below it at a distance representing, to the scale of the diagram, the atmospheric pressure at the time of the test.

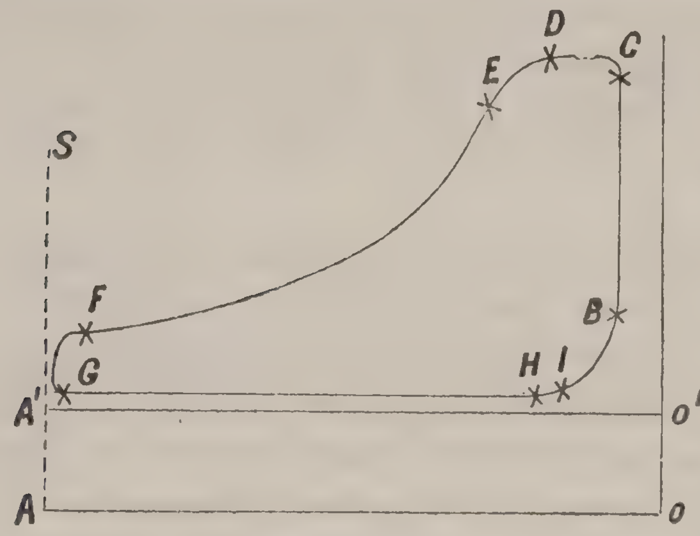


Fig. 180.

In Fig. 180 B is the point of admission of steam to the engine cylinder, D the point at which the slide-valve begins to close, E the point at which it is quite closed. From E to F the admitted

steam is expanding, and at F the release begins, being completed at G. The exhaust valve begins to shut at H and is quite shut at I, the steam still left in the cylinder being compressed between I and B. The rounded corners, such as DE and FG, show the slow acting of the steam valves in closing and opening the steam ports, which is called *wire drawing*. In the ideal engine these rounded corners would become sharp ones.

Determination of I.H.P. *from the diagram.*—Referring to the diagram, Fig. 180, the horizontal distance between the extreme points of the diagram, *i. e.* between the vertical lines $A'S$ and BC, represents the stroke of the engine piston in feet. The ordinates of the diagram perpendicular to the atmospheric line $O'A'$ represent to the scale of the indicator spring used the pressures of the steam in lbs. per $\square''$.

If the scale of the indicator spring used in taking the diagram $=\frac{1}{30}$, each inch of the ordinates represents 30 lbs. pressure per square inch, consequently each square inch of the diagram represents $\frac{30}{12}=2\frac{1}{2}$ ft. lbs. The whole area of the diagram will therefore represent the indicated work in ft. lbs. per square inch of engine-piston, done on one side of it, during one stroke. Since the pressure exerted by the steam on the piston varies at different parts of the stroke, we must know the mean effective pressure for the complete stroke. This is found by *dividing* the *area of the diagram by the base line*, both being reckoned in inch units. The result is the value of the mean ordinate of the diagram or mean effective Pressure (P_m) in lbs. per square inch of piston area.

Hence if L = length of stroke in feet, A = piston area in square inches, and N = number of revs. per min. which the engine is making, then the I.H.P. $= \dfrac{2\ P_m LAN}{33000}$

Apparatus.—Generating set to be tested; engine indicator (p. 531); speed indicator; planimeter (p. 528); ammeter; voltmeter; rheostat for absorbing the load from the generator (p. 467), and a switch.

Observations.—(1) Connect the ammeter, rheostat and switch in series with one another and with the generator, also the voltmeter across the terminals of the machine, and open the switch.

(2) Disconnect the generator and engine and start the latter, running alone for some little time before making a test. Prepare

the engine indicator by first seeing that all the parts are quite clean, well oiled, and work practically frictionlessly. Insert the right spring in the cylinder suitable for the boiler *pressure* and engine *speed* to be *used*, and note its "scale" for future reference.

(3) Blow off steam at the cock which is to carry the indicator for a second or two so as to clear away superfluous water and dirt. Now screw the indicator to it. Place a card on the drum, and make sure the cord which is to actuate the drum will be attached to the proper point on the reducing gear so as to give a suitable length of diagram for the speed of the engine.

(4) Turn the cock so as to admit air under the indicator piston, cutting off all steam. Then hook on the cord so as to rotate the drum and draw the "*atmospheric line.*" Then turn the cock so as to communicate with the engine cylinder and take a full diagram, noting simultaneously the speed of the engine, and in addition which end of the cylinder the diagram is taken from.

(5) Cut off the steam by the cock, unhook the cord, and quickly repeat 3 and 4 at the other end of the cylinder of the engine. This interchange should be repeated two or three times so that an average may be obtained, for the constant speed, when working out the results.

(6) Now run the generator by the engine, absorbing $\frac{1}{2}$, $\frac{3}{4}$, and full load successively in the rheostat, repeating 3–5 at each load for the same speed, both load and speed being maintained constant during the time required for taking the readings. Note the volts and amperes at each load, and tabulate your observations as follows—

NAME . . . DATE . . .

Engine: No. . . . Type . . . Maker . . . Normal I.H.P. = . . . Speed = . . . Pressure = . . .
Dynamo: No. . . . " . . . " . . . " Volts = . . . Amps. = . . . Speed = . . .
Scale of Indicator Spring used = . . . Type of Indicator . . . No. = . . .
Length of Engine Stroke (L) = . . . feet. Area of Piston (A) = . . . sq. ins.

Condition under which the test is made.	Diagram.				Volts V.	Amps. A.	H.P. developed $H = \frac{AV}{746}$.	Combined Efficiency, $\Sigma = \frac{H}{IHP}$.
	End of Cylinder from which Diagram taken.	Speed of Engine (N) Revs. per min.	Mean Effective Pressure P_m lbs. per □″.	Calculated I.H.P. = $\frac{2P_mLAN}{33000}$.				

Note.—The value of P_m in the above table is the *mean* of the *means* of the worked-out results for the two ends of the cylinder of a single cylinder engine. In the case of a compound or triple expansion engine, the I.H.P. can be found from the diagram taken from *either cylinder* as follows—

Sum the products of A and P_m for each cylinder (P_m being given by the diagram for that cylinder found in the usual way) and divide by the A for that cylinder from which the particular diagram under consideration was taken. Thus if A_2 = piston area in sq. ins. of, say, the "*intermediate cylinder*" of a triple expansion engine, the mean effective pressure to be used in the formula, say $P_m' = \frac{\Sigma(P_m'A)}{A_2}$ where Σ indicates the sum of the products for the H.P. intermediate and L.P. cylinders.

The value of the mean effective pressure as obtained from any particular indicator diagram can be obtained by the aid of the planimeter, the use of which in measuring the area of the diagram is described on p. 530.

General Observations on Jointing Electric Lighting Cables.

Good and reliable joints both in *core and insulation* can only be made with practice, care, and attention to the following essential details:—All joints in conductors must be as mechanically and electrically perfect as possible, for they are in most cases a source of weakness in an installation.

The Joint.—That of the metallic core should have a conductivity not less than that of an equal length of the ordinary core, if possible, and to obtain this care must be taken not to *nick* the copper cores of the cables to be jointed either with the paring knife or pliers, which not only reduces the conductivity, but causes the wire of the strand so nicked to break off at once if bent at that point. Before making the joint, all the wires must be *straight* and thoroughly *cleaned* with fine emery cloth, care being taken not to remove the tinning of already tinned wires. The cleaned wires may preferably be re-tinned with the soldering-iron, and must be handled as little as possible, even with clean hands.

No joint will ever solder properly unless it is *quite clean* throughout, and in a *hot, clean,* and well-tinned soldering-iron.

The Soldering-Irons.—These must be properly grooved to take the size of joint to be soldered, and should never be allowed to get too hot and "burn." This always gives rise to an excessive lurid *green* flame, and is not only injurious to the "copper bit," but burns all the tinning off them, thereby giving extra labour and wasting time in re-tinning. Irons may be cleaned with either sal-ammoniac, emery cloth, or carefully with a suitable file. Salts or soldering fluid may be used as a flux in tinning them. Irons should be *well tinned,* and hot enough when used to be unbearable when placed about $1\frac{1}{2}''$ from the cheek. They should be wiped when taken out of the stove before applying to the joint. *Quick soldering* is essential, as continued application of heat seriously weakens copper wire and makes it brittle. Too great a heat causes solder to "rot" and become useless.

Solder.—This should be in thin sticks, and should contain enough tin to enable one to hear it crinkle when bent double close to the ear.

Flux.—Nothing but *resin* (applied in the lump) should be used in soldering copper joints. All liquid fluxes and other substances containing corrosive ingredients should be avoided if the joint is required to remain unimpaired with time.

Insulation.—Great care should be taken to make the insulation of the joint as nearly as possible equal to that of the rest of the cable. In rubber-insulated cables the braiding or taping is removed from the rubber without nicking it, and the pure I.-R. strip wound with *lap winding* over the joint and *tapered* ends of the rubber thus bared. I.-R. solution is now rubbed over the joint, but must never touch the bare joint. The taping should be done tightly, and be quite solid when completed.

(160) Detailed Instructions for Jointing Electric Light Cables.

Introduction.—For successful and efficient jointing the following remarks must be rigidly adhered to—

(1) *In baring* any wire or cable preparatory to making a joint,

great *care must be taken not to nick* any one or more of *the copper wires* forming the core, which would not only cause the wire so nicked to break off on bending it once or twice, but would also diminish the sectional area of the cable and so also its current carrying capacity.

(2) *In cleaning* the copper wires of the core *fine emery cloth* must be used and all dirt removed, but as little as possible (if any) of the original tinning.

(3) Just *sufficient* and *no more* cable must be bared as will make a satisfactory joint, considerations of the cost of insulating materials, and particularly of the ultimate insulation resistance of the joint, making it imperative to keep the dimensions of the joint, in the matter of length, a minimum.

(4) Cleanliness is of vital importance in the actual winding or making of the joint, and a few extra seconds spent in insuring this will almost invariably save many minutes, much solder and soldering flux in the end, and even possibly the necessity for a second attempt at the whole joint.

(5) A badly made joint, or a badly insulated one, is a source of considerable danger in an electric light installation.

(6) Too much attention cannot be paid to the soldering irons, as it is perfectly hopeless to attempt to solder a joint with—a dirty iron, badly tinned iron, or a soldering iron that is not hot enough.

Course in Jointing Electric Light Wires, Cables, and Mains.

The following series of joints constitute a course in the actual practice of "*jointing making*" which the author instituted in his department at The University, Leeds. They comprise practically all the principal distinctive types of joints commonly met with in practice, and which might be required to be made *by any ordinary* wireman—

No.							
1	Twist-Joint between two	No. 18	S.W.G.	insulated	E.L.	wires.
2	,, ,, ,, ,,	No. 14	,,	,,	,,	,,
3	T- ,, of a No. 18 on to a	No. 14	,,	,,	,,	,,
4	T- ,, of a No. 14 on to a	No. 7/18	,,	,,	,,	cable.
5	Britannia-Joint between two	No. 10	S.W.G.	bare	copper	wires.
6	Scarf- ,, ,, ,,	No. 6	,,	,,	,,	,,

No.		
7	Twist-Joint between two	No. 7/18 S.W.G. insulated E.L. cables.
8	T- ,, ,, ,,	No. 7/18 ,, ,, ,, ,,
9	Twist- ,, ,, ,,	No. 7/14 ,, ,, ,, ,,
10	T- ,, ,, ,,	No. 7/14 ,, ,, ,, ,,
11	Twist- ,, ,, ,,	No. 19/16 ,, ,, ,, ,,
12	T- ,, ,, ,,	No. 19/16 ,, ,, ,, ,,
13	Twist- ,, ,, ,,	No. 37/16 ,, ,, ,, ,,
14	T- ,, ,, ,,	No. 37/16 ,, ,, ,, ,,
15	Twist- ,, ,, ,,	No. 7/16 S.W.G. insulated lead-covered E.L. cables.
16	T- ,, ,, ,,	No. 7/16 S.W.G. insulated lead-covered E.L. cables.
17	Twist- ,, ,, ,,	No. 16 S.W.G. insulated gutta-percha-covered wires.
18	T- ,, ,, ,,	No. 16 S.W.G. insulated gutta-percha-covered wires.
19	Twist- ,, ,, ,,	No. 19/16 and a No. 7/18 S.W.G. insulated electric light cables.
20	Slanting T-Joint between two	No. 19/16 S.W.G. insulated electric light cables.
21	Twist-Joint on a large concentric lead-covered electric light main.	

NOTE.—No. 5 is a joint used for aërial telegraph and telephone lines. Nos. 17 and 18 are joints used for telegraph work principally.

TWIST-JOINTS NOS. 1 AND 2.

To prepare.—Carefully bare, with a *sharp knife*, about $1\frac{1}{2}$ inches of the ends of the two wires to be jointed. This must *not* be done by a cut perpendicular to the wire, but by a short slicing motion round the wire, when the piece of insulation will in most cases come off whole with a suitable pull.

Clean each bared wire with fine emery cloth, straighten and place them across each other, then lightly gripping them at the crossing point with a pair of pliers, bend one free end round the other wire. Do this with the other end and finally straighten and trim the ends up close, so that they do not project outwards, as they are then liable to pierce the insulation.

To solder.—Place the joint in a well-tinned groove of the iron containing solder, then when hot just touch with a lump of resin and draw a thin stick of solder over the joint. This usually suffices, but if not, repeat the operation, using very little resin. The soldered joint must leave the iron quite *bright* and without any globules of solder hanging to the underside of it. The soldered joint should appear as in Fig. 181.

To insulate.—When cool, taper the ends of the insulation, and

starting from over the rubber of the wire, wind the joint over with a spiral *half overlap* of pure I.R. strip (para tape) to the other end, gently stretching the tape all the time so as to obtain a *firm* (*not spongy*) layer of I.R., which, since it is wound in half

FIG. 181.

overlap, constitutes a double layer. Apply I.R. solution to the outside of this I.R. lapping (on no account next to the copper joint) and rub evenly all over the lapping with the finger. Next, when the spirit has evaporated out of the rubber solution, wind on a similar layer of black prepared rubber tape, overlapping the outer insulation of the wire at each end, and sealing the ends down with solution. Lastly, paint the outside of the joint with black waterproof varnish and allow it to dry.

T-JOINTS NOS. 3 and 4.

To prepare.—Carefully bare about $1\frac{1}{4}$ to $1\frac{1}{2}$ inches of the wire to be tapped (the larger of the two) and clean with *fine* emery

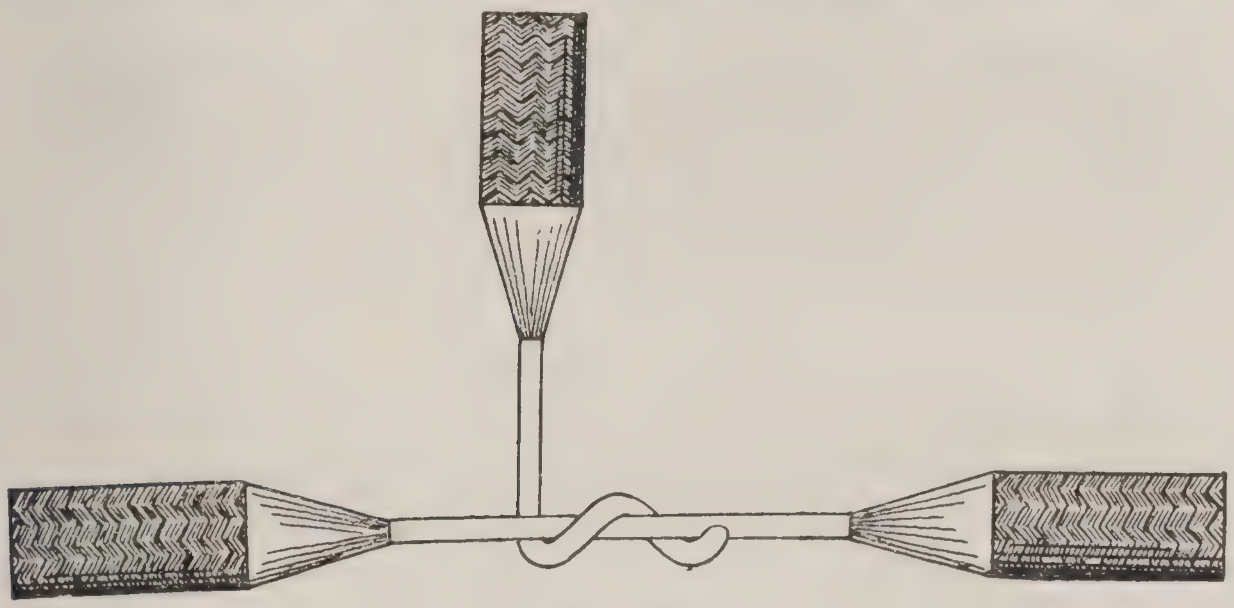


FIG. 182

cloth. Bare about $1\frac{1}{4}$ of the end of the other wire, clean and straighten, then placing this across the other wire twist it round two or three times to produce the joint shown in Fig. 182, and trim

the end so that it does not project to any extent. Solder in the manner described for joints 1 and 2.

To insulate.—Proceed as in these last-named joints, but on arriving at the **T** with each serving, carefully branch off down it and back, stretching the tapes more tightly to allow for increased thickness of insulation; finally, continue along the remaining straight portion of the cable, and varnishing over the last layer of tape. Considerable care is required in insulating a **T**-joint, as it is more difficult to get round the corners (*i. e.* angles) of the **T** with the tapes, and these parts therefore are most liable to imperfect insulation.

Britannia-Joint No. 5.

To prepare.—Gently straighten the ends of the wires to be jointed by lightly tapping them with a mallet on the anvil. Clean each with fine emery cloth, tin them both for a distance of about 2 inches from the end, and bend sharply round the tips of

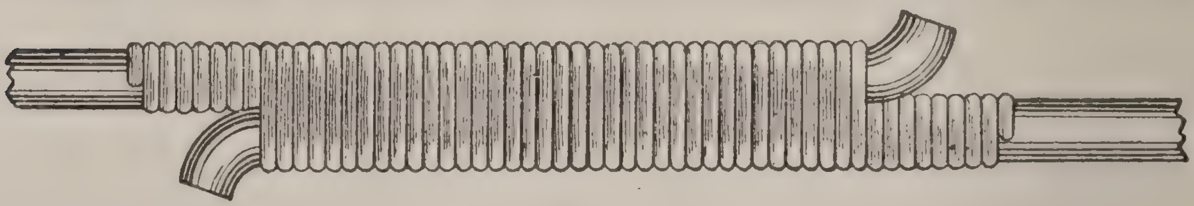


Fig. 183.

their ends. Next place them together with the bent ends pointing in *opposite* directions, and with an overlap of about 2″; then bind them together with about No. 20 tinned copper binding wire as shown in Fig. 183. Trim off the ends of this wire, and solder the whole into one solid mass as described in joints Nos. 1 and 2.

Scarf-Joint No. 6.

To prepare.—Gently straighten the ends of the wires to be jointed by lightly tapping them with a mallet on the anvil. Clean each for a distance of about 1 inch from their ends with emery cloth.

Fig. 184.

Next scarf them with a flat file as shown in Fig. 184, so that

they taper to thin edges and fit. Then tin them both, wiping off nearly all the superfluous solder by a clean cloth. Now warm the ends up, and when the surface solder on the scarfed portions is melted, place them together to form a continuous wire and

FIG. 185.

allow to cool. Bind the joint with No. 18 or 20 tinned copper wire for about $\frac{3}{4}''$ from the centre each way as in Fig. 185. Lastly, trim the ends of the wire and solder into one solid mass, care being taken to *just keep* the scarfed ends in gentle contact while soldering and until set.

TWIST-JOINTS NOS. 7 AND 9.

To prepare.—Carefully bare about 4″ of the two ends to be jointed with a sharp knife and a slicing motion (not a cutting one perpendicular to the cable).

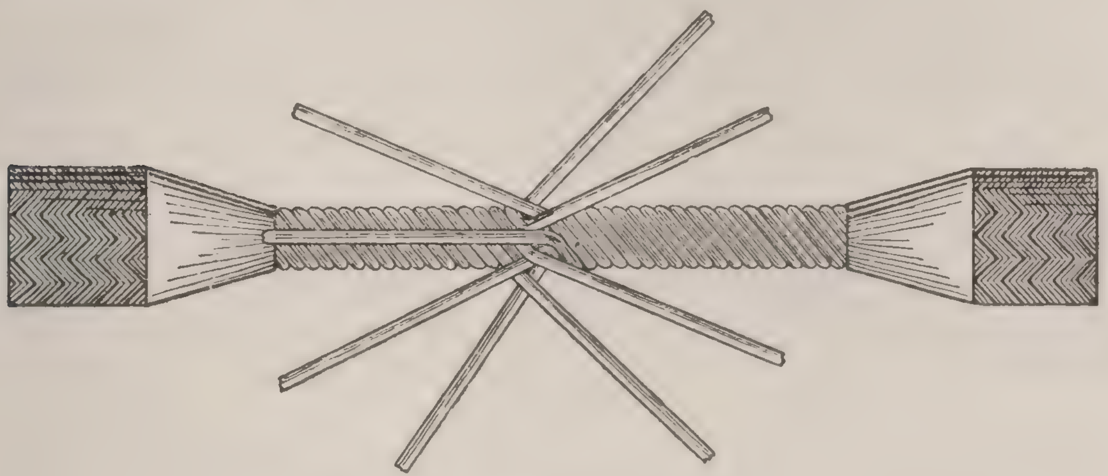


FIG. 186.

Separate out each wire and clean them all with fine emery cloth carefully, so as not to remove any tinning if possible. Straighten each, cutting off half the centre wire of each cable, and then re-twist the outer six up to the end of the centre wire with about the same pitch as the rest of the cable itself in both cases, arranging them so that the six free straight ends form a cone with its apex at the end of the centre wire.

Now push the cones together, so that the six wires of each

interlace alternately, and their apexes touch as shown in Fig. 186, *i. e.* the two middle wires butt against each other. Now press down the left-hand set on to the cable, and hold tightly in the hand. Then wind with the other hand the remaining six wires of the other cable, *one by one*, by, say, half a turn at a time, and evenly to, say, 1 or $1\frac{1}{4}$ inches from the centre, snipping off each what is not wanted, and trimming the ends so as not to project outwards. Repeat these operations with the other half, and

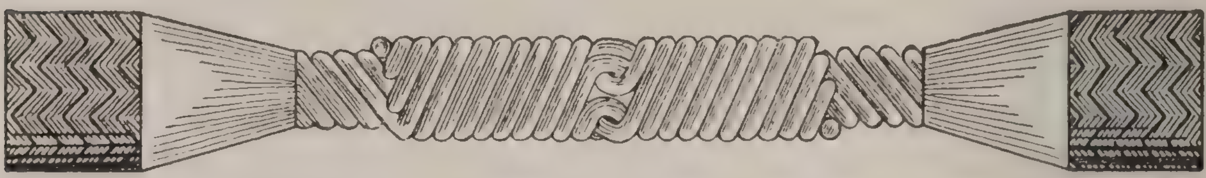


Fig. 187.

finally trim the centre also by pressing with the pliers, but not scraping the cable in so doing.

N.B.—The joint may then appear like Fig. 187 or 188, preferably the latter, which makes the neater joint and is, as seen, wound in the same sense as the main cable.

To solder.—Place in the well-tinned groove of a fairly large soldering iron and run in some solder around it, then when quite hot just touch the joint for merely an instant with a lump of

Fig. 188.

resin, and draw a stick of solder over the joint. This, as a rule, will suffice to cause the whole joint to become tinned ; if it does not, repeat. When properly tinned the joint should leave the iron in a bright state, and with no globules of solder hanging to the underside of it, and should be one solid mass.

To insulate.—When cool taper the ends of the insulation, and starting over the rubber of the cable, wind on spirally with a half overlap two layers of pure I.R. tape in opposite directions and with enough tension to make the insulation firm and solid. The end of this tape is fixed down by I.R. solution, which is also

applied to the outside of the rubber-taping all over the joint by means of the finger. When the spirit has evaporated repeat the above winding process with two layers of black prepared rubber tape overlapping the outer braiding of the cable. Lastly, varnish the joint all over with black waterproof varnish and allow it to dry.

T-Joints Nos. 8 and 10.

To prepare.—Carefully bare about $2\frac{1}{2}''$ to $3''$ of the cable to be tapped. Clean the outside of the stranded core with fine emery cloth and re-tin well.

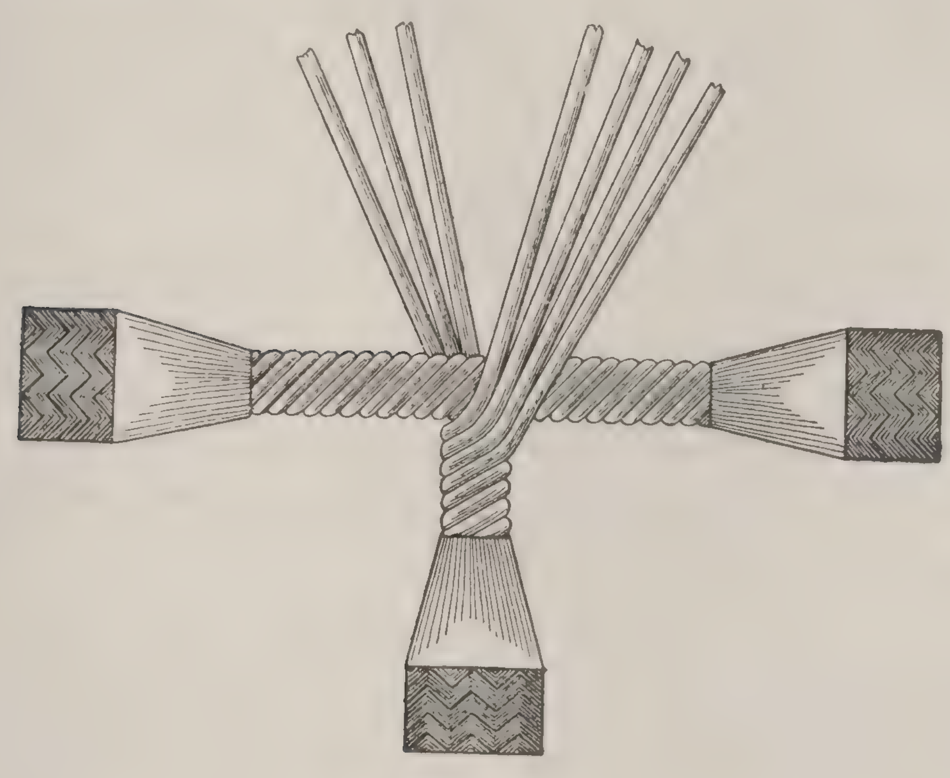


Fig. 189.

Next carefully bare about $3''$ of the end of the other cable, separate out all the wires, clean each with fine emery cloth, straighten and re-twist them up to about $\frac{1}{2}''$ from the end of the insulation in a slightly sharper twist than the ordinary cable. Now spread the 7 wires out to form a **V**, the apex of which is at $\frac{1}{2}''$ from the insulation, 4 wires being one side and 3 the other.

Next press the other cable into the **V** as shown in Fig. 189, then holding the two tightly together, wind *one by one* the 4 wires round in one direction, and the 3 in the opposite direction to, say, $\frac{3}{4}''$ either side of the centre. Clip off what is not wanted

of each wire, and trim so as to not project outwards, when the joint should be as in Fig. 190.

To solder.—Repeat the operation described for joints 7 and 9.

To insulate.—Repeat the operation described for joints 7

Fig. 190.

and 9, except that when the T is reached, wrap carefully round the angles, down the T and back, stretching the tapes tighter here to allow for the extra lapping this part will receive, then finish off the joint as in 7 and 9 above.

Twist-Joint No. 11.

To prepare.—Carefully bare about $4\frac{1}{2}''$ of the two ends to be jointed, and separate out the outer layer of 12 wires, clean them with fine emery cloth and straighten. Without unwinding the inner 7 solder them into a solid mass and cut half off in both cables alike. Now re-twist the outer 12 up to the end of the inner 7 with about the same pitch as the ordinary cable, and arrange them to form a cone with the apex at the end of the inner 7. Then pushing the two cones together and interlacing alternately, proceed to finish exactly as in joints 7 and 9.

T-Joint No. 12.

To prepare.—Carefully bare about $3''$ of the cable to be tapped, clean the outside of the stranded core with fine emery cloth, and re-tin it well.

Next carefully bare about $3\frac{1}{2}''$ of the end of the other cable. Separate out each wire, clean with fine emery cloth, and straighten. Next re-twist both inner and outer sets up to about $\frac{3}{4}''$ from the insulation with a slightly sharper pitch than the ordinary cable, then *carefully* spread out all the wires in the best possible manner to form a **V** with 10 one side and 9 the other. Lastly, press the other cable into this **V**, and finish the joint precisely as in Nos. 8 and 14.

Twist-Joint No. 13.

To prepare.—Carefully bare about $5\frac{1}{2}''$ of the ends of the cables to be jointed, unwinding the outermost layer of 18 wires, and proceed precisely as in No. 11, except that half the inner 19 must now be cut off after soldering them together.

T-Joint No. 14.

To prepare.—Carefully bare about $4\frac{1}{2}''$ to $5''$ of the cable to be tapped and about $5\frac{1}{2}''$ of the end of the other, and proceed exactly as set forth for joint 12, excepting that the **V** will now have 19 wires on one side and 18 the other. Finish it off as there indicated.

Twist-Joint No. 15.

To prepare.—Carefully cut the lead sheathing away, without in any way nicking the copper core of the cable, for about $4''$ of the ends of the cables to be jointed. Next slip on to one cable, to some little distance from the joint to be made so as to be out of the way, a lead sleeve consisting of a length of lead piping, a little larger than the size of the lead-covered cable and some $2''$ longer than the finished joint will be. Then proceed to make the joint and insulate it precisely as in Nos. 7 and 9, taking extra care to get the insulating tapes on tightly and efficiently. Now slip back the loose sleeve over the joint and either carefully "solder" or "solder-wipe" the ends, thus completely sealing in the cable.

Note.—If lead piping to the right size is not available, a sleeve may be cut out of lead sheet, bent round the joint and finally sealed along the edge; this, however, does not make so neat a joint as that with the pipe.

T-Joint No. 16.

To prepare.—Carefully cut away about 4″ of the lead sheathing of the cable to be tapped, great pains being taken to avoid nicking the copper core. Next remove about $3\frac{1}{2}''$ of the lead sheathing from the end of the other cable and slip over this latter a short sleeve of lead piping slightly larger than the ordinary lead-covered cable, and of sufficient length to cover the insulation of the joint and overlap the end of it. Now make and insulate the joint precisely as described in Nos. 8 and 10 and slip back the small sleeve over the insulation; also cut out a piece of sheet lead to form a sleeve over the rest of the joint, its ends overlapping those of the lead covering on the cable by about 1″; lastly, train and trim these lead coverings to fit closely, and solder or solder-wipe the seams to make a neat water-tight joint.

Twist-Joint No. 17.

Prepare and make the joint precisely as in Nos. 1 and 2, when it will have the appearance shown in Fig. 191, and it may preferably be kept as short as possible in order to facilitate insulating it.

To insulate.—Warm up the G.P. covering on either side of the joint, then work and draw down with moistened fingers that on one side half-way over the joint as in Fig. 192, and next that on the other side, giving the form shown in Fig. 193; work the two draw-downs together and sear all the joint with a hot searing iron.

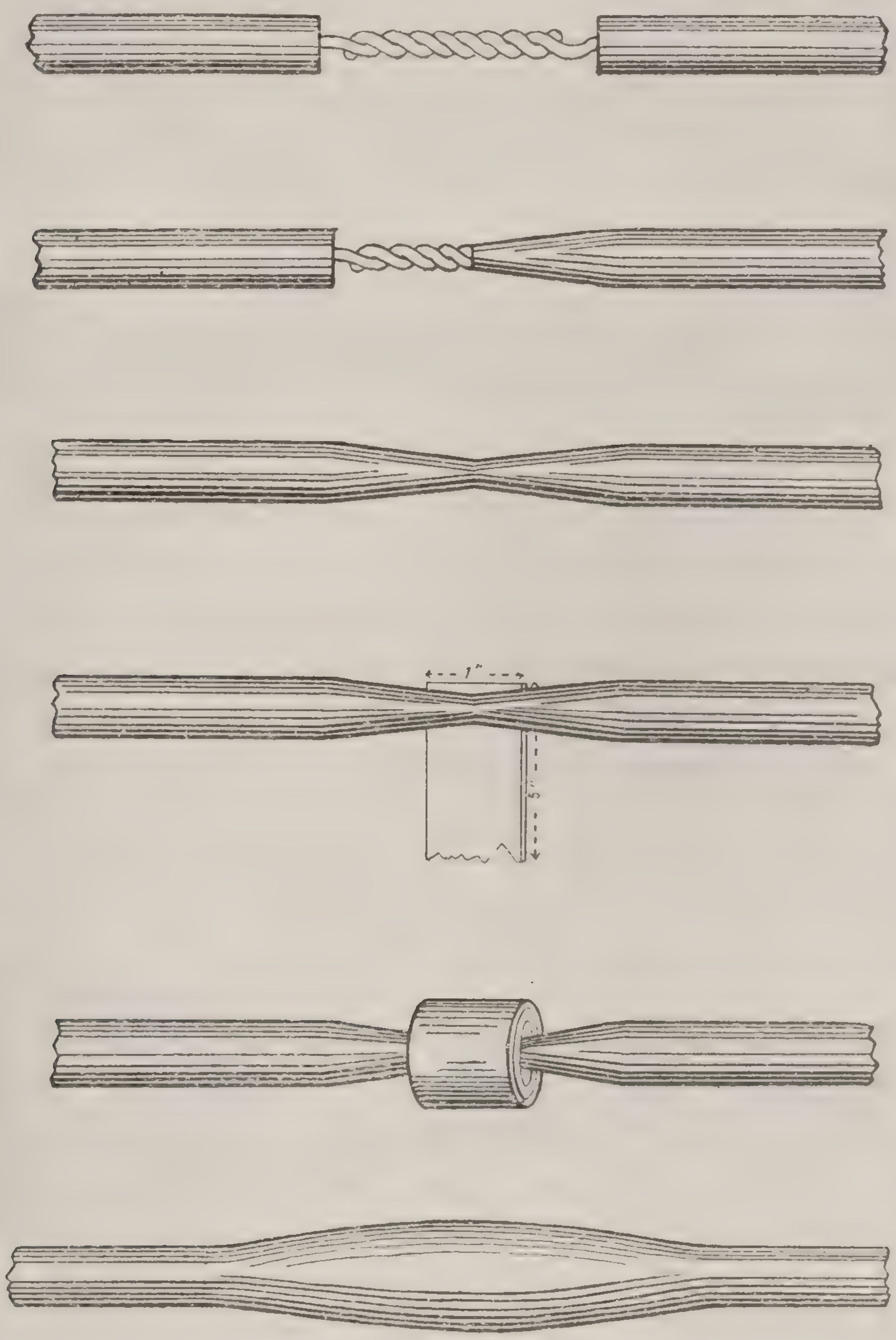
Wrap a strip of G.P. well warmed over a flame round the centre of the draw-down as in Figs. 194 and 195; now work this roll both ways with moistened fingers until it is uniformly distributed over the joint as seen in Fig. 196, finally smoothing all over with a searing iron, and lastly with wetted fingers so as to leave the whole joint quite smooth.

T-Joint No. 18.

Prepare and make the joint precisely as in Nos. 3 and 4, keeping its dimensions small.

Insulate in a somewhat similar manner as in No. 17, working the three branch G.P. coverings into each other at the T. Warm

narrow G.P. strip must now be wrapped round the T, first one way and then the other, and drawn down in the three directions so as to leave a clean, smooth, insulated joint.



Figs. 191—196.

Twist-Joint No. 19.

This joint is made in precisely the same way as No. 11, and is insulated in the same manner, and is shown in Fig. 197.

Slanting T-Joint No. 20.

The cable to be tapped is prepared exactly as in No. 12, the other cable having its inner seven wires soldered together, cut half off and scarfed to the desired angle or slant. The remaining twelve wires are then wound as before in opposite directions, six one way and six the other, round the other cable, giving the joint shown in Fig. 198. The scarf should butt up against the tapped cable and be soldered to it in the final sweating.

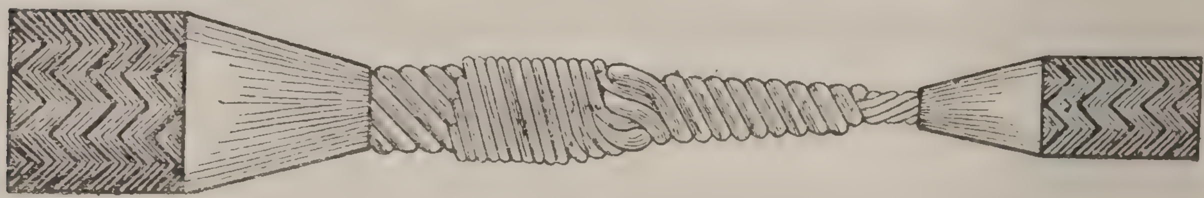


Fig. 197.

Straight Concentric-Joint No. 21.

We will assume that the cable or main is a 37-stranded, lead-covered and armoured one, the gauge being any one of the usual sizes in practice. Treat each of the two ends to be jointed as follows :—Unwrap the outer serving of yarn or hemp for a distance of something like 13 inches from the end, but do not cut it off. Next unwrap the strip armouring for about the same, or a slightly less distance, say 11 or 12 inches, without cutting it off. Then unwrap the inner serving of yarn, which separates the armour and lead sheathing, for some 10 inches, without cutting it off.

Now remove the lead sheathing for about 9 inches altogether.

Joint in Inner Main.—Carefully bare the outer conductors for about 7 inches from the end with a sharp knife. Spread out and clean them each with fine emery cloth and straighten, leaving them outspread. Next bare, carefully, the inner conductors for about 5 inches. Spread out and clean each conductor with fine emery cloth and straighten each. Re-twist up the innermost 19 as they were originally, and cut $\frac{1}{3}$ of this inner 19 strand off. Remove any jagged edges and solder so as to form them into a solid mass.

Each main will now appear as in Fig. 199, except that the unwrapped ends of the yarn and armouring are not shown. Now cut off $\frac{2}{3}$ of every *alternate* wire immediately surrounding

the inner 19, and bringing the two ends of the cables thus prepared together, so that the inner 19's butt up to each other. Interlace these outer wires so that the respective pairs also butt though alternately on either side of the centre. Then bind this

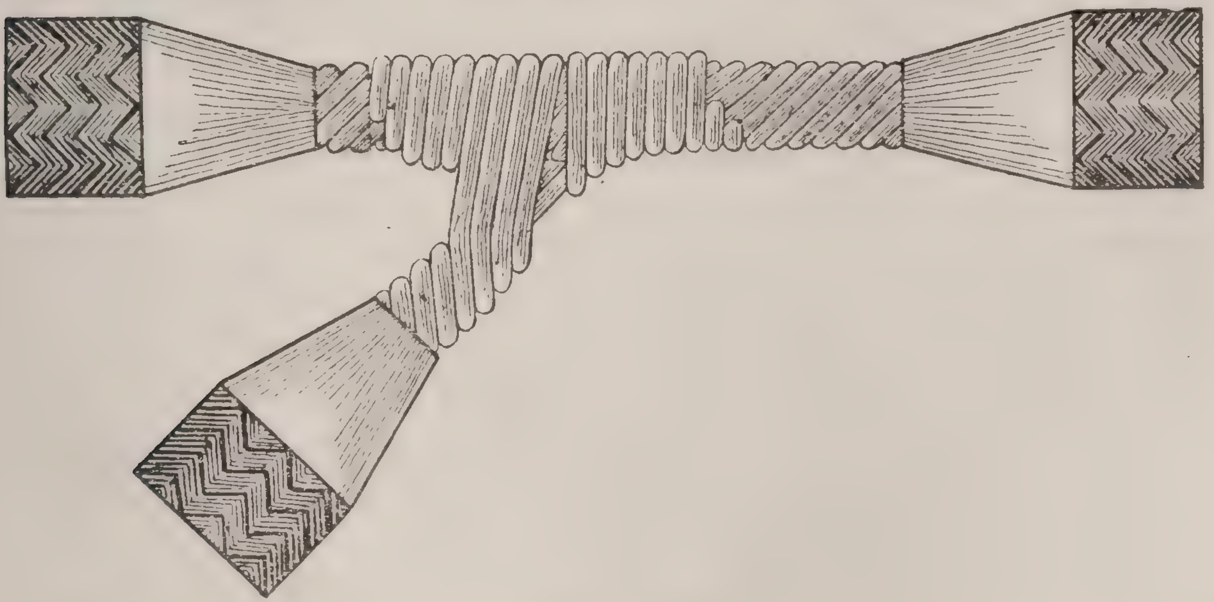


FIG. 198.

first joint of the inner cable with four strips of tinned copper binding wire, each of some ten turns, and the strips equally spaced over the joint, so that the three sets of butts come in between the four strips of binding wire. Lastly, by means of two

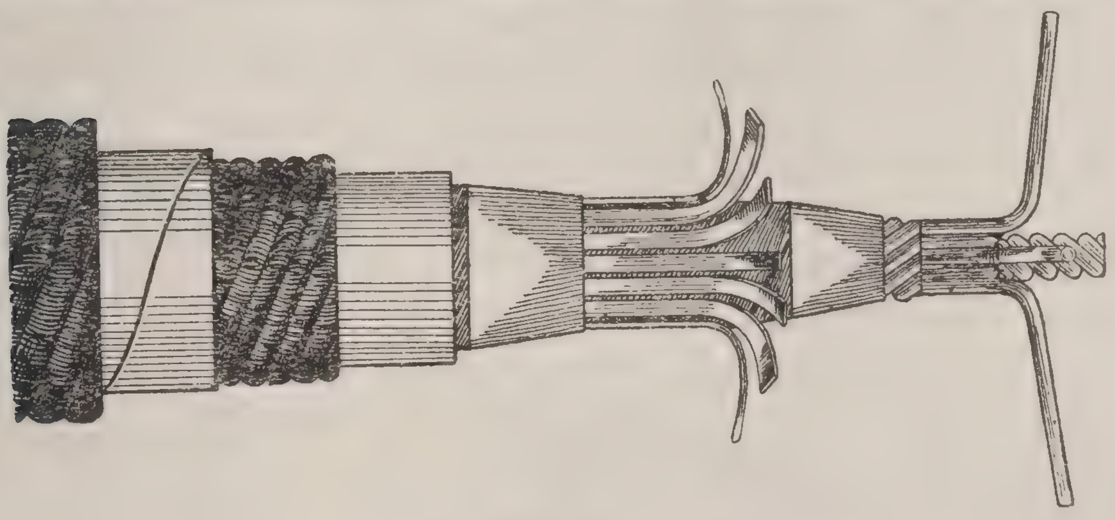


FIG. 199.

ladles, keep pouring melted solder over the joint, catching it in the ladle held underneath and using it over again and again until the joint *takes* the solder and becomes tinned and soldered into a solid mass. Fine powdered resin must be used as a flux in just sufficient quantity.

When cool enough, taper the end of the first insulating covering, when the joint should then present the form shown in Fig. 200. Now wrap on tightly in the usual way, first pure rubber strip, with the application of rubber solution between layers, and then prepared tape up to a thickness slightly exceeding the other insulation.

Joint in Outer Main.—Wrap a sheet or sleeve of copper plate which has been previously cleaned on the outside carefully with emery cloth, and which is of such a thickness as to be comfortably pliable. This sleeve must be the length of the outer conductors, and make one turn or wrap fitting the insulated cable closely. Now cut off half of every *alternate* wire of each outer main and interlace, after cleaning and straightening each as before. Then bend them closely over the sleeve of copper by tinned binding wire in, say, four strips about ½″ wide, the butts of the conductors being between the pair of strips, either end.

Solder as before with the ladles. The joint now has the appearance shown in Fig. 201.

When cool enough insulate up in the usual way, tapering the ends of the old insulation of the cable beforehand.

Protections to insulated joint.—Wrap a piece of sheet lead over the last insulation, so as to form a sleeve of just one complete turn and overlapping the ends of the ordinary lead sheathing of the cable. Then *solder wipe* the ends and down the seam so as to make a good water- and air-tight joint.

Note.—Any part of the lead may be previously painted if desired so as to prevent the lead solder wiping from taking to that part. Next re-serve the inner yarn as far as it will go over the lead joint, adding more to complete the serving.

Then repeat this operation with the strip armouring, and lastly with the outer yarn, when the joint may finally be well tarred over and is then *complete* for *laying in.*

Generally speaking, all joints larger than about $\frac{7}{18}$ can be more expeditiously and effectively soldered by using flux and pouring molten solder out of one ladle over the joint and catching it in a second and larger ladle underneath. The reason is that by this means the joint can be raised to, and kept at, the proper temperature, which is difficult to obtain with soldering irons.

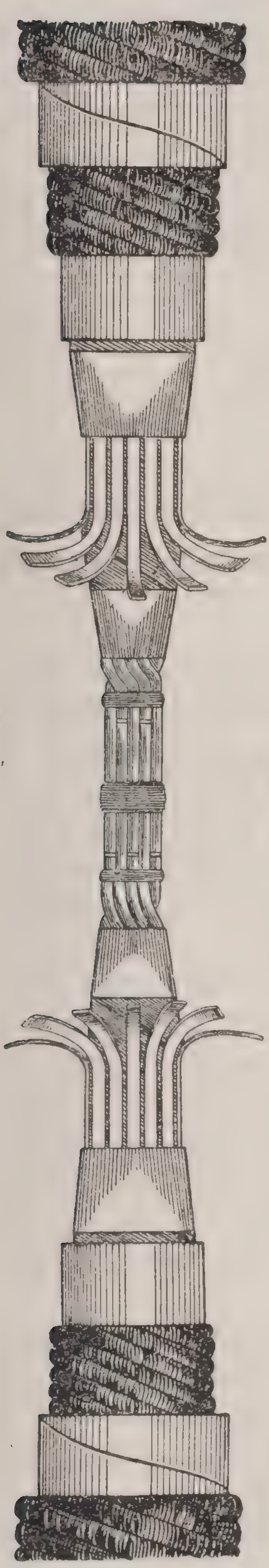


FIG. 200.

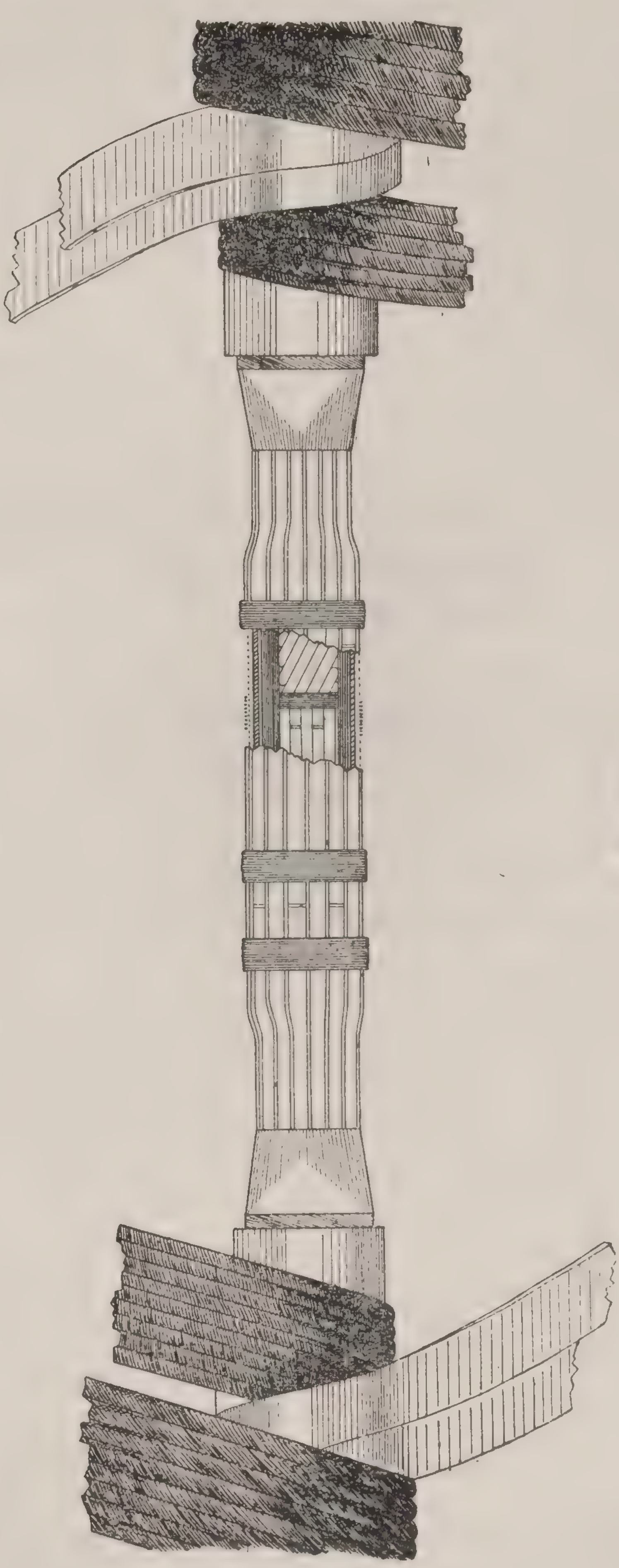


FIG. 201.

APPENDIX

PROOFS OF FORMULAE

Deviation of the deflections of Reflecting Galvanometers from the direct proportional law.

It has been stated on p. 6 that the scale deflections of a D'Arsonval galvanometer are *directly proportional* to the currents producing them. Though this is not rigorously correct, it is sufficiently true for most practical purposes in the usual forms of instruments belonging to this class. For very accurate work, however, it is necessary to apply a correction, usually amounting to a small fraction of 1% for deviation from this law, and this we now proceed to indicate.

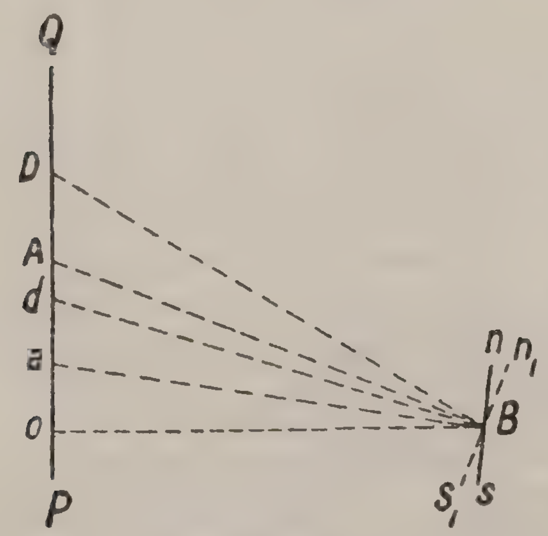


Fig. 202.

Let O be the zero of the scale PQ, presumably in the centre, though, for convenience, only one-half of the scale is shown, and let Od and OD be the scale deflections of the spot of light on PQ from zero for currents C_1 and C_2 through the galvanometer coil.

If B is the centre of the needle ns, then OB is the incident ray of light from some source at O and Bd, BD the reflected rays for the two positions of the mirror and its attached needle ns.

By drawing the normals to the mirror in each position we get

Ba and BA respectively, and it can at once be shown that the angle $dBD = 2aBA$, or that the angular motion of the mirror is half that of the reflected ray.

Now since ns is contained by the plane of its coil for *no* current and is parallel to PQ, we shall always have (for the small angular motion of ns usually obtained in mirror galvanometers)—

$$C_1 : C_2 = \tan. \tfrac{1}{2}\, OBd : \tan. \tfrac{1}{2}\, OBD.$$

$$= \tan. \tfrac{1}{2} \frac{Od}{OB} : \tan. \tfrac{1}{2} \frac{OD.}{OB.}$$

$$= \frac{\sqrt{1+\left(\frac{Od}{OB}\right)^2} - 1}{\frac{Od}{OB}} : \frac{\sqrt{1+\left(\frac{OD}{OB}\right)^2} - 1}{\frac{OD}{OB}} \text{ (accurately),}$$

or if Od and OD are not very different and are small, then—

$$C_1 : C_2 = Od : OD \text{ (approximately).}$$

Measurement of the Internal Resistance of Secondary Cells. (Fall of Potential Method.)

Proof of Formula.—Referring to p. 76, let E = the total E.M.F. of the cell or battery, and V the potential difference at its terminals, when sending a current A.

If then B is the internal resistance of the cell and R the resistance of the external circuit, we have by Ohm's Law—

Fall of Potential round external circuit $= AR = V$,
and „ „ in the cell itself $= AB$.

Hence we must have $E = AR + AB = A\,(R + B)$.

But the Fall of Potential in the cell itself is also $= E - V$,

$$\therefore\ E - V = AB,$$

and
$$\therefore\ B = \frac{E-V}{A} \text{ ohms;}$$

or thus
$$\frac{E}{V} = \frac{A\,(R+B)}{AR}$$

Hence
$$B = \frac{E-V}{V} R, \text{ but } AR = V, \text{ or } R = \frac{V}{A}$$

$$\therefore\ B = \frac{E-V}{A} \text{ ohms.}$$

Measurement of Resistance. (Wheatstone Bridge Method.)

Proof of Formula.—Referring to Fig. 33 I., p. 82, let V_1, V_2 and V_3 = the Potentials of the points P, N and T respectively; then the point Q will also be at the potential V_2, since when the bridge is "balanced" no deflection will occur on the galvanometer, owing to there being *no difference* of potential between N and Q to cause a current to flow. Now let C_1, C_2, C_3 and C_4 be the currents passing through the resistances r_1, r_2, r_3 and r_4 respectively of the arms of the bridge.

Then $$V_1 - V_2 = C_3 r_3 = C_4 r_4,$$
and $$V_2 - V_3 = C_1 r_1 = C_2 r_2;$$
but since on balance being obtained no current flows through the galvanometer, *i. e.* between the points N and Q, we get
$$C_1 = C_4 \quad \text{and} \quad C_2 = C_3,$$
whence by division $$\frac{C_3 r_3}{C_2 r_2} = \frac{C_4 r_4}{C_1 r_1}$$
or $$\frac{r_3}{r_2} = \frac{r_4}{r_1}$$
and $$\therefore\ r_1 r_3 = r_2 r_4.$$
The resistance in the galvanometer and battery circuits is immaterial, so long as it is not great enough to diminish the sensitiveness of the tests; consequently it may vary without vitiating the results at all.

Measurement of Low Resistance. (Potential Difference Method.)

Solution of Inferences.—Referring to Fig. 34, p. 85, let A = current flowing through the resistances r and R in series, and v, V = Potential Differences across their extremities respectively.

Then from Ohm's Law we have
$$A = \frac{V}{R} = \frac{v}{r}$$
but since the galvanometer resistance is high compared with

either r or R, we have its deflections d_r and d_R across these resistances respectively proportional to v and V, whence

$$\frac{d_R}{R} = \frac{d_r}{r}$$

and $\therefore$ the unknown resistance, assumed to be (r), is

$$r = \frac{d_r}{d_R} R \text{ ohms.}$$

Assumptions.—(1) That no fluctuations of the current A have occurred in the interval of time between the observations d_R and d_r of any particular pair.

(2) That the introduction of the galvanometer across the resistance has not altered the P.D. at their respective terminals.

(3) That the deflections are proportional to the currents, which is very nearly true for ordinary reflecting instruments.

Errors may arise from the warming up of the resistances to be compared, due to the passage of the current, and the consequent alteration of resistance. The current should not be strong enough to do this.

Also from the presence of thermo currents caused by the warming up of a junction of two dissimilar metals.

Lastly, from the inconstancy of the current, which can be minimized by taking readings with one of the resistances *before* and after that with the other, and noting the mean of the two.

Measurement of very high or Insulation Resistance. (Direct Deflection Method.)

Solution of Inferences.—Referring to p. 101—

Let b = the internal resistance of the battery,

g = the galvanometer resistance,

$C_R\, C_r$ = the currents through the galvanometer, causing deflections $d_R\, d_r$, when the resistances R and r are in circuit respectively and K be a constant converting these deflections to actual currents.

Then if E is the E.M.F. of the battery we have by Ohm's Law

$$C_R = Kd_R = \frac{S_R}{S_R + g} \times \frac{E}{R + b + \dfrac{S_R g}{S_R + g}} = \frac{S_R E}{(R+b)(S_R+g) + gS_R}$$

Similarly $C_r = Kd_r = \frac{S_r}{S_r+g} \times \frac{E}{r+b+\frac{S_r g}{S_r+g}} = \frac{S_r E}{(r+b)(S_r+g)+gS_r}$

whence by division

$$\frac{d_R}{d_r} = \frac{(r+b)(S_r+g)+gS_r}{(R+b)(S_R+g)+gS_R} \times \frac{S_R}{S_r}$$

Now b will always be negligibly small compared with R or r.

Hence $$\frac{d_R}{d_r} = \frac{r(S_r+g)+gS_r}{R(S_R+g)+gS_R} \times \frac{S_R}{S_r} = \frac{r\left(1+\frac{g}{S_r}\right)+g}{R\left(1+\frac{g}{S_R}\right)+g}$$

or $$d_R\left[R\left(1+\frac{g}{S_R}\right)+g\right] = d_r\left[r\left(1+\frac{g}{S_r}\right)+g\right] \quad . \; . \; (1)$$

When no shunts are used S_R and S_r both = infinity and

$$d_R(R+g) = d_r(r+g)$$

Lastly, if (g) is negligibly small compared with R and r, then

$$d_R R = d_r r,$$

and $$\therefore r = \frac{d_R}{d_r} R \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (2)$$

The assumption made in both formulas 1 and 2 above are that—

(*a*) The E.M.F. of the testing battery remains constant throughout the tests, which is justifiable owing to its working through such high resistances.

(*b*) The internal resistance of the battery is so small compared with R and r as to be quite negligible.

In the relation (2) above, it is assumed that—

(*c*) No shunts are used at all with the galvanometer.

(*d*) The galvanometer resistance (g) is so small compared with R and r as to be quite negligible.

Insulation Resistance of Electric Light Installations while working.

Solution of Inferences.—Let A and B be the two points of the circuit most convenient for attaching the wires to, that come from the two-way key K. Let r_v = resistance of the voltmeter, and $R_1 R_2$ = insulation resistances of the $+^{\text{ve}}$ and $-^{\text{ve}}$ sides, respectively,

of the network or installation, in ohms. Then that of the whole system, everything included, is $R=\frac{R_1R_2}{R_1+R_2}$

In Fig. 203, E represents the earth or nearest gas or water-pipe.

If then V_1 and V_2 are the voltmeter readings when placed between the $+^{ve}$ and $-^{ve}$ sides of the circuit and earth, and V the voltage between the mains, we have

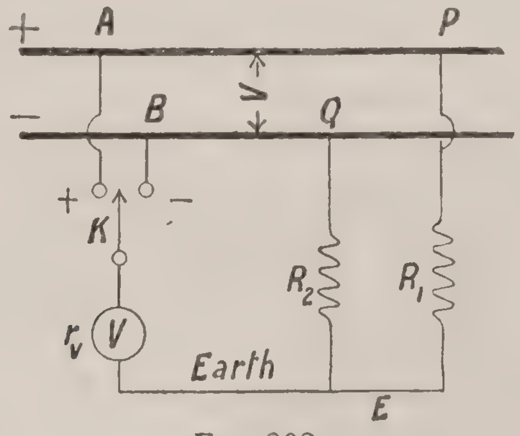


FIG. 203.

Fall of P.D. between A and E, viz. $V_1=V\dfrac{\dfrac{R_1\,r_v}{R_1+r_v}}{\dfrac{R_1\,r_v}{R_1+r_v}+R_2}$

Similarly the fall of P.D. between B and E, viz. $V_2=V\dfrac{\dfrac{R_2\,r_v}{R_2+r_v}}{\dfrac{R_2\,r_v}{R_2+r_v}+R_1}$

Since EPA and EVA are in parallel, and the combination in series with EQB.

$$\therefore\ V_1+V_2=V\frac{(R_1+R_2)\,r_v}{(R_1+R_2)\,r_v+R_1R_2}$$

$$\therefore\ \frac{1}{V_1+V_2}=\frac{1}{V}\left(1+\frac{R_1R_2}{R_1+R_2}\cdot\frac{1}{r_v}\right)$$

or
$$R=r_v\left(\frac{V}{V_1+V_2}-1\right)$$

also
$$R_2=R\,\frac{V_1+V_2}{V_1}=\frac{r_v\,[V-(V_1+V_2)]}{V_1}$$

$$R_1=\frac{r_v\,[V-(V_1+V_2)]}{-V_2}$$

(V_1+V_2) representing the absolute sum of the two voltages, assuming they are to opposite sides of zero.

Insulation Resistance of a Storage Battery.

Solution of Inferences.—Referring to Fig. 50, p. 128—

Let $P_1\,P_2\,P_3\,\ldots$ be points on the battery which are partially earthed,

$R_1\,R_2\,R_3\,\ldots$ be the resistances of these earths,

$C_1\,C_2\,C_3\,\ldots$ be the currents flowing from $P_1\,P_2$ and $P_3\ldots$ to earth when G is connected, K closed, and $r=0$.

$E_1\,E_2\,E_3\,\ldots$ the E.M.F.s between P and $P_1\,P_2\,P_3\,\ldots$ respectively.

Then if all the currents flow from battery to earth

$$C_1R_1 - C_G g = E_1, \qquad C_G g - C_8 R_8 = E_8,$$
$$C_2R_2 - C_G g = E_2, \qquad C_G g - C_9 R_9 = E_9,$$
$$\text{etc.} \qquad\qquad \text{etc.}$$

Also
$$C_1 + C_2 + C_3 + \ldots + C_G = 0.$$

Hence—

$$\frac{E_1+C_Gg}{R_1}+\frac{E_2+C_Gg}{R_2}+\ldots+\frac{C_Gg-E_8}{R_8}+\frac{C_Gg-E_9}{R_9}+\ldots+C_G=0.$$

Now when C_G falls to C_r by increasing g to $(g+r)$ we have

$$\frac{E_1+C_r(g+r)}{R_1}+\frac{E_2+C_r(g+r)}{R_2}+\ldots+\frac{C_r(g+r)-E_8}{R_8}$$
$$+\frac{C_r(g+r)-E_9}{R_9}+\ldots+C_r=0.$$

Hence from the difference of these two equations we have

$$\{C_Gg-C_r(g+r)\}\left(\frac{1}{R_1}+\frac{1}{R_2}+\ldots\right)+C_G-C_r=0,$$

whence
$$R=\frac{C_Gg-C_r(g+r)}{C_r-C_G}$$

Insulation Resistance of Dynamos and Motors.

Solution of Inferences.—The proof of the formula employed in this test is precisely the same as that given on p. 494 for the insulation resistance of electric light installations. It will not therefore be repeated here.

Efficiency of Direct-Current Generators. (Hopkinson's Electrical Method.)

Solution of Inferences.—Referring to Fig. 85, p. 224, let $V_1\ V_2\ V_3$ be the terminal voltages of α, β and γ respectively, and A the main current through them all. Then we have—

Power developed by the generator α $= A V_1$ Watts.
,, supplied by the auxiliary γ $= A V_3$,,
,, given to the motor β $= A V_2$,,

Hence $A V_3 = A V_2 - A V_1 = A\ (V_2 - V_1)$,

which is the power lost in both machines together.

If V = normal working pressure of α and β when running as dynamos, r_α and r_β = normal resistances of their shunts.

Then $\frac{V^2}{r_\alpha}$ and $\frac{V^2}{r_\beta}$ = Watts wasted in their respective shunt circuits, neglecting any extra resistance such as R_1 and R_2. Now let the total internal loss in α = that in β. Then—

Efficiency of dynamo α

$$\Sigma = \frac{\text{useful power developed}}{\text{total power put in}} = \frac{A V_1}{\frac{1}{2} A V_3 + A V_1 + \frac{V^2}{r_\alpha}}$$

Efficiency of motor β

$$\Sigma_\beta = \frac{\text{useful B.H.P. developed}}{\text{total E.H.P. put in}} = \frac{-\frac{1}{2} A V_3 + A V_2 + \frac{V^2}{r_\beta}}{A V_2 + \frac{V^2}{r_\beta}}$$

$$= \frac{\frac{1}{2} A \left(V_2 + V_1\right) + \frac{V^2}{r_\beta}}{A V_2 + \frac{V^2}{r_\beta}}.$$

If now we neglect the shunt losses $\frac{V^2}{r_\alpha}$ and $\frac{V^2}{r_\beta}$ in comparison with the outputs of α and β, then

$$\Sigma_\alpha = \frac{2 V_1}{V_1 + V_2} \qquad \text{and } \Sigma_\beta = \frac{V_1 + V_2}{2 V_2}$$

$$\therefore\ \Sigma_\alpha \Sigma_\beta = \frac{V_1}{V_2}$$

Lastly, if we assume that the machines are so alike that

$$\Sigma_\alpha = \Sigma_\beta = \Sigma \text{ (say)}$$

Then $$\Sigma^2 = \frac{V_1}{V_2} \qquad \text{or } \Sigma = \sqrt{\frac{V_1}{V_2}}$$

which is therefore the efficiency of either machine.

Self-Induction by the Impedence Method using Single-Phase Alternating Currents.

Solution of Inferences.—Referring to Fig. 132, p. 360, let V_a and V_D be the readings of the voltmeter when an alternating and direct current of the same strength A is successively passed through the self-induction whose value L is to be determined.

Let n = frequency of the alternating current in $\sim$ per sec., then its angular velocity $p = 2\pi n$.

If R = the ohmic resistance of the self-inductive circuit we have for the direct current, by Ohm's Law,

$$R = \frac{V_D}{A} \text{ ohms,}$$

and for the alternating current we have

$$\frac{V_a}{A} = \sqrt{L^2p^2 + R^2} \text{ the impedence of the circuit.}$$

Hence by substitution $$\frac{V_a}{A} = \sqrt{(Lp)^2 + \left(\frac{V_D}{A}\right)^2}$$

or $$\left(\frac{V_a}{A}\right)^2 = (Lp)^2 + \left(\frac{V_D}{A}\right)^2$$

whence $$L^2p^2 = \left(\frac{V_a}{A}\right)^2 - \left(\frac{V_D}{A}\right)^2 = \frac{1}{A^2}\left\{V_a^2 - V_D^2\right\}$$

and $$\therefore L = \frac{1}{pA}\sqrt{V_a^2 - V_D^2} \text{ henries.}$$

Self-Inductions in Series and Parallel Laws of Combination.

Proof of Formula.—Suppose that we first take the arrangement represented in Fig. 135 γ, p. 367, in which all the self-inductions L_1 L_2 L_3 and L_4 are in parallel.

Let them possess ohmic resistance R_1 R_2 R_3 and R_4 respectively, but no mutual induction.

Also let L_c and R_c be the combined or equivalent self-induction and ohmic resistance of the combined parallel circuit such that if substituted for the parallel branches the same potential difference V would exist at the terminals, causing the same current A to flow through.

If then the angular velocity of the alternating current be denoted as usual, by $p=2\pi n$ where n is the periodicity, we can apply the solution obtained by Lord Rayleigh for the impedence of parallel circuits and given in a paper by him "On Forced Harmonic Oscillations of Various Periods."—*Phil. Mag.*, May 1886.

Thus
$$L_c=\frac{B}{A^2+B^2p^2} \text{ and } R_c=\frac{A}{A^2+B^2p^2}$$

where
$$A=\Sigma\left(\frac{R}{L^2p^2+R^2}\right) \text{ and } B=\Sigma\left(\frac{L}{L^2p^2+R^2}\right)$$

Hence
$$\frac{V}{A}=\sqrt{L_c^2p^2+R_c^2}=\sqrt{\frac{1}{A^2+B^2p^2}}$$

Substituting the values of A and B given above we get

$$\frac{V}{A}=\sqrt{L_c^2p^2+R_c^2}=\sqrt{\frac{1}{\left(\Sigma\,\frac{L}{(L^2p^2+R^2)^2}\right)^2p^2+\left(\Sigma\,\frac{R}{(L^2p^2+R)^2}\right)^2}}$$
$$=\sqrt{\frac{1}{\left(\Sigma\,\frac{L}{I^2}\right)^2p^2+\left(\Sigma\,\frac{R}{I^2}\right)^2}}$$

Next take the arrangement shown in Fig. 135β, in which the inductions are connected up, two in series and two in parallel.

Here, since self-inductions in series sum up like resistance in series, we have, applying the last equation for two parallel branches, that

$$\frac{V}{A}=\sqrt{L_c^2p^2+R_c^2}$$
$$=\sqrt{\frac{1}{\left(\frac{R_1+R_2}{(I_1+I_2)^2}+\frac{R_3+R_4}{(I_3+I_4)^2}\right)^2+\left(\frac{L_1+L_2}{(I_1+I_2)^2}+\frac{L_3+L_4}{(I_3+I_4)^2}\right)^2p^2}}$$
$$=\frac{(I_1+I_2)\,(I_3+I_4)}{\sqrt{(L_1+L_2+L_3+L_4)^2p^2+(R_1+R_2+R_3+R_4)^2}}$$

$$=\frac{\sqrt{(L_1+L_2)^2p^2+(R_1+R_2)^2}\ \sqrt{(L_3+L_4)^2p^2+(R_3+R_4)^2}}{\sqrt{(\Sigma L)^2p^2+(\Sigma R)^2}}$$

which is the combined or effective or equivalent impedence of this particular branched circuit.

Electrostatic Capacity of Electrical Cables. (Multicellular Voltmeter Method.)

Proof of Formula—Let C_a = capacity of the standard air condenser ; C_L = capacity of the cable, and K_1K_2 = capacities of the voltmeter at the potentials V_1 and V respectively.

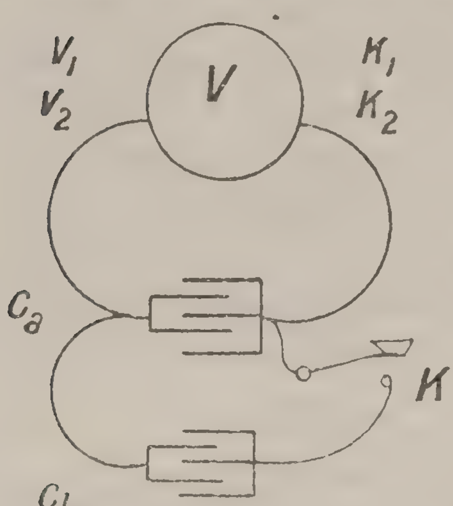


FIG. 204.

Then if C_a is first charged in parallel with V to a potential V_1, the quantity of electricity in the charge is $Q_1=V_1(C_a+K_1)$, since the capacities C_a and K_1 are in parallel.

If now K is closed so as to put C_L in parallel with the first two and the potential falls in consequence to V_2, the quantity of the charge is still the same assuming no leakage, and we have

$$Q_1=V_2\ (C_a+C_L+K_2).$$

Hence $$V_1\ (C_a+K_1)=V_2\ (C_a+C_L+K_2),$$

or $$\frac{V_2}{V_1}=\frac{C_a+K_1}{C_a+C_L+K_2}$$

and by a well-known rule in proportion we have

$$\frac{V_2}{V_1-V_2}=\frac{C_a+K_1}{C_a+C_L+K_2-C_a-K_1}=\frac{C_a+K_1}{C_L+K_2-K_1}$$

$$\therefore\ V_2C_L+V_2K_2-V_2K_1=V_1C_a-V_2C_a+V_1K_1-V_2K_1.$$

Hence $$V_2C_L+V_2K_2=C_a(V_1-V_2)+V_1K_1$$

and $$\therefore\ C_L=\frac{C_a(V_1-V_2)+V_1K_1-V_2K_2}{V_2}$$

or neglecting the capacity of the voltmeter in comparison with C_a and C_L we have $\quad C_L=C_a\dfrac{V_1-V_2}{V_2}.$

Capacity of Concentric Cables. (Standard Magneto Inductor Method.)

Proof of Formula.—Referring to the test, p. 377, let N be the number of turns of wire on the standard inductor and F the total number of lines of magnetic force threading the gap, then the interlinking of turns and field $= FN$.

If now $R =$ the total resistance in ohms of the inductor circuit, the whole quantity of electricity Q_1 producing the throw d_I is

$$Q_1 \propto d_I \propto \frac{NF}{R}.$$

Again, let $C =$ the capacity of the cable in microfarads, $V =$ the potential difference in volts to which it is charged, then the quality of electricity Q_2 causing the throw d_c is

$$Q_2 \propto d \propto CV,$$

and 1 microcoulomb $= \dfrac{10^{-1}}{10^6} = 10^{-7}$ C.G.S. units of quantity,

also 1 ohm $= 10^9$ C.G.S. units of resistance.

Hence $$\frac{d_I}{d_c} = \frac{FN}{10^{-7} \times 10^9 \, RVC} = \frac{FN}{100 \, RVC}$$

and $$\therefore \; C = \frac{FN}{100 \, RV} \frac{dc}{d_I} \text{ microfarads.}$$

For the greatest accuracy we ought to have

$$d_I = d_c.$$

Three Voltmeter Method of Measuring the "True Electrical Power" in Single-Phase Alternating Current Circuits.

Solution of Inferences.—Referring to Fig. 140, p. 382, let the $\sqrt{\text{mean square}}$ value of the voltages, as given by either a hot wire or electrostatic voltmeter, be denoted by V, V_1 and V_2 for the positions shown in the figure; also let $(r) =$ the value of the non-inductive resistance QR in ohms.

Then we require the value of the *mean* or True Power in Watts W given to the whole circuit PR in which the whole power is absorbed; let v, v_1 and v_2 be the instantaneous values of the

voltages corresponding to the $\sqrt{\text{mean square}}$ values V, V_1 and V_2 respectively at any instant (t), then $v = v_1 + v_2$; also if (a) = instantaneous current in amps. flowing through PR at this same instant t, we have the instantaneous value of the power in Watts (w) given to PR at that instant as $w = va$;

but we also have $\quad a = \frac{v_2}{r}, \quad$ since QR is non-inductive,

whence $\quad w = va = v\frac{v_2}{r} = \frac{vv_2}{r}$

or $\quad vv_2 = wr;$

now since $\quad v = v_1 + v_2$

$$\therefore \; v_1 = v - v_2$$

and $\quad 2v_1v_2 = 2v_2(v - v_2) = 2vv_2 - 2v_2^2.$

Hence by substitution $2v_1v_2 = 2wr - 2v_2^2$;

but $\quad v^2 = v_1^2 + 2v_1v_2 + v_2^2 \quad$ by squaring,

$\therefore \; v^2 = v_1^2 + 2wr - 2v_2^2 + v_2^2 \quad$ by substitution.

Hence $\quad w = \frac{1}{2r}\left\{v^2 - v_1^2 + v_2^2\right\}.$

Integrating this equation for the period T we get

$$\int_0^T wdt = \frac{1}{2r}\left\{\int_0^T v^2dt - \int_0^T v_1^2dt + \int_0^T v_2^2dt\right\}$$

and finally $\quad W = \frac{1}{2r}\left\{V^2 - V_1^2 + V_2^2\right\}.$

Efficiency of Transformers. (Blakesley's Three Dynamometer Method.)

Solution of Inferences.—Referring to p. 423, on which will be found a formula derived by Mr. Blakesley for expressing the power given to the primary of a transformer in his method of measuring the efficiency of such an appliance, Professor Ayrton and Mr. Taylor have deduced the following general proof of this relation, which makes no assumption whatever as to the nature of the current, whether sinusoidal or otherwise, but only that there is no magnetic leakage between primary and secondary windings. This is approximately true for "closed circuit" though not for "open circuit" transformers, so that the method cannot be considered a very good one.

Let $a_1\, a_2$ = instantaneous values of the currents and $v_1\, v_2$ those of the E.M.F.s in the primary and secondary windings having $N_1\, N_2$ turns respectively, and B = the mean density of lines in the core, then if $R_1 R_2$ = ohmic resistances of the primary and secondary

we have
$$v_1 = R_1 a_1 + \frac{N_1}{10^8}\frac{dB}{dt}.$$

But
$$\frac{N_2}{10^8}\frac{dB}{dt} = R_2 a_2 \text{ since we assume no leakage,}$$

$$\therefore\ v_1 = R_1 a_1 + \frac{N_1}{N_2} R_2 a_2;$$

multiplying all through by a_1 we get

$$v_1 a_1 = R_1 a_1{}^2 + \frac{N_1}{N_2} R_2 a_2 a_1.$$

Integrating this last equation between the limits of the period T we get

$$\frac{1}{T}\int_0^T v_1 a_1 dt = \frac{R_1}{T}\int_0^T a_1{}^2 dt + \frac{N_1}{N_2}\frac{R_2}{T}\int_0^T a_1 a_2 dt.$$

Hence if A is the split dynamometer reading we finally have

$$W_P = A_1 R_1 + \frac{N_1}{N_2} R_2 A$$

APPARATUS

Preparation of the Clark Standard Cell.

Definition of the Cell.—The cell consists of zinc and mercury in a saturated solution of zinc sulphate and mercurous sulphate in water, prepared with mercurous sulphate in excess, and is conveniently contained in a cylindrical glass vessel.

Preparation of Materials.—*The Mercury.*—To secure purity it should be first treated with acid in the usual way, and subsequently distilled in vacuum.

The Zinc.—Take a portion of a rod of pure zinc, and solder to one end a piece of copper wire. Clean the whole with glass paper, carefully removing any loose pieces of zinc. Just before making up the cell, dip the zinc into dilute sulphuric acid, wash with distilled water, and dry with a clean cloth or filter paper.

The Zinc Sulphate Solution.—Prepare a saturated solution of pure (re-crystallized) zinc sulphate by mixing in a flask distilled water with nearly twice its weight of crystals of pure zinc sulphate, and adding a little zinc carbonate, in the proportion of about 2 per cent. by weight of zinc sulphate crystals, to neutralize any free acid. The whole of the crystals should be dissolved with the aid of *gentle heat*, i. e. not greater than 30° C., and the solution filtered while still warm into a stock bottle. Crystals should form as it cools.

The Mercurous Sulphate.—Take mercurous sulphate sold as pure, which is white, and wash it thoroughly with cold distilled water by agitation in a flask; drain off the water, and repeat the process at least twice, but after the last washing, drain off as much

water as possible. Mix the washed sulphate, in the proportion of about 12 per cent. by weight of $ZnSO_4$, crystals, with the zinc sulphate solution, adding sufficient crystals of zinc sulphate from the *stock bottle* to ensure saturation, and a small quantity of pure mercury. Shake them well up together to form a paste of the consistency of cream. Heat the paste sufficiently to dissolve the crystals, but *not above* 30° C. Keep the paste for one hour at this temperature, agitating it from time to time, and then allow it to cool.

Crystals of zinc sulphate should then be distinctly visible throughout the mass. If this is not the case, add more crystals from the stock bottle, and repeat the process. This method ensures the formation of a saturated solution of zinc and mercurous sulphates in water. The presence of the free mercury throughout the paste preserves the basicity of the salt, and is of the *utmost importance.* Contact is made with the mercury by means of a platinum wire about No. 22 B.W.G., which is prevented from making contact with the other materials of the cell by being sealed into a glass tube, the ends of the wire projecting beyond those of the tube. One end forms the terminal; the other end, and part of the glass tube, dip into the mercury.

To set up the Cell.—The cell may be conveniently set up in a small test tube of about 2 cms. in diameter and 6 or 7 cms. deep.

Place the mercury in the bottom of this tube, filling it to a depth of, say, 1·5 cms.

Cut a cork about 0·5 cm. thick to fit the tube. At one side of the cork bore a hole through which the zinc rod can pass tightly; at the other side bore another hole for the glass tube which covers the platinum. At the edge of the cork cut a nick through which the air can pass when the cork is pushed into the tube.

Pass the zinc rod about 1 cm. through the cork. Carefully clean the glass tube and platinum wire, then heat the exposed end of the wire red hot, and insert it in the mercury in the test tube, taking care that the whole of the exposed platinum is covered.

Shake up the paste, and introduce it without contact with the upper part of the sides of the test tube, filling the tube above the mercury to a depth of rather more than 2 cms.

Now insert the cork and zinc rod, allowing the glass tube to pass through the hole in the cork made for it.

Push the cork gently down until its lower surface is nearly in contact with the liquid. The air will thus be nearly all expelled, and the cell should be left in this condition for at least twenty-four hours before sealing, which should be done in the following way—

Melt some marine glue until it is fluid enough to pour by its own weight into the test tube above the cork, using enough to cover completely the zinc and soldering. The glass tube should project above the top of the marine glue.

The cell thus set up may be mounted in any desirable way; do it so that the cell is immersed in a water-bath up to the level, say, of the upper surface of the cork. Its temperature can then be determined more accurately than is possible when the cell is in air.

Instruments for Standard Measurements of the Highest Accuracy.

The potentiometer in general must rank as one of the first of this kind, not solely from the point of view of accuracy, but also because of the ease and rapidity with which the measurements possible with it can be taken. The principle underlying its use is contained in the Clark-Poggendorff method of comparing E.M.F.s, and an elementary application of this principle in Poggendorff's method of calibrating a voltmeter, a detailed description of which will be found in the author's work entitled *Practical Electrical Testing* for first and second year students. It may, however, here be remarked that when using the potentiometer for measuring current, resistance, and high voltages, the principle, as is well known, consists in reducing any of the three electrical quantities which are to be measured, to the form of electrical pressure, or E.M.F., so that it can be compared by means of a potentiometer with a standard pressure such as that of the Clark cell.

The N.C.S. Potentiometer.

This is a somewhat new type of potentiometer, in which there is *no slide wire* to get injured or deteriorate with time. It works entirely by means of adjusted resistances of perfectly definite values, the ends of which are permanently attached to circular rows of contact studs.

Fig. 205 shows a general view of the potentiometer in its containing case with the lid slightly raised. The internal

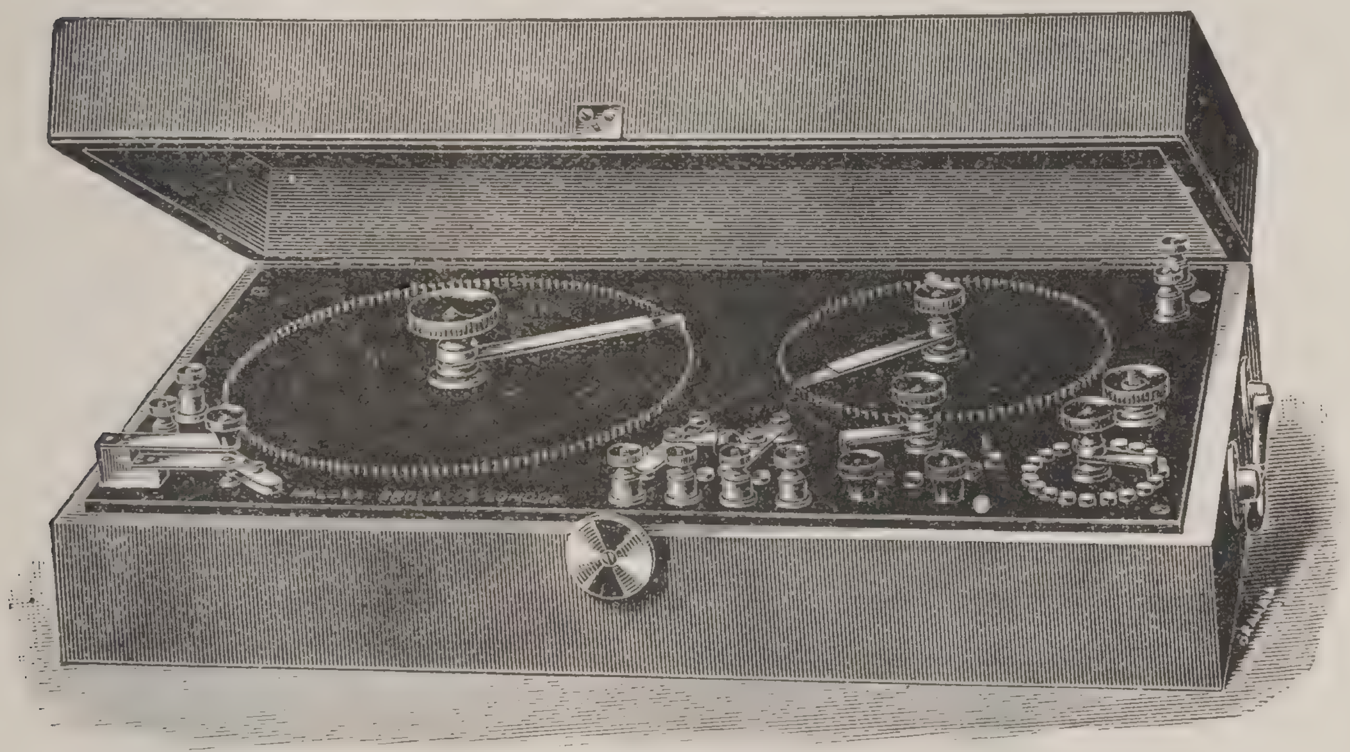


Fig. 205.

arrangements and connections are seen in the symbolical diagram, Fig. 206, with reference to which the working of the instrument will be understood more clearly. The two diagrams exactly correspond with one another so far as the relative positions of the various parts are concerned.

A secondary battery is joined to the two terminals *F*, and sends a current through the two dials *C* and *D* in series and then through adjusting resistances *K* and *H*. The *C* dial has 150 exactly equal coils in it, numbered from 0 to 150. The *D* dial has 100 equal coils in it, the whole dial being exactly equal to the one coil in *C*. The two together are therefore

equivalent to a slide wire with 15,000 fixed contacts at equal distances. The dial K contains nineteen equal small wire resistances, and H is a carbon resistance for fine adjustment of the current, so that working with one secondary cell, the potentiometer can be set to read volts direct with a Clark cell.

A standard cell of known E.M.F. is joined up to $A+$ and $A-$, and the switch L is put over to A.

Any convenient galvanometer, the most convenient form being a D'Arsonval, is joined up to the galvanometer terminals.

Say the voltage of the standard cell, at the temperature used, is 1·4412; the arm of the C dial is then set to 144, and that of the D dial to 12.

The galvanometer key is then depressed on to its first stop (taking care that the catch is underneath so that it cannot go right down) and the outside resistance K is adjusted till—with the switch on one stop—the deflection is one way, and, with it on the next, in the opposite direction. The head H is then turned until an exact balance is obtained. A slight pressure should be put on the head H when turning it; it may not be found necessary to use H at all the next step.

Push aside the catch on the galvanometer key and depress fully. This cuts out a ½ megohm which was previously in circuit to protect the cell.

An exact balance can now be obtained by further adjustment of H or possibly K.

The instrument is now set so that each division on C is equal to ·01 of a volt, and each division on D is equal to ·0001 volt.

Any low E.M.F. to be measured is joined to BB terminals and the switch L set to B; for example, a Leclanché cell or the terminals of a low resistance for current measurement.

The galvanometer key is then again fully depressed and a balance obtained by moving C and D.

The E.M.F. between B and B is then read direct on the two dials; for example, if the reading on C is 83 and that on D is 67, the volts between B and B is ·8367. Any volts higher than 1·6 have to be measured on the terminals marked VOLTS + and −, and with the switch L turned to V. The switch M is then turned to any convenient multiplying power, 3, 10, 30, etc., and the readings obtained from C and D (when the balance is

obtained as before) have to be multiplied by this multiplying power; for example, if C is 103 and D 26, and M is standing at 30, the volts on the terminals are $1{\cdot}0326 \times 30$ or say 30·98.

The resistances in H and K are arranged so that any voltage up to 3 volts can be used at F, thus allowing either a secondary cell or two large Leclanchés to be used; it is well to leave the battery joined up to the instrument for some ten minutes or so, so as to steady down before beginning work.

A set of coils are required to potentiometer down high voltages amounting to 300 so as to bring them within range of the dials.

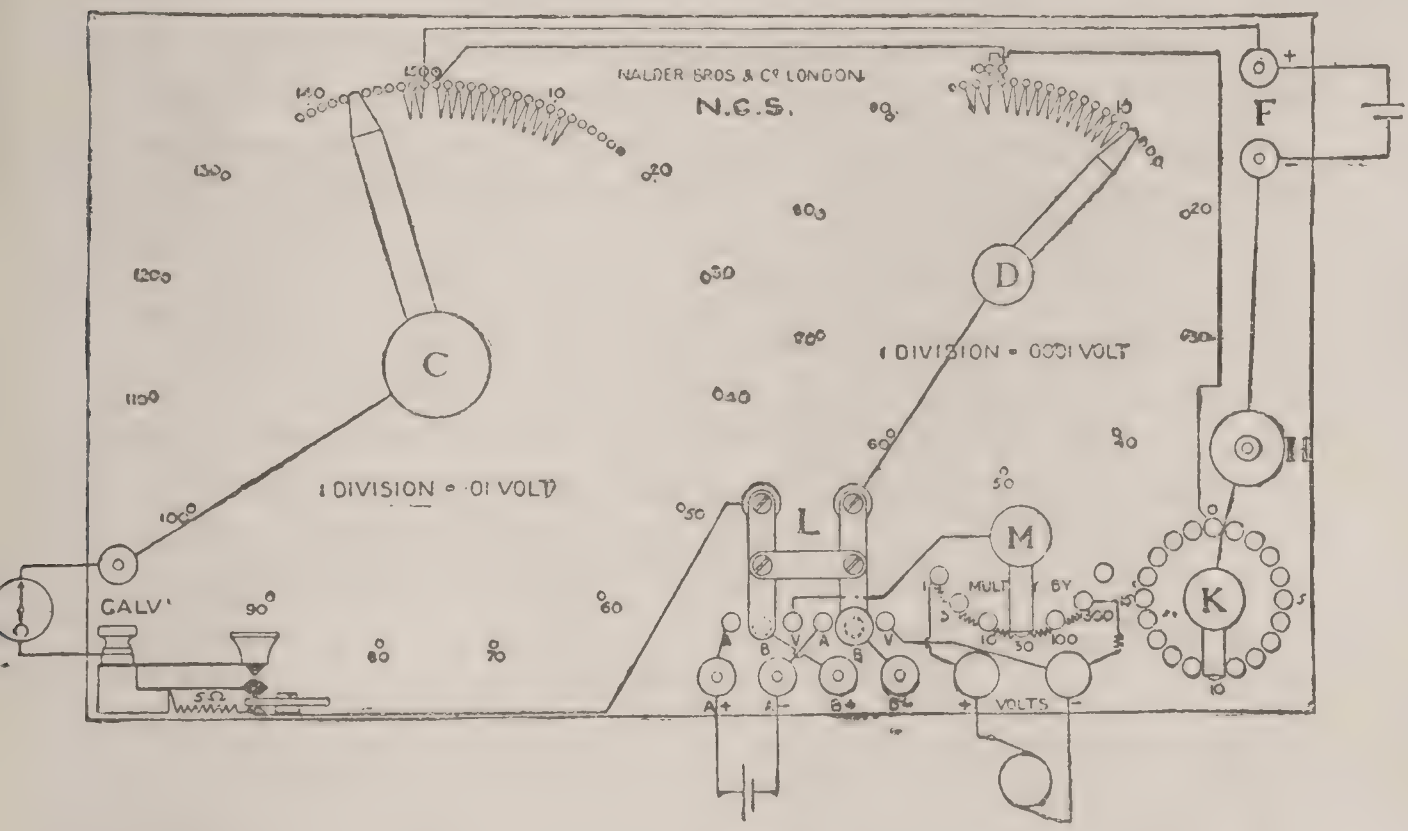


Fig. 206.

The three-way switch L according to its position inserts the known fraction of the high voltage, the standard cell, or any third unknown E.M.F. in circuit with the instrument.

The following points should be noted—

When using a standard cell, get a balance with the catch under the key before removing the catch.

Be careful to join up any cells, etc., to their right terminals and not + to −.

Do not screw the head H down too tight. It draws up a rod which compresses a carbon resistance; there is plenty of range

to cover the difference between two stops K without squeezing it at all tight.

Press head H down when turning; it keeps the resistance steadier.

The total resistance in the M dial is 100,000 ohms; not more than 200 volts should be applied to VOLTS terminals for any length of time. For higher values a known resistance can be added outside and allowed for.

Crompton's Potentiometer.

This potentiometer is shown diagrammatically in Fig. 207. It consists of a wire AB stretched over a scale and through which a constant current is maintained from one or more secondary cells of sufficient size to keep it fairly constant for three or four

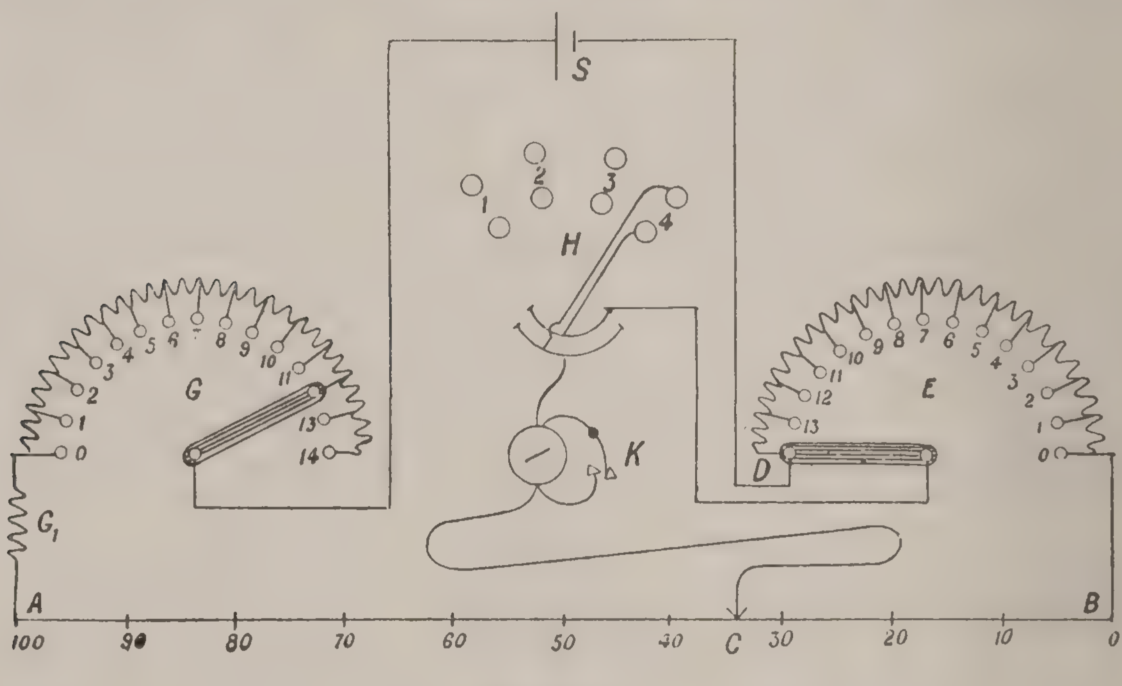


FIG. 207.

days' working. A variable resistance G and rheostat G_1 is introduced in series with the cell and wire, arranged so as to adjust this current to give a certain difference of potential at the two ends of the wire. The circuit which includes the E.M.F. to be measured is connected to one end D of the effective potentiometer wire, and through a galvanometer to a sliding saddle C, which carries a knife edge to make contact on the wire AB at any desired point of its length. This circuit is so coupled

up that the E.M.F. to be measured is *opposed* to the E.M.F. in the portion of the slide wire BC, in order that the position of C on the slide wire can be adjusted until the two E.M.F.s balance one another so that no current passes through the galvanometer. In order to compare the E.M.F. to be measured with the standard E.M.F., a Clark cell is put in in circuit with the galvanometer, and the position of the slide C is noted when the above-mentioned balance has been obtained. The electrical pressure or E.M.F. required to be compared is then substituted for the Clark cell, and a similar balance found by adjusting the slider C a second time. Then the comparison between the two E.M.F.s can be made by comparing the respective lengths of the slide wire included between the points B and C in the first and second case. By dividing the slide wire AB into a suitable number of parts, and adjusting the variable resistance until the galvanometer comes to zero when the Clark cell is in circuit and the contact C is at a point on the scale corresponding to the temperature value of the Clark cell, say 1·434 at 15° C., the instrument becomes direct reading in volts or fractions of volts.

Dr. Fleming in the year 1883 first called attention to the advantages of this system of measuring, and since that time the system has been steadily developed, and refinements have been introduced. As the following description will apply to instruments such as are suitable for a municipal standardizing laboratory, it is here necessary to specify what are the requirements and limits of accuracy within which these instruments may be reasonably expected to measure. First comes the verification of voltmeters. This is of importance on account of the disputes that are likely to arise as to the pressure supplied from electric-lighting stations. Voltmeters for standard pressures of 150 to 200 and 220 volts are principally used for noting the pressures in the consumers' houses, and for such cases it is desirable that their readings at about the standard pressure should be verified to one-tenth of a volt or within one part in 1000. Next it should be possible to verify and certify the constant of the various kinds of meters by which the electrical supply is measured to the consumers. They also ought to be verified to one part in 1000. Next comes the verification of ordinary ammeters, or current instruments used for trade

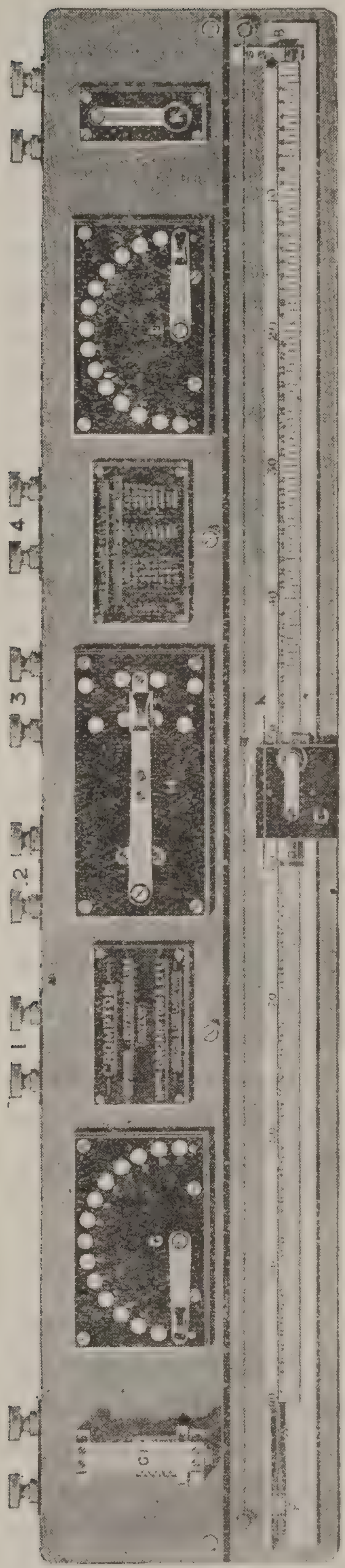


FIG. 208.

purposes to about the same degree of accuracy. In both these last cases the range through which instruments would have to be compared is very considerable; for instance, the instruments sent for verification may be those used for testing electric lamps or for telegraphic purposes, measuring currents of one milliampere, up to those used for metallurgical or electrolytic purposes up to 5000 amperes or more.

The correct comparison of resistance standards as well as the correctness of the ratios of resistance boxes ought to be capable of verification to one part in 10,000. In the comparison of resistances must be included the testing of the conductivity of various metals, and the insulation resistance of various insulating materials.

In the first form of instrument proposed by Dr. Fleming, the potentiometer wire was made of an alloy of platino-iridium four metres long, and had a resistance of about 23 ohms. The wire was divided into two parts and stretched over a scale; the whole of the wire, being required for measuring purposes, had to be carefully calibrated, that is to say, its electrical resistance made equal per unit length throughout its entire length. This was a very tedious and expensive process. It was found very difficult, if not impossible, to obtain wire as it finally left the drawplate, which was suffi-

ciently homogeneous to be used without further adjustment. In most cases the whole of the wire had to be carefully scraped or rubbed until the required equality of resistance was obtained. Such a wire was very valuable, and if it was broken or accidentally melted, its loss was a serious matter.

In a later form of instrument (Fig. 208) Messrs. Crompton have abandoned the use of this expensive material, and at the same time have arranged so that only one-fifteenth part of the wire AB is stretched over the scale and subject to the wear of working. AB in the drawing shows this portion of the wire 25 in. long, stretched over a scale divided into 1000 parts; therefore each division on the scale being one-fortieth of an inch, represents one fifteen-thousandth of the pressure at the two ends of the wire; in other words, if the instrument being standardized to 1·5 volts at the terminals of its wire, each of the smallest divisions of the scale represents $\frac{1{\cdot}5}{15{,}000}$ or one ten-thousandth part of the volt. The resistance of this wire is usually about 2 ohms, and the remaining wire is divided into fourteen coils, each of about 2 ohms, so that the resistance of the whole is 30 ohms. These coils are naked spirals, the terminals of which are fixed to the underside of the fourteen contact blocks shown at E. The swinging arm shown can make contact with any one of these blocks. It is an easy matter to adjust these fourteen coils when the instrument is first made so that their resistance will accurately equal one another, and the resistance of the working wire AB can then be adjusted by slightly stretching it until it is also exactly equal to any of them. As the fourteen coils are protected they never need further adjustment, but if in the course of time the exposed portion of the wire AB becomes worn or is in any way damaged, a new piece of wire can be substituted in a few minutes and stretched by means of the stretching screw shown at F, until its resistance over the portion from 0 to 100 exactly balances that of any one of the fourteen coils. C is the sliding saddle carrying the contact. This consists of an ebonite box arranged to slide smoothly on the scale. It is provided with a knife-edge spring contact so that the contact may be made with a regular pressure which is independent of the pressure of the hand of the user; it also has a micrometer adjustment for

accurate work. The semi-circular switch G and the cylindrical rheostat G_1 shown to the left, which latter gives a graduation of resistance, form part of the variable resistances above described, and which are required to reduce the difference of potential between the terminals of the wire from that of the secondary cell 2 volts to any desired value. The value that has been chosen in this instrument is 1·5 volts. The instrument is provided with four pairs of terminals, 1, 2, 3, 4. A Clark cell can be put on to one of these, and any three other electrical pressures to be measured can be connected on to the others. The switching arrangement H shown in the centre brings the galvanometer into series circuit with any one of these pairs of terminals. The contact key K shown to the extreme right is used for short-circuiting the galvanometer.

Whenever it is required to measure an electrical pressure greater than that on the terminals of the potentiometer, that is to say, in this case, greater than 1·5 volts, the pressure to be measured is applied to certain terminals of a resistance box, and the terminals of the potentiometer are connected to other terminals on this box, which include between them a resistance which is an even part, say, one-tenth, one hundredth, or one thousandth of the entire resistance of the box.

When, however, the potentiometer is employed for measuring currents, this is done by measuring the difference of pressure at two points on standard resistances which, in order to make the instrument direct-reading, must be either 1 ohm, one-tenth, one-hundredth, or one-thousandth and so on. As these low resistances have often to carry very high currents, they have to be designed so that they do not heat to a sufficient extent to introduce errors. A description of a few different forms of them will be found on p. 604.

Method of "Setting Potentiometer" by Standard Cell.

One secondary cell being connected direct to the extreme left-hand pair of terminals, the galvanometer to those on the extreme right and a standard Clark cell to terminals IV suppose. Note the *temperature* of the Clark cell on its own thermometer, and from the table showing its E.M.F. for different temperatures

affixed to the instrument or by calculation, using the formula on p. 17, or table p. 643, obtain its present E.M.F. corresponding to its present temperature. For example, suppose this to be 15° C., then the E.M.F. = 1·4340 volts.

Next place the double switch H on studs 4, the lever on stud 14 of E, and the slider key C at 340 on the scale, then pressing this latter, adjust the resistance G and rheostat G_1 so that the galvanometer comes back to the zero, at which it was originally set. Now every one of the scale divisions will be equal to one ten-thousandth of a volt, and the potentiometer is thus "*set*." To use the instrument for voltage measurements H is turned to the unknown E.M.F., while G and G_1 remain untouched, only E and C now being varied to obtain balance with the unknown E.M.F. in circuit with the galvanometer.

Precautions.—A high resistance, such as 10,000 ohms, should always be connected up in series with the standard cell before placing this across the terminals of the potentiometer, but when balance in the "setting" is practically obtained, the resistance can be cut out of circuit to make the balance as sensitive as possible. By doing this the cell will not be able to send any but an extremely small current when the balance is *far from perfect*, and only under such conditions of use can the cell be relied on as an accurate and constant standard of E.M.F. of the value set forth in the table above mentioned.

In taking a series of readings with a potentiometer, care should be taken, at intervals, to see that the "*setting*" with the Clark cell remains constant, and if it doesn't, to adjust so that it is.

Sources of Error.—The current from the secondary or "*Working cell*" through the stretched wire and resistances E *altering*, owing to the E.M.F. of this cell varying. To prevent this it is advisable *not* to use a newly-charged cell, the E.M.F. of such being liable to frequent alteration, but to use one amply large enough, that has already been $\frac{1}{4}$ discharged. An extremely constant E.M.F. and current will then be obtained.

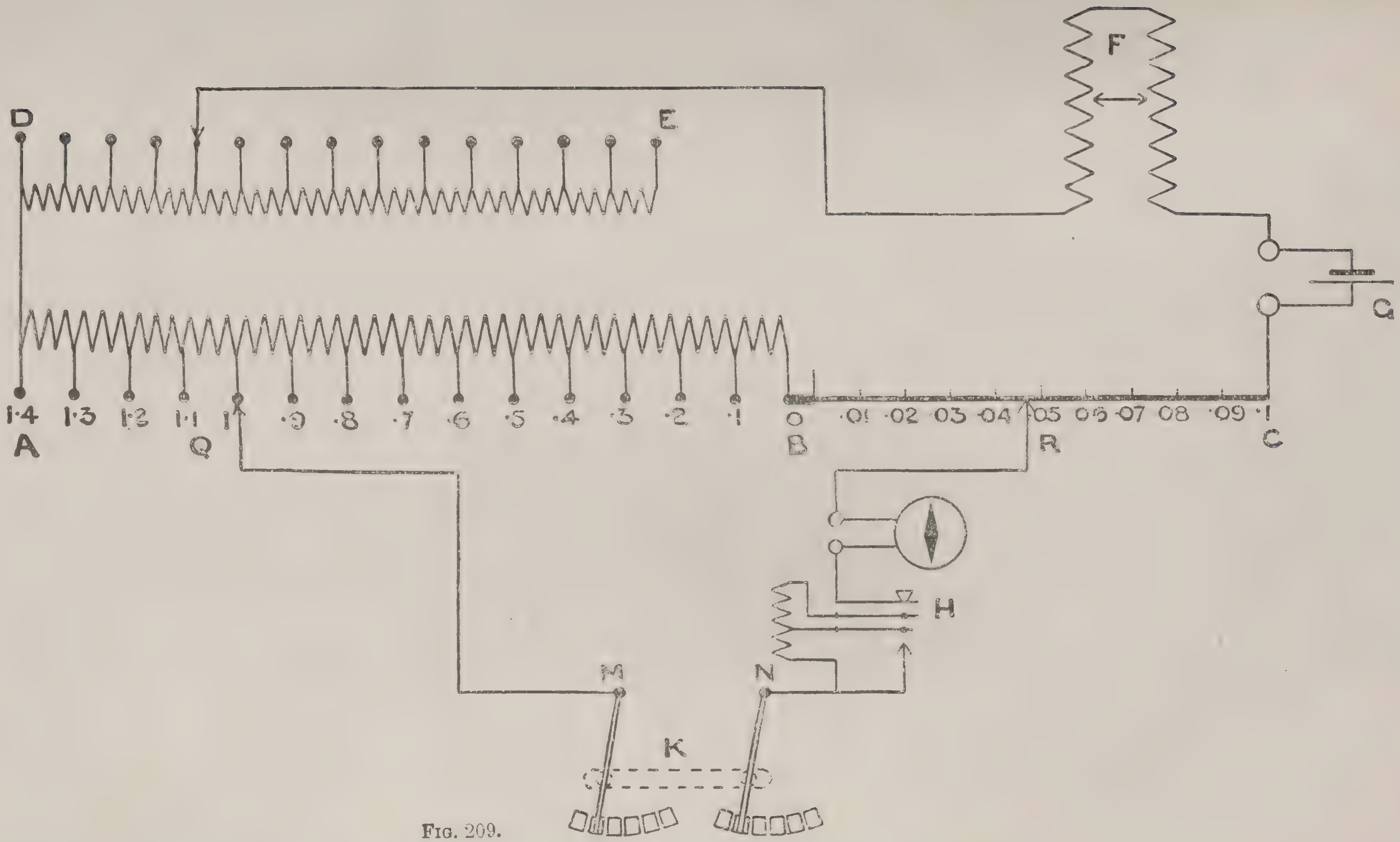
Again, a further error will be introduced by the stretched wire not being uniform, and great pains should be taken to ensure that it is uniform. This may be proved by calibrating it carefully in the manner employed in thus testing a metre bridge wire, a

full description of which is given in the author's work on *Practical Electrical Testing*; or by sending a perfectly steady and constant current through the wire and measuring the fall of potential down equal lengths throughout the scale.

Still another error may be caused by leakage causing the galvanometer to deflect when an accurate balance *has* been obtained. To avoid this, the insulating of the various pieces of apparatus should be attended to.

The above form of potentiometer has been much improved by the makers, and that made at the present time differs from the above in several constructional details. These, together with the appearance and connections of the latest type of instrument, will be easily understood from the following description by the makers.

The construction of the potentiometer itself is shown diagrammatically in Fig. 209. The calibrated wire is arranged in fourteen coils, called potentiometer coils, lettered AB, and a straight section BC, called the scale wire, the resistances of the several coils and of the straight section being equal. One sliding contact Q moves over the terminals of the fourteen coils, and another R along the straight wire. The reading of the instrument in the position shown is 1·046. The pairs of points whose potential differences are to be compared are connected to the blocks of the double-pole switch K, whose levers, MN, connect them, one pair at a time, to the sliding contacts QR through the galvanometer. The galvanometer key H is arranged to complete the circuit through two resistances, which are short-circuited in succession as the key is depressed. The current required is derived from a small secondary battery G. An adjustable resistance, consisting of a set of coils DE, and a continuous rheostat F is placed in the circuit. By adjusting these the resistance of the circuit and the current passing through it from the storage cell, and consequently the fall of potential along the scale wire can be continuously altered, and the operator is able to obtain a galvanometer balance against a standard cell when the reading of the sliders is that of the known E.M.F. of the cell at its actual temperature. If, for example, the temperature of the cell be 15°, so that its E.M.F. is 1·434 volts, the sliders may be set to that reading, and the galvanometer brought to zero by



D
E
F
G
1·4
1·3
1·2
1·1
1
·9
·8
·7
·6
·5
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·3
·2
·1
0
·01
·02
·03
·04
·05
·06
·07
·08
·09
·1
A
Q
B
R
C
H
M
N
K

Fig. 209.

adjusting the resistance *DE* and the rheostat *F*. When this has been done the scale readings at all points are direct readings in volts.

A view of the potentiometer is given in Fig. 210, and a diagram of its internal connections in Fig. 211. Here *a b* is the scale wire; *c* the set of equal potentiometer coils in series with it; *d* is the double-pole switch connecting the six pairs of terminals *ABCDEF* in succession to the slide contacts; *e f* are the resistance coils and rheostat respectively, and *G* is the galvanometer key. All the moving contacts are under glass, and the coils and the scale wire are inside the box. The box itself is completely closed, but the inside can be inspected by removing a sliding bottom. Nearly all the measurements made, involve the use of a standard cell, and one pair of terminals, the pair *A*, is assigned to its connections to save confusion in working. Fuses of fine wire are inserted at all terminals except those for the galvanometer to save the instrument coils in the case of an accidental connection to a source of high pressure.

Two scales are engraved for slide wire readings. One is a series of even divisions from 0 to 105, the resistance of the scale wire between 0 and 100 being the same as that of each potentiometer coil. It has been found convenient to be able to take readings a little beyond the 100 mark without having to move the potentiometer coil switch, and the scale is extended to 105 to admit of this. The other scale gives the values of the Clark cell at different temperatures, and is used in the following way :—The potentiometer coil switch is set to 14, and the slide to the temperature of the Clark cell taken from the thermometer attached to it. The potentiometer reading is then the correct value in volts of the Clark cell at that temperature. By adjustment of the rheostat the galvanometer is balanced, and when this has been done the current in the potentiometer wire is such that readings at all points give correct values in volts, and the instrument is a direct-reading voltmeter. Its maximum range is then 1·5 volts, reading in thousandths of a volt, and by inspection to ten thousandths.

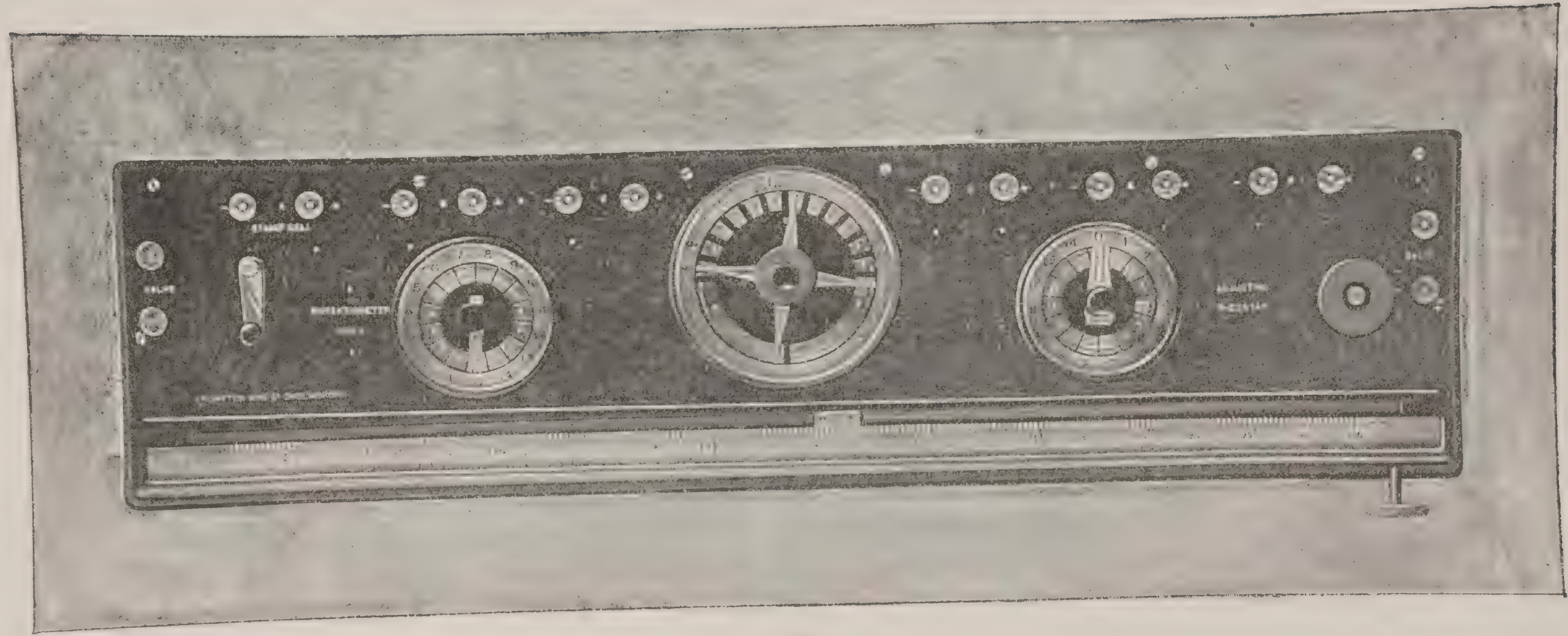


Fig. 210.

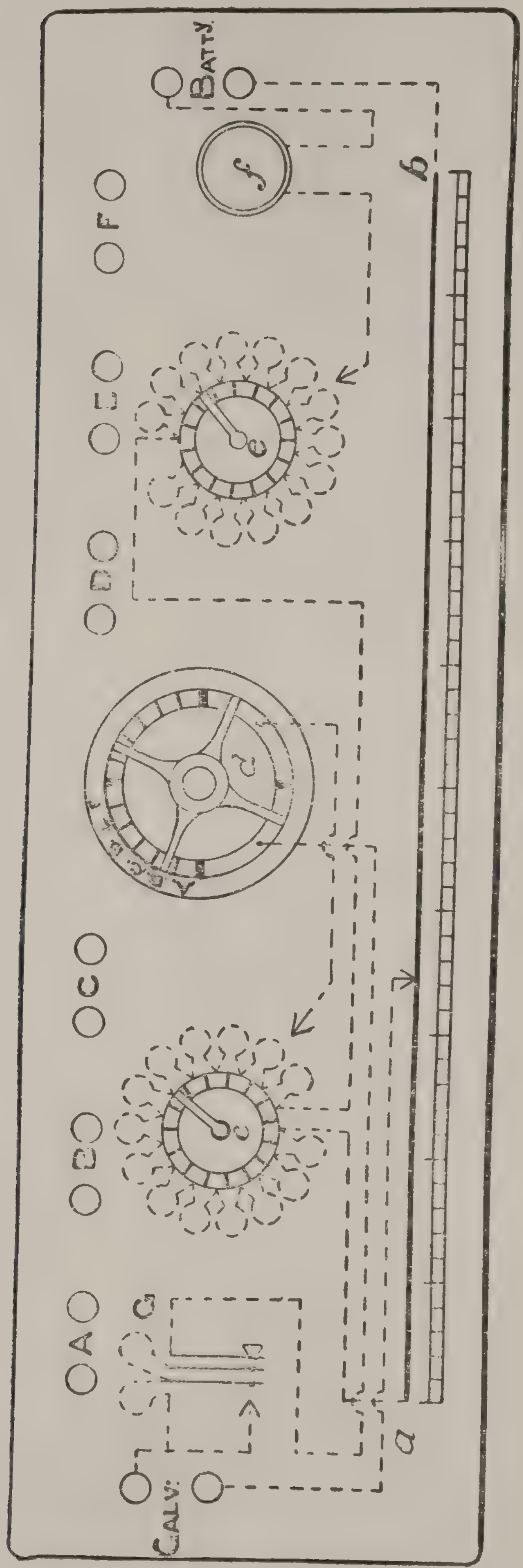


Fig. 211.

Potentiometer Volt Box.

This is a simple and most convenient piece of apparatus by means of which a known definite fraction of any voltage can be easily and quickly obtained. Hence such an instrument can be employed with a potentiometer for the measurement of high voltages, enabling the small requisite voltage to be taken off and used for measurement in the potentiometer. Fig. 212 shows a general view and Fig. 213 (p. 523) a plan of the internal connections of a variable volt box designed by the author for use with a Crompton potentiometer. It consists of two sets of resistance coils, each set having its own separate semi-circular row of contact studs over which the spring contact levers work and make contact. Two pairs of terminals are provided, the high E.M.F. being directly connected to those marked *M*, which are larger terminals than those marked *B* to distinguish them and avoid mistakes. *B* is the side at which the known fraction of the total P.D. is obtained, and of course go to the potentiometer direct. With the levers as shown, Fig. 213, $\frac{100}{9900+100}$ or $\frac{1}{100}$th of the total P.D. across *M* will be taken from *B*.

Low Resistance Measurer.

The following instrument affords a simple and convenient means of measuring very low resistances, such as are met with in the armatures of large dynamos, motors, and electric light mains. These cannot be obtained by an ordinary Wheatstone Bridge, owing to the difficulty of making, and the resistance introduced by, the contacts, and other reasons which go to vitiate and make the results worthless (see Fig. 214).

The working of the instrument is as follows—The resistance to be measured is joined in series with a battery and the slide wire. One coil of a differential galvanometer is joined by two leads to the two ends of the resistance to be measured, and the other coil across more or less of the slide wire.

The wire is divided into 1000 parts, and the whole is so arranged that the fall of potential over the whole wire, through the one galvanometer circuit, exactly balances that over $\frac{1}{10}$th ohm in the other.

The readings are then proportional throughout the scale. By

FIG. 212.

shifting two plugs the values may be multiplied by 5 or divided by 2, thus making the top read $\frac{1}{2}$ or $\frac{1}{20}$ of an ohm.

Directions for use.—Place the instrument on a fairly level table or bench.

Free the galvanometer needle by turning the screw at the back.

Turn the galvanometer on its shank till the needle points to zero.

Join the large terminals of the instrument in series with the resistance to be measured, and a cell capable of giving, say, 5 amperes; introducing somewhere in the circuit a resistance, to bring the current down to about 5 amperes.

Any odd bit of iron or German silver or other wire will do. A piece of G.S. suitable for use with a two-volt cell is sent with the instrument.

Join the long leads on to the small pair of terminals.

Depress the key on the contact arm to touch the wire; close the circuit switch, and the direction of the deflection.

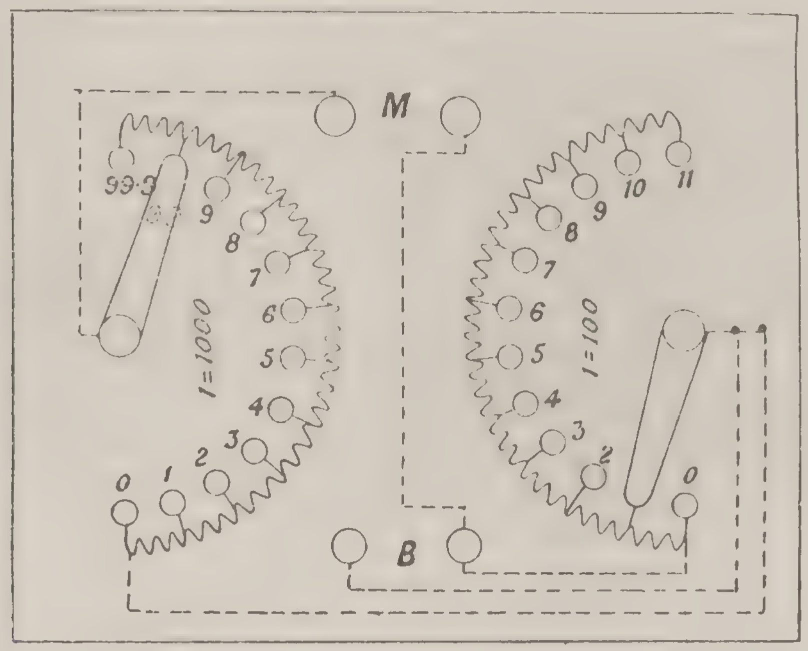


Fig. 213.

Press the two contact spears at the ends of the long leads on to the two points between which the resistance is to be measured—the contact arm being up. Again close the circuit switch and note direction of deflection. If in same direction as before, reverse the two contact spears.

Then, keeping the spears pressed on the resistance to be measured, depress the key on the contact arm.

If the deflection reverses it shows that the fall of potential over R (the unknown) is less than that over the wire, and the arm must be moved back towards the zero end until the galvanometer needle points to zero.

The arrow on the contact arm then points to the resistance on

the scale direct in ten-thousandths of an ohm, thus a reading of 517 is $\frac{517}{10000}$, or ·0517.

When the plugs are in 2 and 3 the instrument reads as above described. When in 1 and 2 the reads must be multiplied by 5, thus a read of $274 = {\cdot}0274 \times 5 = {\cdot}1370$ ohm. When in 1 and 3 the reads must be divided by 2, thus $98 = {\cdot}0098 \div 2 = {\cdot}0049$ ohm.

The spears should make fair metallic contact, but nothing more is necessary.

The galvanometer turns on its pillar, and can thus be set to zero. It should be set to zero with the current switch closed,

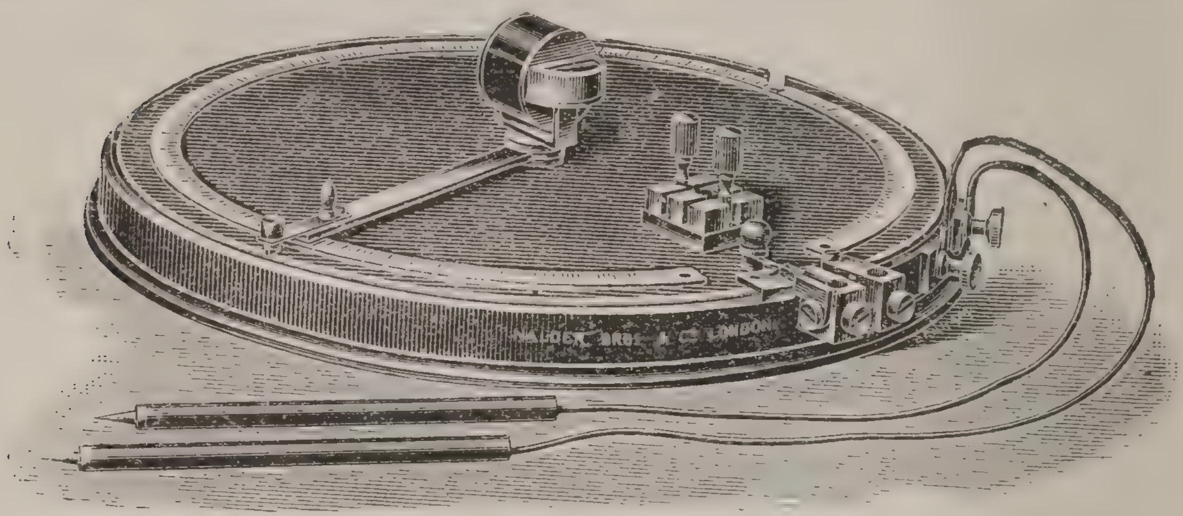


FIG. 214.

but without the contact arm being pressed or the spears in contact. This avoids all disturbances from the heavy current leads.

The contacts in the current leads do not need any care, their resistance, even if variable, does not affect the result.

Approximate Tests for very Low Resistances.

In armatures it is sometimes desirable to test each bar separately, to see that they are all the same, but if tested as above the reading would be so very small that there would be no certainty. A comparison of the bars can be taken by putting a shunt of any

convenient size, say 6″ of No. 8 platinoid, across the heavy terminals $T\,T$ at the end of the slide iron; using a larger current and working as before. The results will be of course only comparative, but if the shunt is firmly fixed, the readings will be quite sufficiently accurate for practical purposes. In the same way any two very low resistances can be compared by putting the two in series with the instrument and then transferring the point contacts from one to the other.

Siemens Low Resistance Bridge.

Instructions for use.—Connect up as shown in Fig. 215, which is a symbolical representation of the bridge and its connections, the general view being shown in Fig. 216.

In each of the arms of the branch box marked ×, unplug equal resistances and also on the ÷ side, the resistances being chosen according to the magnitude of the resistance to be measured.

The slide wire resistance is carefully calibrated and is compensated for all changes of the air temperature so that 0 to 100 = 0·01 standard ohm. (At any time should the wire require cleaning, only chamois leather should be used for the purpose.)

The contact slide vices, etc., are arranged for testing conductivity, but any other resistance may be tested on jointing it by four connection leads to $L1$, Ly and $L2$, Lz; $L1$ and $L2$ being the "current connections" and Ly and Lz the "potential connections," it must be remembered that any excessive resistance in the leads from the potential contacts to the branch box would increase the value of the branches and should be allowed for; they should therefore be as low as conveniently possible. The *battery* used should be one of *low internal resistance;* the galvanometer serves to indicate that there is a current flowing in the circuit.

Resistance Test.—The left-hand vice, etc., being clamped at zero, insert the specimen and clamp it at both ends; the length corresponding to the resistance measured will be given by the pointer on the meter scale, and is the distance between the two knife edges or "potential contacts." Alter the position of the sliding roller contact on the resistance wire until after closing

the battery circuit by means of the key B, and then pressing key M, no motion of the galvanometer needle is observed; the

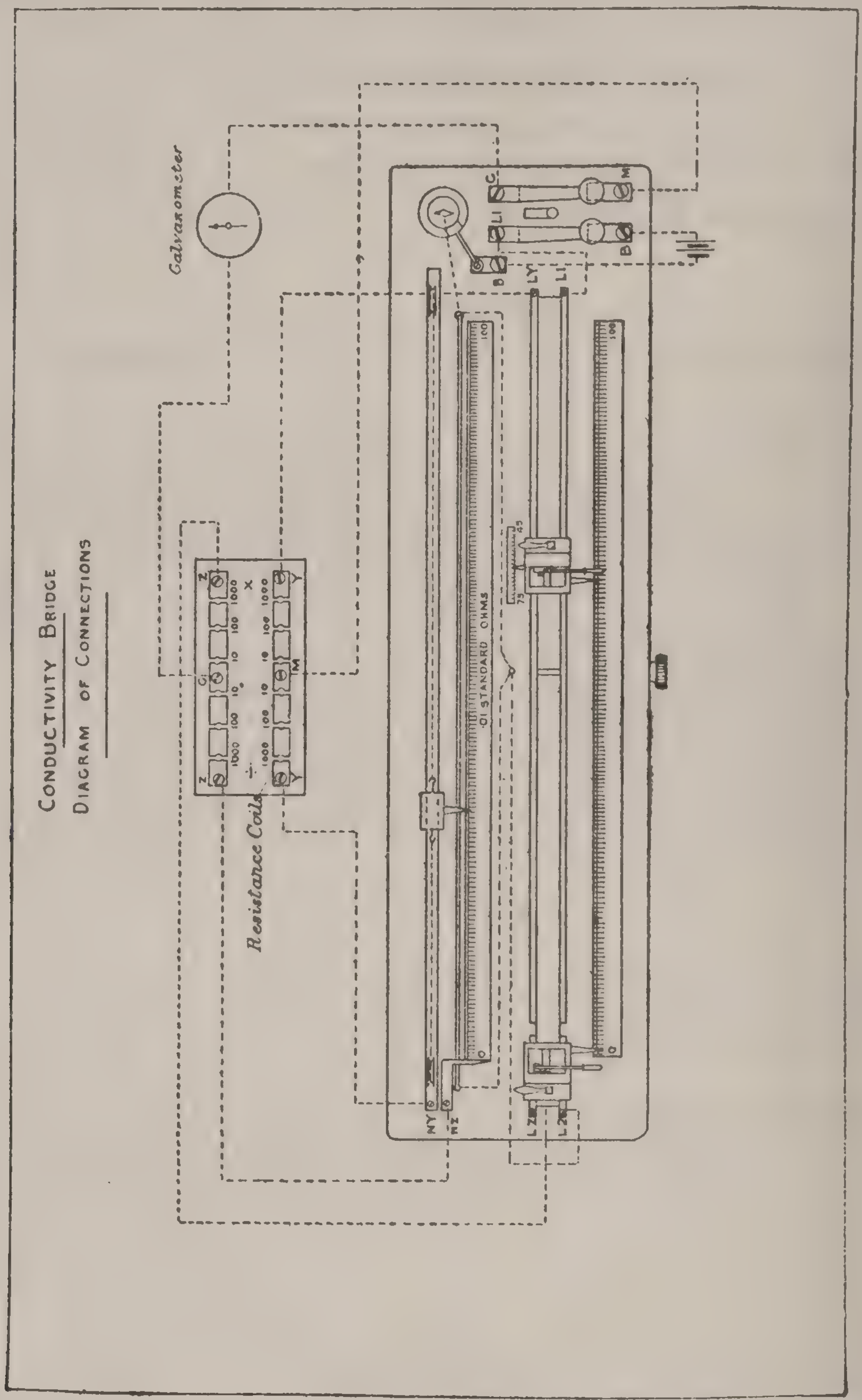


Fig. 215.

position of the roller contact being then read off, will give the resistance on the slide wire (S).

Then the resistance of the specimen is $X = S \cdot \frac{\times}{\div}$ ohms.

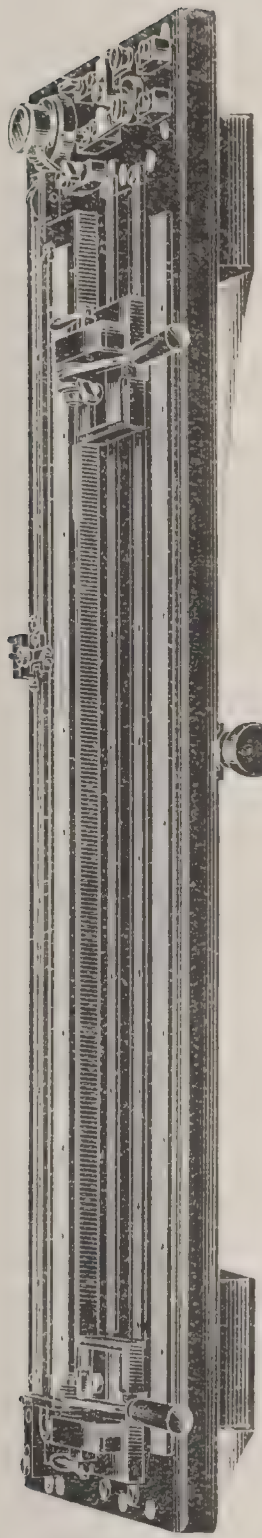


FIG. 216.

Example.—If the two tens be unplugged on the $\times$ side and the two thousands on the $\div$ side and $S = 45{\cdot}5$, then

$$X = \tfrac{45 \cdot 5}{100} \times \tfrac{10}{1000} \times 0{\cdot}01 = 0{\cdot}0000455$$
standard ohm.

Conductivity Test of Copper Wire.—For determining the conductivity of copper wire, the wire where it will come in contact with the vices and knife edges must be cleaned from all oxide. Then insert the end in the left-hand vice and clamp it. The right-hand vice must now be set by the pointer opposite the temperature of the specimen, as shown by the small scale, which is graduated from 45° to 75° F. The distance between the knife edges when the pointer is opposite 60° F. is 816·06 m.m., and the scale is calculated so that the difference in length compensates for the difference of resistance due to temperature of the copper.

Proceed as before to measure the resistance (X), taking note of the exact length in millimetres as shown by the metre scale. Then cut off by means of the knives and ascertain its weight in grammes, and by simple proportion determine the weight of 816·06 m.m. (W). The resistance of a pure copper wire 816·06 m.m. long weighing 1 gramme is 0·1 standard ohm @ 60° F. Then the conductivity per cent. of pure copper is

$$\frac{0{\cdot}1}{X \times W} \times 100.$$

Example—

Length cut off	Weight in grammes	Weight of 816·06 m.m.	Resistance X	Value of $X \times W$	% conductivity
809 m.m.	14·63.	14·76.	0·00696.	0·1027.	97·33.

Amsler's Planimeter.

If any figure on paper is measured in the ordinary way with compass and rule, the figure is first divided into triangles the area of which can be calculated, and the sum of their areas will give the area of the figure.

This method was shortened very much in 1827 by Mr. Oppenkoffer, a Swiss engineer, who invented an instrument called the "Planimeter," which measured the area of plain surfaces by following the outlines of the figure with a pointed tracer, which, being connected with a dial-plate, showed the area of the figure. This instrument soon came into general use, although somewhat awkward and expensive.

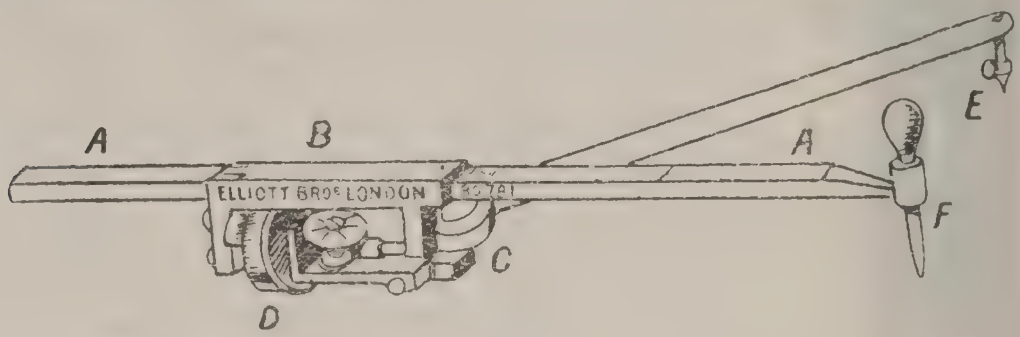


Fig. 217.

In 1849 an improvement was made by Mr. Welty, another Swiss engineer, which was still rather clumsy. No further improvement was made till 1854, when Mr. J. Amsler, Professor of Mathematics at Schaffhausen, introduced the Planimeter which is now in use; the construction is simple, the instrument can be carried about without fear of damage. One experiment showed that an area which after dividing it into triangles took nearly three hours, was done by Amsler's Planimeter in about two or three minutes.

Instructions for working the Planimeter.

(1) Before working with the instrument, adjust the screw centres upon which the index roller D revolves, so that the roller works freely, and does not touch the vernier. The same care must also be taken with the centre pin C. It is good to grease

the screw centres now and then, so that they work easily. Care should be taken to prevent the tube B, the tracer F, and the point E from being bent, and also to see that the barrel D is kept uninjured.

(2) To find the area of any figure, set the roller D and the counting wheel G to zero; the square rod A must be pushed into the tube B, and the line on A marked 1 sq. dcm., or 0·1 sq. ft. etc., must come even with the small line on the bevelled part of the tube B; when this is done, place the instrument on the paper, and see that the roller D, the tracing point F, and the needle point E touch the paper. Press the point E slightly into the paper, and put the small German silver weight on the hole over the point E; the instrument is then ready for work.

(3) Take any point P on the outline of the figure about to be measured, set the tracing point F to that point, and when it is marked, read off the index roller D and counting wheel G. For example, suppose the counting wheel G shows 2, the roller D 91, and the vernier 5, the number will be 291·5. Follow the outline of the figure with the point F as accurately as possible to the right, until you come to the starting-point. Straight lines can be followed along a ruler; then read off the numbers on wheel and roller; say it is the second time 476·7.

(4) When these two numbers are obtained, there are two cases to be observed—

(1st) If the point E is outside the figure, subtract the first reading 291·5 from the second 476·7, the remainder is 185·2, which shows that the area contains 185·2 units. Of course the units depend entirely on the regulation of the bar A; if they are 0·1 sq. ft. we have $185{\cdot}2 \times 0{\cdot}1 = 18{\cdot}52$ sq. feet, as the area of the figure measured on the paper.

The rule therefore is, when the point E is outside, multiply the difference of the two readings by the number on the bar to the right of the corresponding division.

(2nd) When the point E is inside the figure, before making the subtraction, the number engraved on the top of bar A, above the corresponding line of division, must be added to the second reading. In this instance, suppose the number on top of bar A is 20·985, the second reading is 4·767, the calculation would be as under—

2nd reading =	4·767
Number over 0·1 sq. ft. =	20·985
	25·752
Deduct 1st reading =	2·915
Remainder	22·837

The area is therefore 22·837 units or $22{\cdot}837 \times 0{\cdot}1 = 2{\cdot}2837$ square feet. It is of no consequence whether the roller is inside or outside the figure, provided it is on the same level.

(5) In measuring large figures, it may sometimes happen that the wheel G goes through one or two or more entire revolutions. If such is the case, 10,000 or 20,000, etc., must be added to the difference of the two readings before multiplication.

There is another form of planimeter which measures surfaces in square inches only; it is more simple than the other in construction, and can be worked with the above directions, always bearing in mind that the result is shown on the counting wheels in square inches and not as in the other instrument in 0·1 square decimetres, or 0·1 square feet, etc.

Amsler's Planimeter for Determining the Mean Pressure in an Indicator Diagram.

By the use of this instrument a great saving of time is effected in calculating large numbers of indicator diagrams, and the results obtained are more accurate than by any other method.

Directions for using the Planimeter.

The diagram is carefully pinned to a perfectly flat board. The points 00 of the planimeter are adjusted so that the distance between them is equal to the length of the diagram projected on to the atmospheric line. The instrument is then placed upon the board, the point b being brought into agreement with any fixed point of the diagram, while the weighted point c is placed in any convenient position outside the diagram. The point b is then passed once round along the lines of the diagram. The indication

on the wheel P and disc S, when the point b has again reached the starting-point, divided by 40 gives the mean height of the diagram in inches. Supposing, for instance, that the wheel P was adjusted to stand at zero before using the instrument, and that, after tracing the diagram, the disc S—which advances by one figure for every ten revolutions of the wheel P—stands between 1 and 2, while the wheel P indicates 21·2, a vernier being provided for reading the last figure, then the resulting number is 121·2, and this divided by 40 gives 3·03, which represents the mean height of the diagram in inches. Now, supposing that the diagram was taken by means of a No. 7 Richards Indicator Spring with a scale of 32 lbs. to the inch, then the mean pressure amounts to $3{\cdot}03 \times 32 = 96{\cdot}96$ lbs. per square inch.

In practice the calculation is somewhat simplified, as the springs used are mostly of such scales that instead of dividing by 40 and multiplying by the vertical scale, the mean pressure may be obtained by simply multiplying by a factor corresponding to the scales used.

In order to secure accurate results, the instrument must be carefully cleaned before being used, and the board must be perfectly flat.

Thompson's Indicator.

The chief distinguishing feature of this Indicator consists in a novel parallel motion which is preferable to the motion employed in the Richards Indicator on account of its greater lightness and rigidity. The irregularities in the diagram due to the inertia of the moving parts are consequently greatly reduced, and a figure is obtained which forms the nearest possible approximation to the correct diagram. The parallel motion is carefully designed to ensure that the pencil point describes a straight line, and that the motion of the pencil point is precisely proportional to the displacement of the indicator piston throughout the stroke.

Owing to its general efficiency, this indicator is also particularly applicable for high speeds, and it has been successfully employed at a speed of 400 revolutions per minute.

The piston is rigidly connected with the piston rod, which is guided in the cylinder cover, and in this way a perfect guide is

obtained for the piston. The tension of the spring in the drum

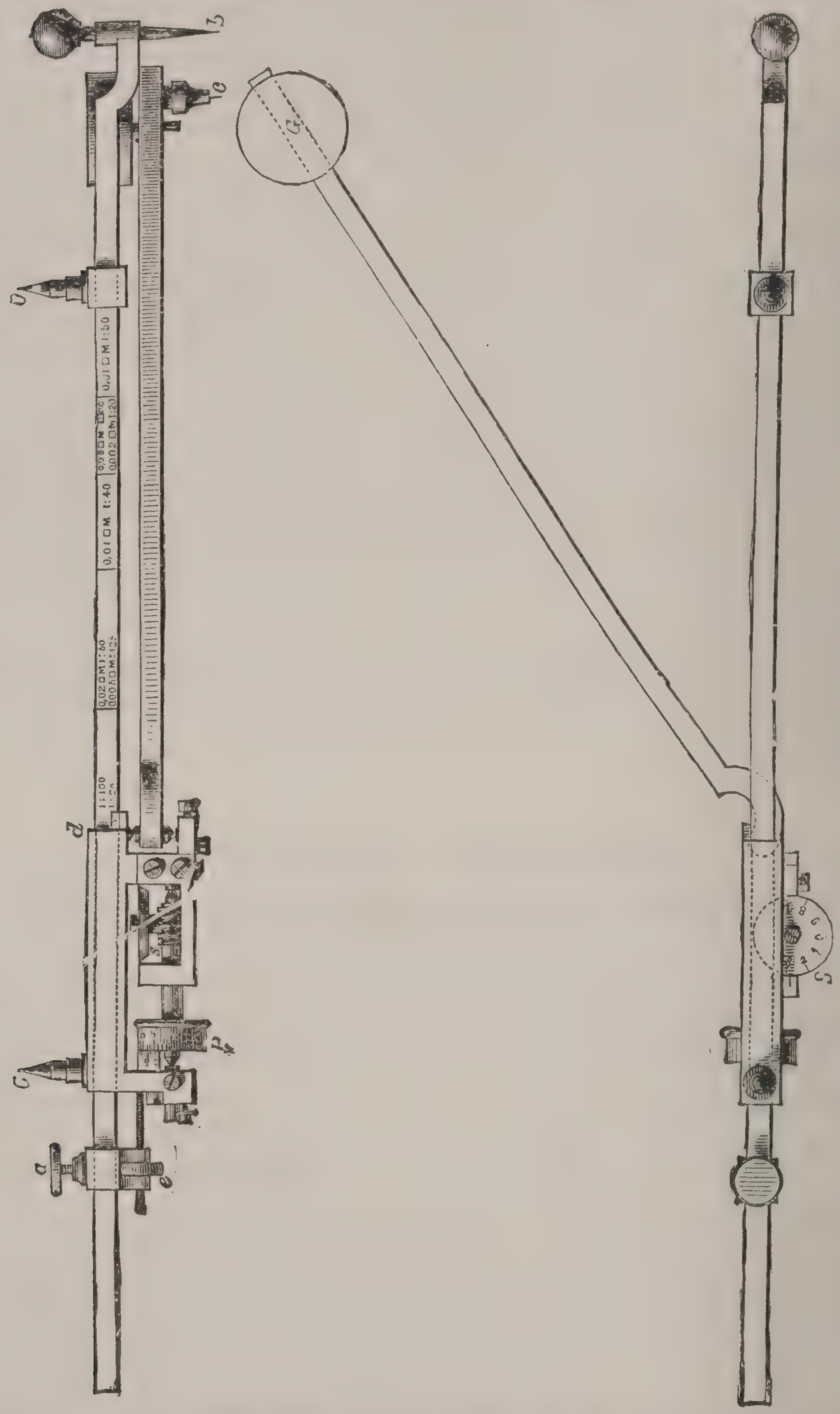


Fig. 218.

can be varied to suit the speed of the engine by loosening the

nut on the top of the drum spindle and by turning the disc hold-

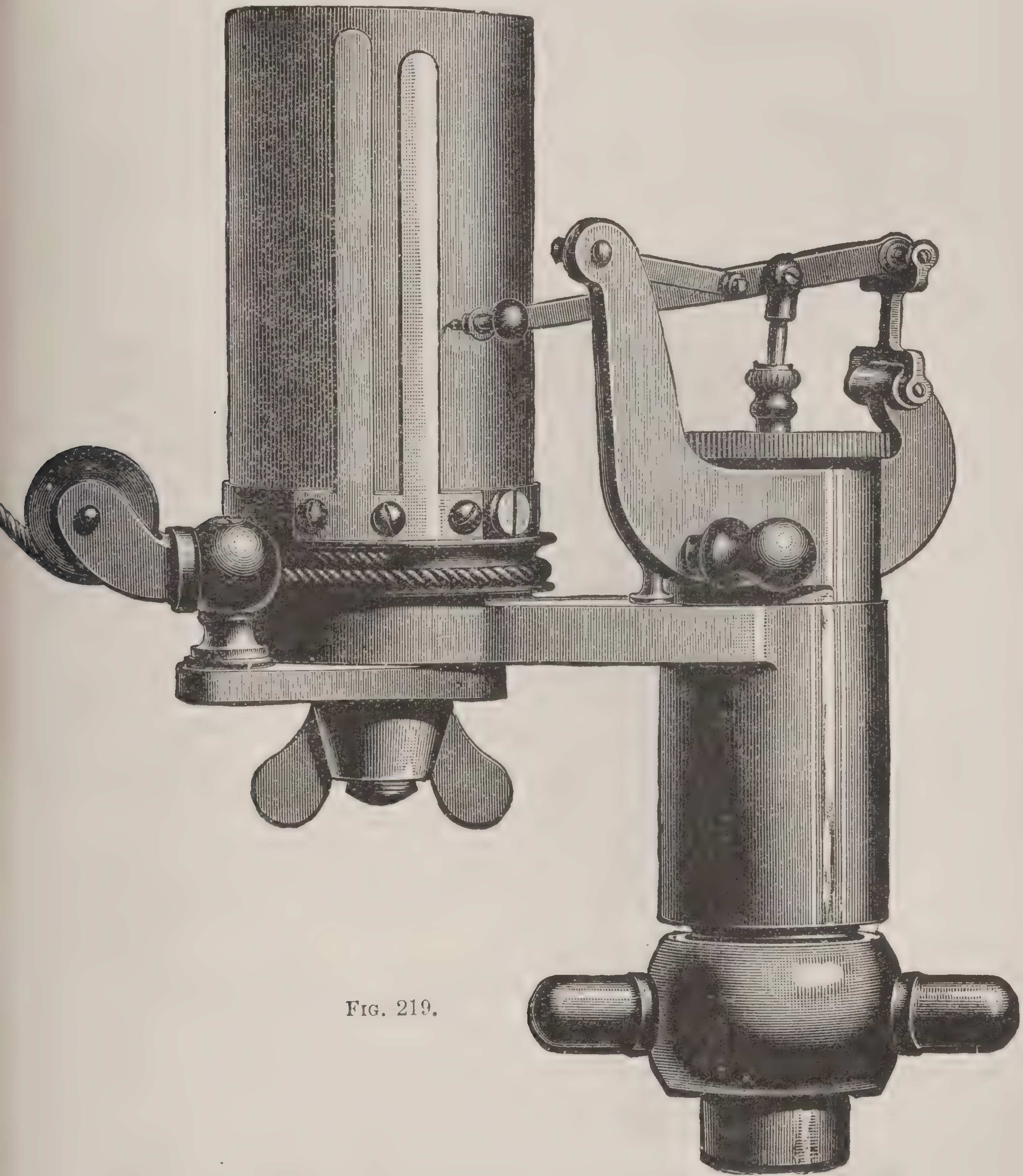


FIG. 219.

ing the spring until the required tension is obtained, the nut being then again screwed home.

TABLE VIII.

LIST OF SPRINGS FOR THOMPSON'S INDICATOR.

Sizes, No.	0	1	2	3	4	5	6	7	8	9	10	11	12
For Pressures from (Lbs. per sq. inch.)	–15	–15	–15	–15	–15	–15	–15	–15	–15	–15	–15	–15	–15
„ „ up to (Lbs. per sq. inch.)	8	12	18	30	40	50	68	75	95	120	150	180	200
Scale: 1 lb. per sq. inch equals . . . . inch	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{16}$	$\frac{1}{20}$	$\frac{1}{24}$	$\frac{1}{30}$	$\frac{1}{32}$	$\frac{1}{40}$	$\frac{1}{48}$	$\frac{1}{60}$	$\frac{1}{72}$	$\frac{1}{80}$

Small Thompson Indicator.

This indicator is specially adapted for indicating high-speed steam engines, gas engines, etc., and will give correct diagrams at all engine speeds occurring in practice without the necessity of taking special precautions, and has the further advantage of being very portable. It is, therefore, eminently suited to the requirements of the engineer or engineering student. The apparent disadvantage of a somewhat smaller diagram obtained from this indicator at slow speeds is more than compensated for by increased accuracy.

TABLE IX.

LIST OF SPRINGS FOR THOMPSON'S SMALL INDICATOR.

Sizes, No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
For Pressures from (Lbs. per sq. inch.)	–15	–15	–15	–15	–15	–15	–15	–15	–15	–15	–15	0	0	0	0
„ „ up to (Lbs. per sq. inch.)	6	15	20	30	45	60	70	80	100	125	150	200	250	300	375
Scale: 1 lb. per sq. inch equals . . . . . . inch	$\frac{1}{12}$	$\frac{1}{16}$	$\frac{1}{20}$	$\frac{1}{24}$	$\frac{1}{32}$	$\frac{1}{40}$	$\frac{1}{48}$	$\frac{1}{56}$	$\frac{1}{64}$	$\frac{1}{80}$	$\frac{1}{90}$	$\frac{1}{120}$	$\frac{1}{160}$	$\frac{1}{200}$	$\frac{1}{240}$

The Silvertown Portable Testing Set.

This is a small collection of the necessary instruments for testing electrically such insulated conductors as are used in telegraph, telephone, or electric-light work.

The whole set is contained in two small wooden boxes, of which one holds the batteries and the other the galvanometer, resistance coils, key, and commutators required for making the two most important measurements on such circuits. These are measure-

ments of the resistance of the conductor, and the efficiency of the insulation.

The battery consists of two parts: one—commonly called the

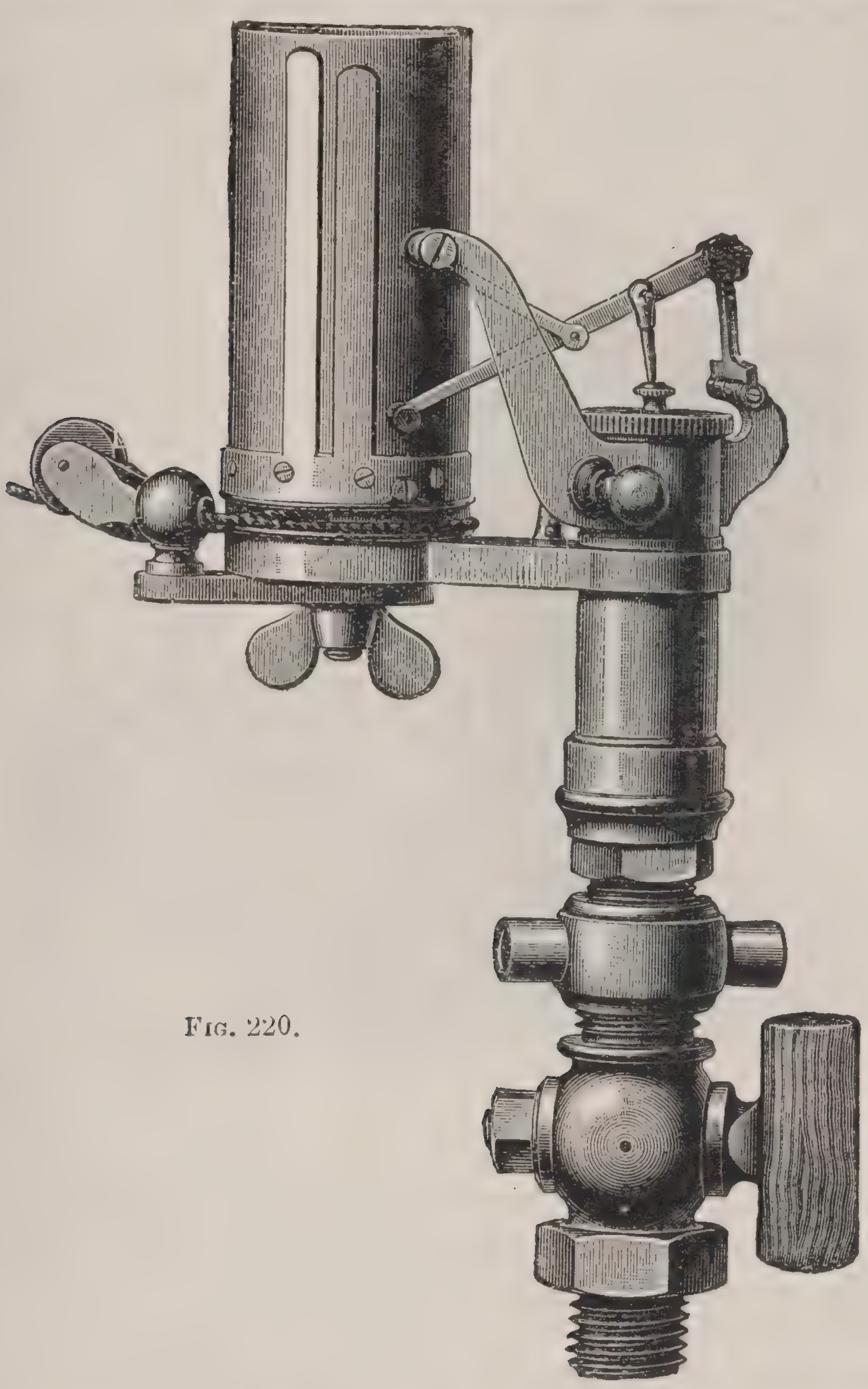


FIG. 220.

bridge battery—is a set of three Leclanché cells of low resistance intended to be used in testing conductor resistances only, a purpose for which currents of electricity of sensible magnitude are

required. The other part is a set of 36 small Leclanché cells having a total electro-motive force of 55 volts, intended exclusively for measuring insulation resistances, or other resistances, of considerable magnitude. These cells are designed to give only very small currents of electricity, and care should be taken not to connect them inadvertently to the Wheatstone Bridge or otherwise put them on a circuit of low resistance. This battery, called

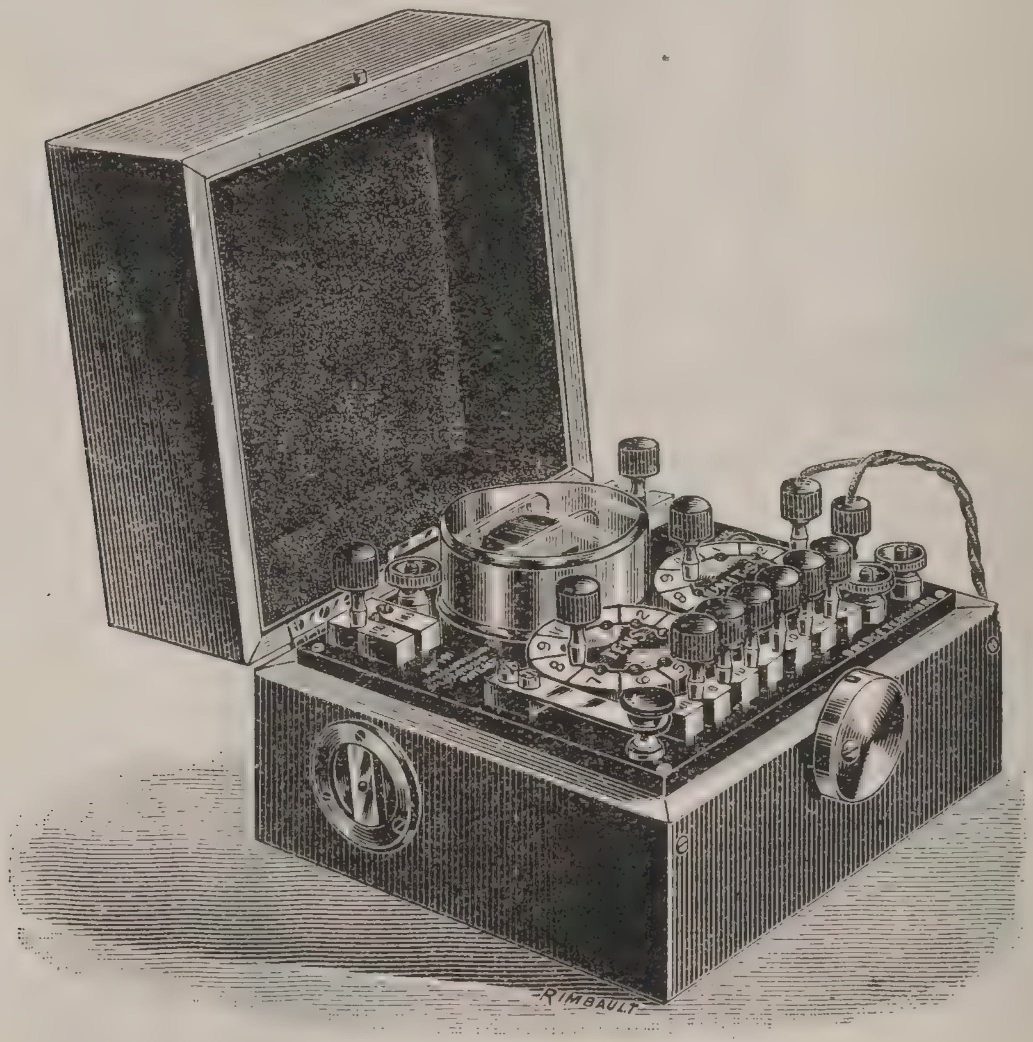


Fig. 221.

the insulation battery, is subdivided into three sections of 3, 15, and 39 cells, so that electro-motive forces of about 5, 25, or 60 volts can be employed as may be found convenient.

For connecting the battery to the testing instruments convenient leads are provided terminating in brass plugs with insulated handles for inserting in the proper plug-holes. The instruments shown in perspective in Fig. 221 are connected up together in their own box in such a way as to secure the greatest

portability and economy of space, and to enable the two tests to be taken with the greatest readiness.

A plan of this box showing the general arrangement of all the connections, resistance coils, and galvanometer is seen in Fig. 222.

The galvanometer consists of a coil of fine wire on a brass bobbin, in the centre of which a small magnetic needle with an aluminium pointer is hung in the same way as is usual in compasses. The pointer projects through the opening in the end of the coil, and the excursions of the needle are limited by the size of the opening to about 45° on each side of the centre. On removing the glass cover the needle on its point may be taken out by withdrawing the slide on which it is pivoted from inside the coil. The scale, which is a scale of equal currents, is approximately a scale of tangents, and is obtained empirically by calibrating the instrument. The north end of the magnetic needle points to the left-hand side of the box when it is swinging freely in its zero position.

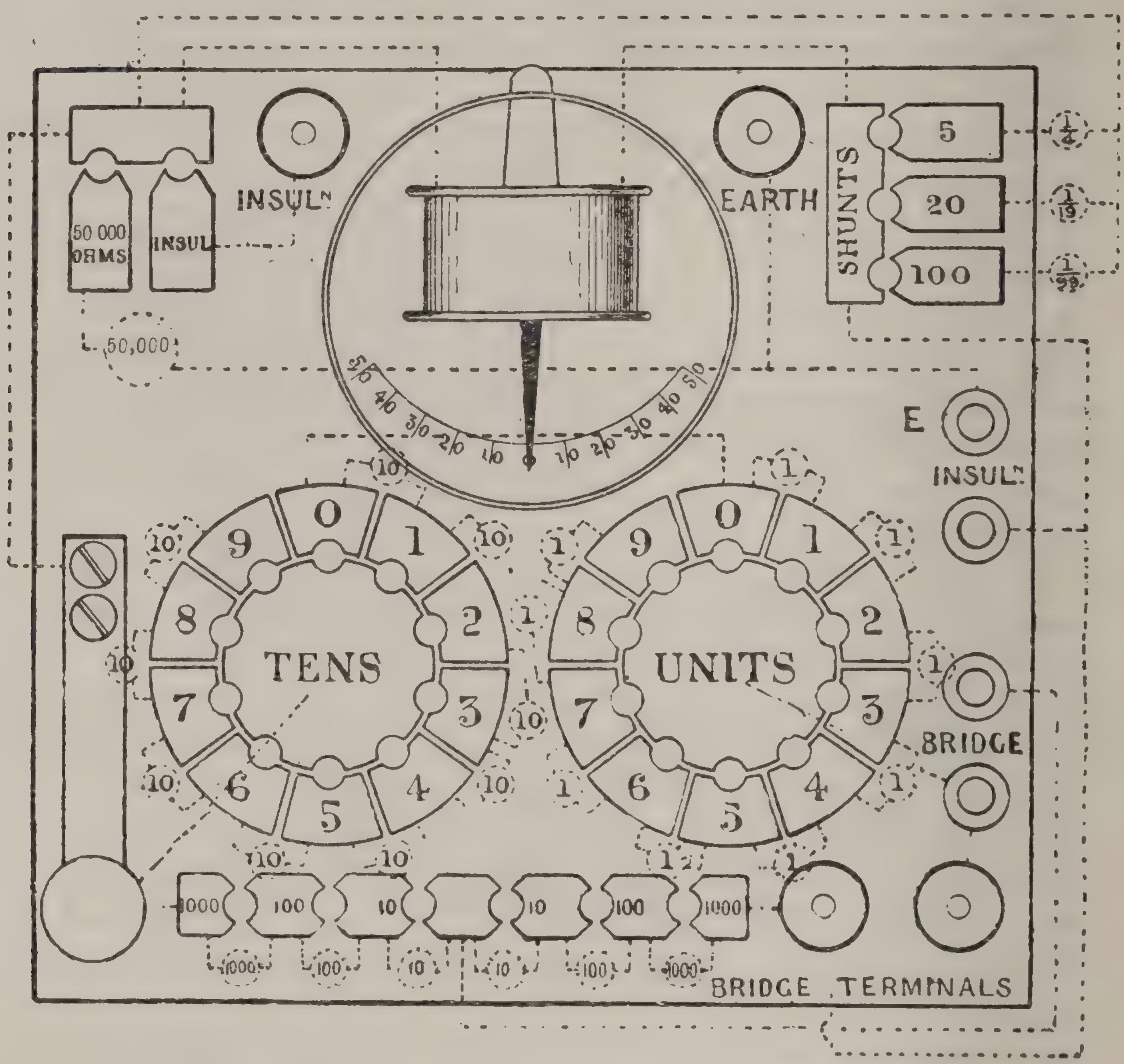
On the left-hand side of the box is placed the controlling magnet, and the position of this affects the sensitiveness of the galvanometer. When the north pole of the controlling magnet is uppermost, the galvanometer will be most sensitive; on turning the magnet round, so that the south pole is uppermost, the deflection of the needle due to any given current will be reduced by about 40 per cent. Generally in testing the insulation of well-insulated wires, the galvanometer is required to be as sensitive as possible, and the north pole of the controlling magnet should be at the top; but for measuring conductor resistances, for which the galvanometer is generally amply sensitive, it will be found more convenient to bring the south pole uppermost, thereby causing the galvanometer needle to oscillate more rapidly.

Besides thus affecting the sensitiveness of the galvanometer, the magnet is also used to adjust the needle to the zero in its position of rest by turning it slightly in one direction or the other.

The shunts shown to the right of the galvanometer are also for the purpose of diminishing its sensibility by shunting definite known fractions of the main current past the galvanometer when the plug is inserted in the desired hole.

If at any time the galvanometer needle should become insensitive and sluggish, it may be due to one of several causes, namely—

(*a*) That the needle has become demagnetized. This can be remedied by withdrawing and re-magnetizing it with an ordinary



General arrangement showing all connections.

FIG. 222.

horse-shoe magnet, care being taken that this is done in the same direction as before.

(*b*) That some dirt has found its way into the jewel. This may be removed with a piece of soft wood cut to a fine point.

(*c*) That the jewel or the needle point is injured. In this case the slide should be removed and sent with the needle and pointer

to the makers for repair. This will probably have occurred either through the whole instrument having received a blow when the lid is open and the jewel resting on the needle point, or through the brass spring in the lid of the box being bent so that it no longer presses on the lifter when the lid is closed, and the needle has consequently been resting on the point while the box has been carried about.

The remainder of the box consists of the two-way plug switch on the left of the galvanometer, by means of which either a known standard or unknown high-resistance can be separately inserted in series with the galvanometer.

The spring tapping key seen on the left-hand lower corner of the box is for closing the galvanometer circuit when ordinary resistance other than that of insulation is being measured.

The two circular dials, marked TENS and UNITS, form the adjustable resistance-arm of the Wheatstone Bridge arrangement, and consist of two sets of 9 coils each, totalling 99 ohms when both plugs are in the "9" holes, 0 when both plugs are in the "0" holes, and infinity when both plugs are out altogether.

A double set of proportional coils or resistances, each consisting of 10, 100, and 1000 ohms coils, completes the bridge. These are connected to the row of blocks seen at the bottom of Fig. 222.

The terminals shown are for connecting the battery and unknown resistance to.

Evershed "Megger" and "Bridge-Megger" Testing Sets.

The "Megger" Insulation Set.—The general principle underlying the construction, as well as the internal connections, of this set will be seen by a reference to Fig. 223. As seen, the instrument is a combination of a magneto-generator on the right-hand side, with the ohmmeter portion on the left, in a somewhat unique form of magnetic circuit, common to both and consisting of two pairs of field poles braced by strong bar magnets *NS*, *NS*, and forming two bi-polar field magnets in series. In the right-

hand one, and rotated by a folding handle and spur gearing *D*, is the armature of the generator with its brush gear *B*, and terminal bars marked + and −. In the left-hand field is the current coil *A*, pressure coil *P*, and compensating coil *C* of the ohmmeter, connected to resistances *Q* and *R*, a "guard plate" *G*, and the only two external terminals *L* and *E* (marked Line and

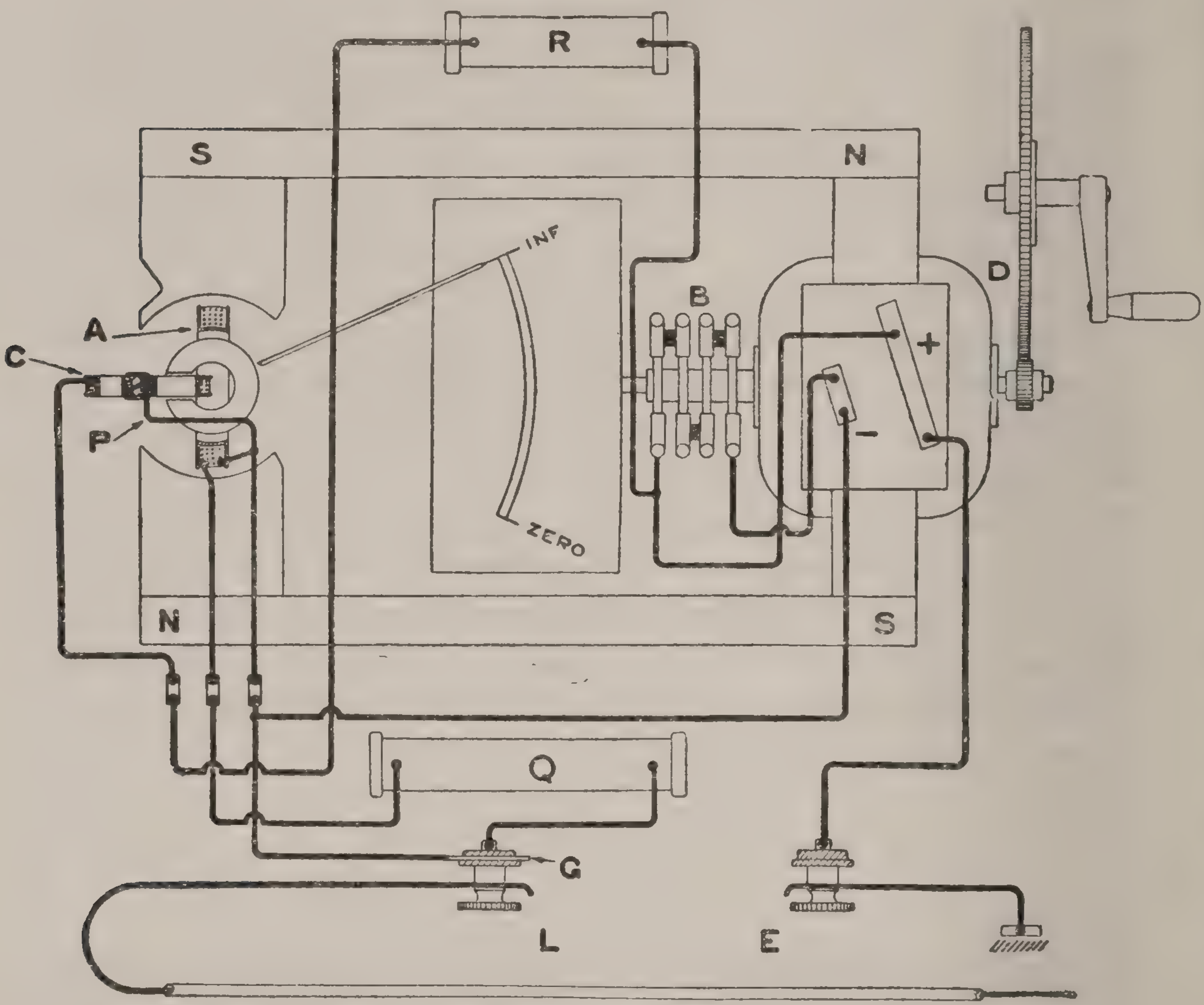


Fig. 223.

Earth in Fig. 224), which shows the general view of this set ready for use, with one end of the carrying strap detached from its spring cleat, the scale lid lifted and driving handle unfolded ready for use.

The arrangement and connections of the moving coil system of the ohmmeter of this set are shown in Fig. 229, and that of the generator armature in Fig. 231, the coils of which are numbered consecutively in order of their winding, No. 1 being

next to the core. The generator of this set may be either of the variable or constant-pressure type.

The "Bridge-Megger" Testing Set is available for use both as an insulation testing set and as a specialized Wheatstone

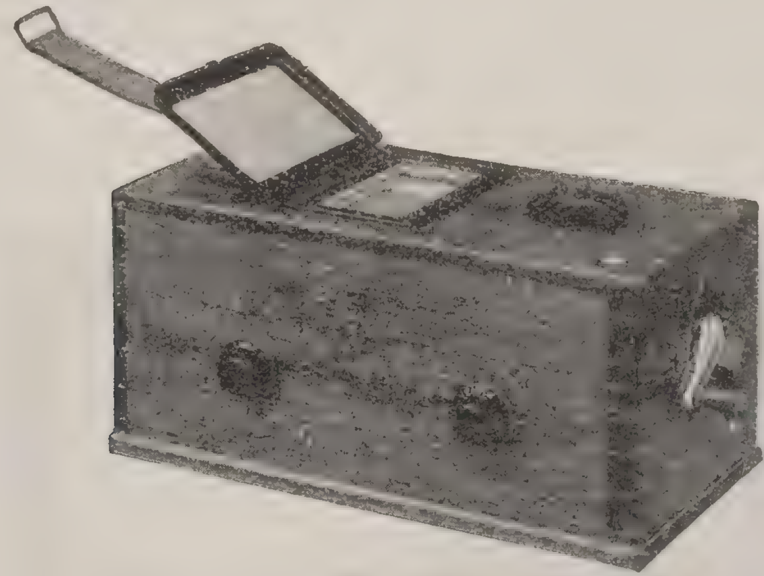


FIG. 224.

bridge. It differs from the above set in outward appearance only by the addition of two pairs of terminals at the left hand end, and of two switches near the top right-hand corner, as will

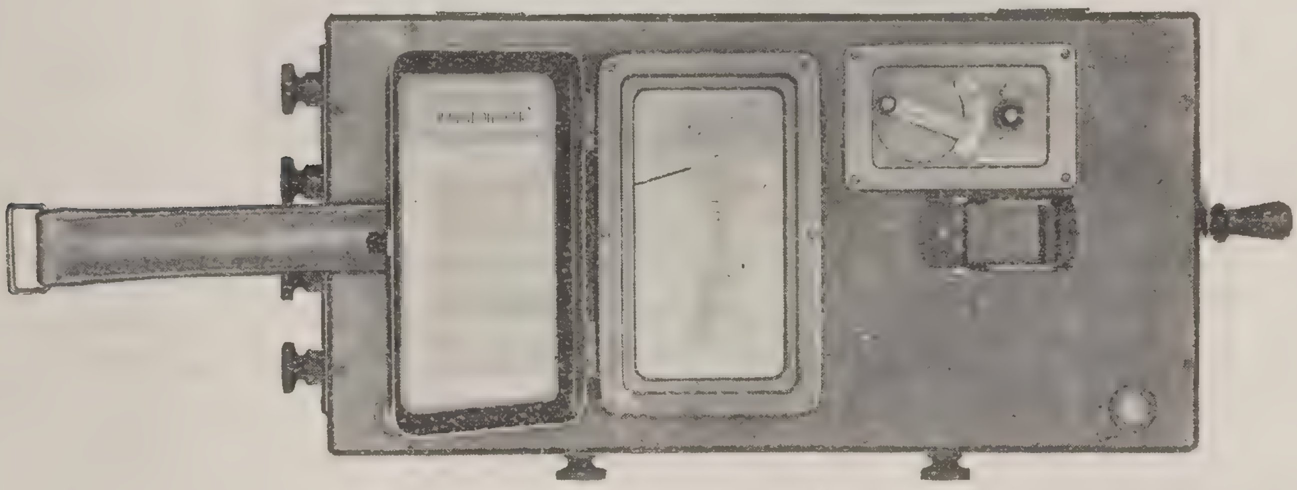


FIG. 225.

be seen by Fig. 225, showing a plan photograph of the box containing the ohmmeter and generator.

One of the two switches is a *Ratio Switch* for varying the proportion of the two ratio arms, when the instrument is used as a Wheatstone bridge, so as to make the unknown resistance (X)

under test either *equal to, or* $\frac{1}{10}$th, *or* $\frac{1}{100}$th of that of the standard resistance box R, thus providing a wide range of measurement which can be again increased by merely *interchanging* positions of the standard R and unknown resistances X relatively to the two pairs of terminals at the end (as seen in Figs. 44 and 45), which gives the unknown (X) now in terms of $R \times 10$ (or $\times 100$) according to the ratio employed.

The other, or *two-way change-over switch*, when *set to* "*Megger*," prepares the instrument for measuring large metallic resistance

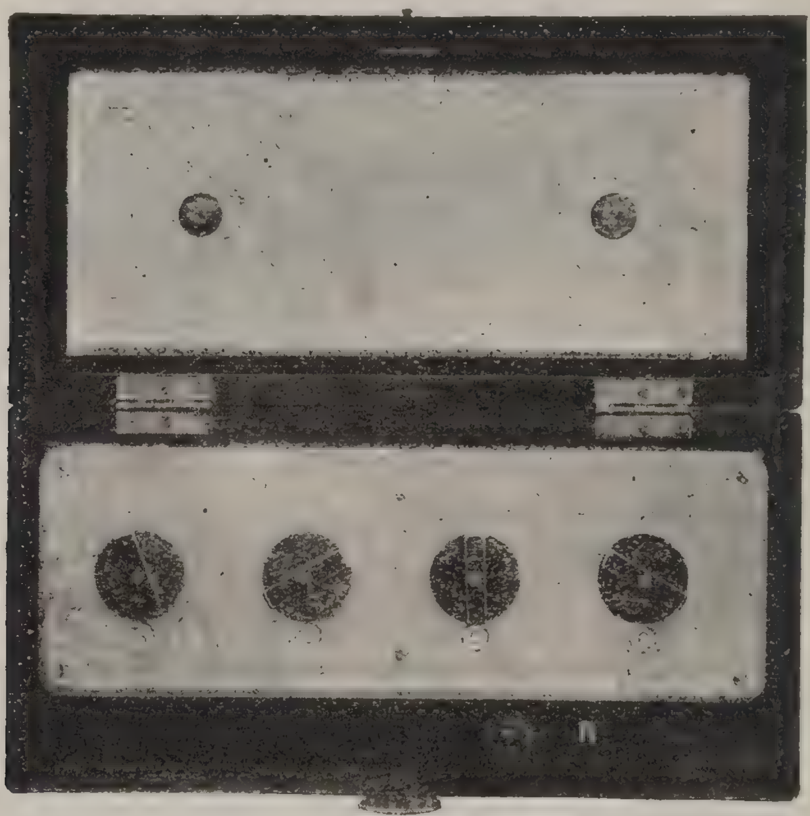


Fig. 226.

(*vide* p. 118), though principally insulation resistance, by coupling the two windings of the constant-pressure generator in series and making the two front terminals, marked *Line* and *Earth*, the only two available for connection.

When *set to* "*Bridge*," the instrument is converted for Wheatstone bridge work—the two windings of the generator being now coupled in parallel in order to increase the current obtainable from it; the ohmmeter part being changed into a galvanometer for the bridge; and the arms of the bridge being switched into their appropriate places in circuit and connected to the only available terminals (namely, the two pairs marked R and X at the end) for use now on the instrument.

The variable standard direct-reading resistance box (R) for use in bridge measurements is shown in plan, Fig. 226, and is of

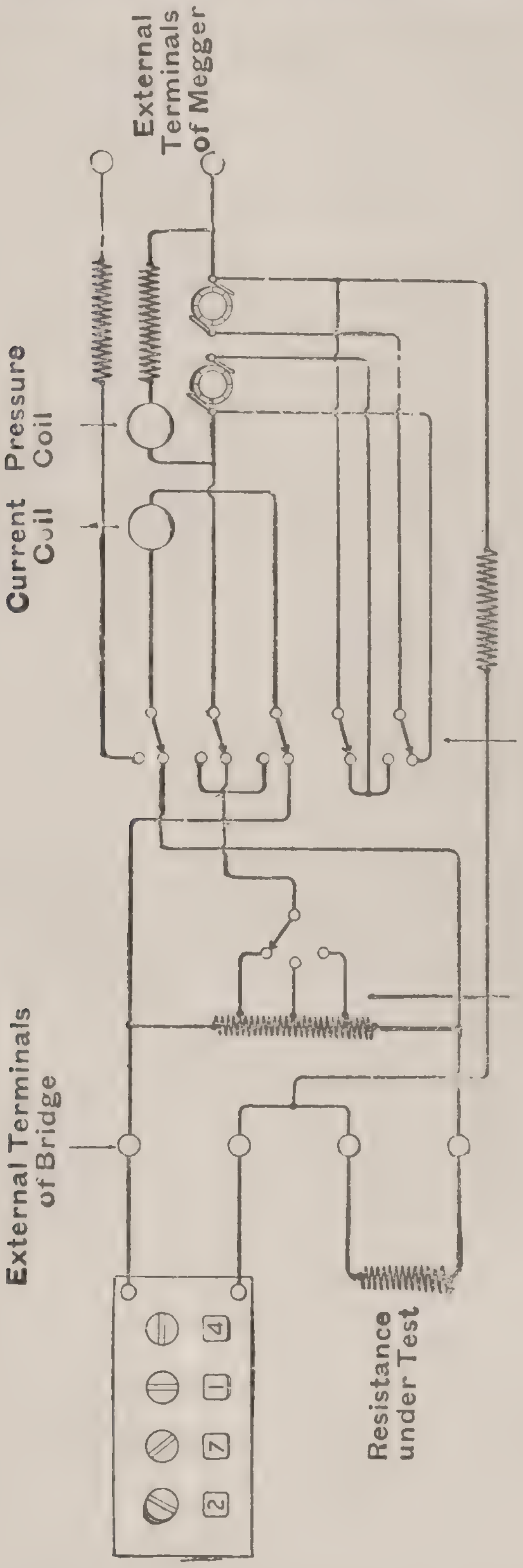


Fig. 227.

the sliding-contact type, operated by turning ebonite handles. The figure appearing for each position of any handle is the resistance of that dial, and the total shown in Fig. 226 is 8,306 ohms.

The complete internal connections of a "Bridge-Megger" set

with change-over switch set to "Bridge" are given in Fig. 227, while the connections forming the usual bridge circuits are depicted in Fig. 228.

The connections of the moving-coil systems are shown separately for the "Megger" set, Fig. 229—and "Bridge-Megger" set, Fig. 230—while the armature connections of the generator are shown for these sets respectively in Figs. 231 and 232.

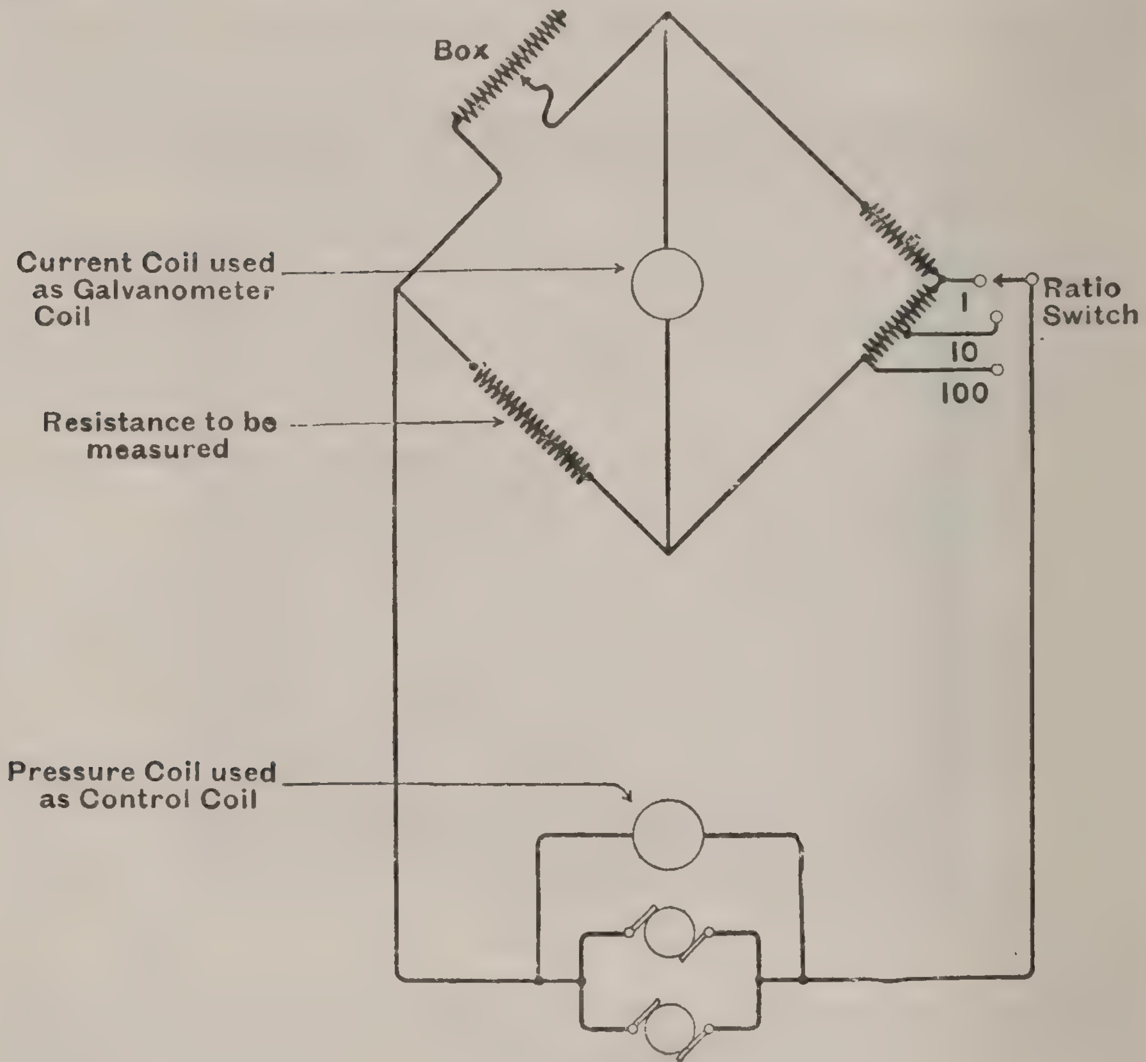


FIG. 228.

The Constant Pressure Generator for the "Bridge-Megger" set is shown in Fig. 233, the permanent bar magnets *NS* of Fig. 223 being removed for clearness.

The free-wheel attachment prevents the armature and gearing being damaged by any sudden stopping of the folding handle, and permits the armature to be driven in one direction only. Between the armature and gearing is interposed a centrifugal friction clutch comprising a drum driven by gearing, on which two arms, attached to the armature and fitted with pads at their

ends, are urged by springs. When the driving handle is turned above slipping speed, the speed of the armature, and hence its E.M.F., is extremely constant—varying as little as 1 part in 1000.

The Index Adjuster, fitted to the latest constant-pressure sets, consists of a small piece of soft iron rod, mounted parallel

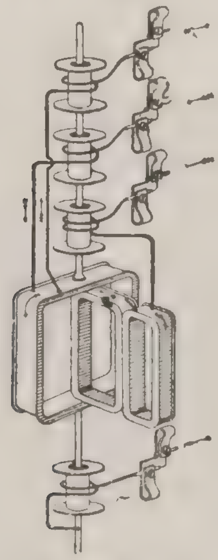


FIG. 229.

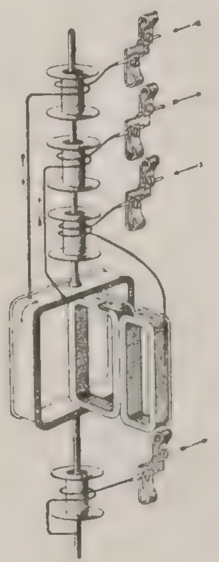


FIG. 230.

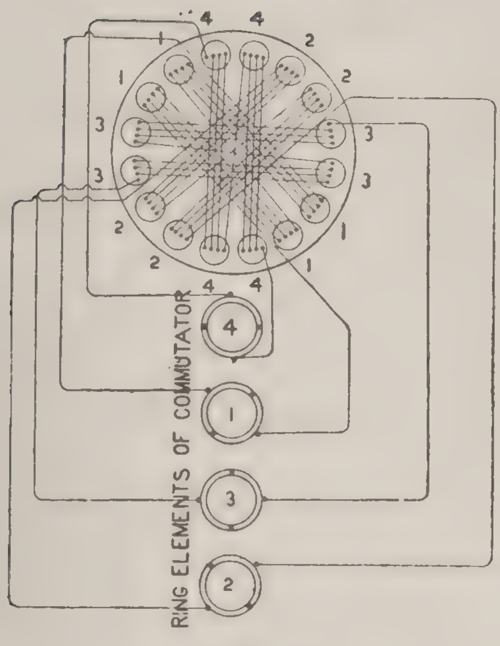


FIG. 231.

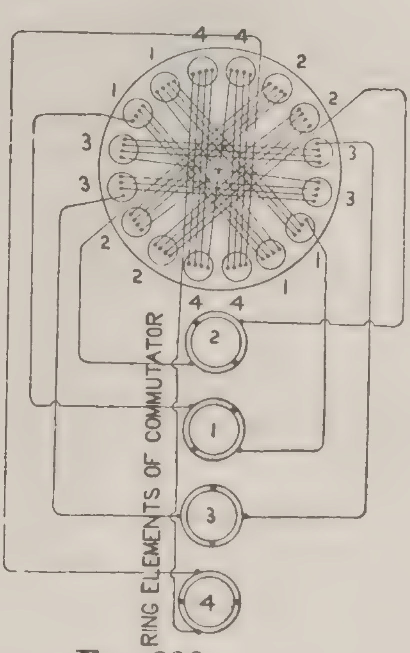


FIG 232.

to the line joining the centres of the field poles and close to the moving coil C, Fig. 223. It is capable of being moved sideways in one direction or the other (relatively to C) by means of a knob.

The rod becoming magnetized inductively by the polar field causes some of its field to pass through the coil C, and hence, if the rod is moved, its field also moves slightly, which at the "infinity" position of the moving systems causes a slight deflec-

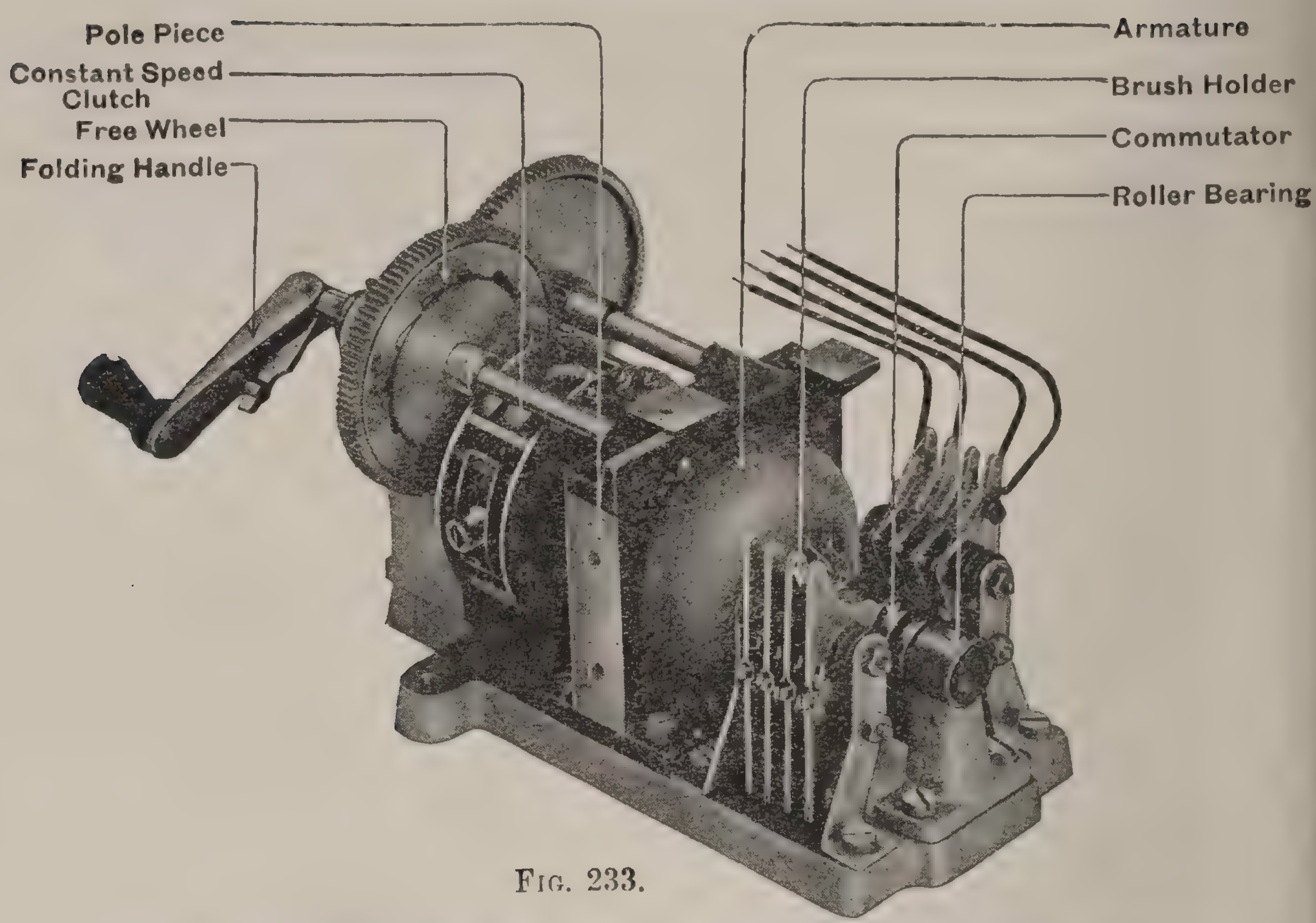


FIG. 233.

tion of the pointer one way or the other, facilitating accurate adjustment to infinity.

The adjustment does not affect the law of the instrument, which is altered only by the centre coil P.

Standard Direct-Reading Electric Balance.

Fig. 235 shows the general appearance (with glass cover removed), and Fig. 234 a part symbolical elevation of Lord Kelvin's centi-ampere balance.

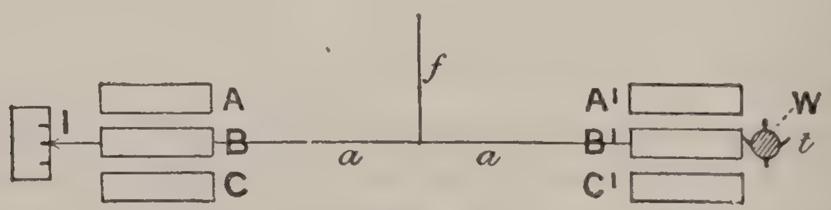


FIG. 234.

(1) The instrument is founded on the principle of action of the mutual forces, discovered by Ampere, between movable and

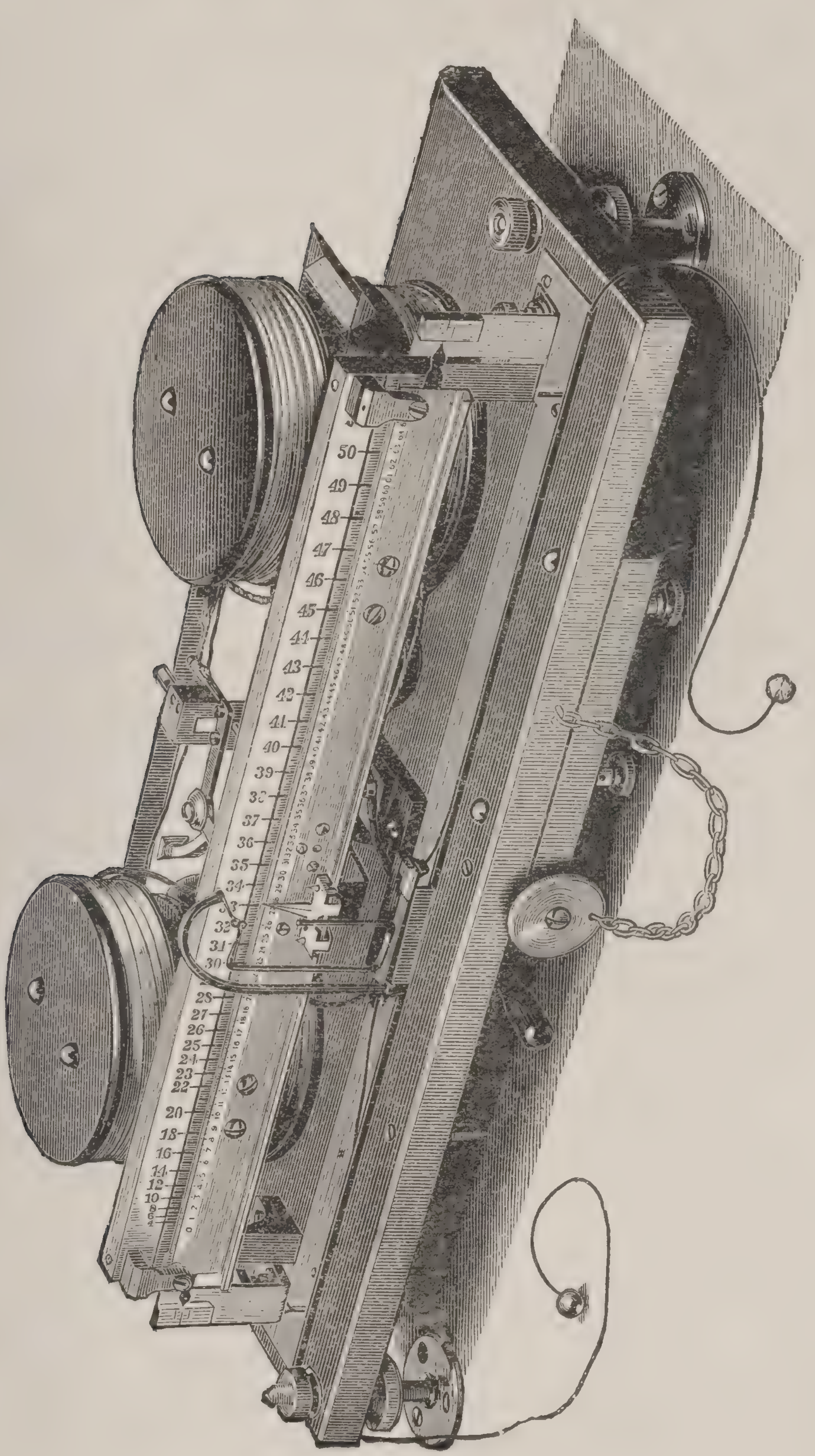


Fig. 235.

fixed portions of an electric circuit. The shape chosen for the mutually influencing portions is circular, and each such part will be called for brevity an ampere ring, whether it consists of only one turn or of any number of turns of the conductor.

(2) In this balance, each movable ring, B and B', is actuated by two fixed rings, AC and $A'C'$—all three approximately horizontal. There are two such groups of three rings—two movable rings attached to the two ends of a horizontal balance arm pulled, one of them up and the other down, by a pair of fixed rings in its neighbourhood. The current is in opposite directions through the two movable rings to practically annul disturbance due to horziontal components of terrestrial or local magnetic forces.

(3) The balance arm is supported by two trunnions, each hung by an elastic ligament of fine wire f, through which the current passes into and out of the circuit of the movable rings.

(4) The mid-range position of each movable ring is in the horizontal plane nearly midway between the two fixed rings which act on it.

(5) The current goes in opposite directions through the two fixed rings, so that the movable ring is attracted by one of the fixed rings and repelled by the other. The position of the movable ring, equi-distant from the two fixed rings, is a position of minimum force, and the sighted position, for the sake of stability, is above it at one end of the beam and below it at the other, in each case being nearer to the repelling than to the attracting ring by such an amount as to give about $\frac{2}{10}$ per cent. more than the minimum force.

(6) The balancing is performed by means of a weight which slides on an approximately horizontal graduated arm attached to the balance; and there is a trough t, fixed on the right-hand end of the balance, into which a proper counterpoise weight W is placed, according to the particular one of the sliding weights in use at any time (sect. 9 below). For the fine adjustment of the zero a small metal flag is provided, as in an ordinary chemical balance. This flag is actuated by a fork having a handle below the case outside, as shown at the bottom of Fig. 235. *To set the zero* the left-hand weight is placed with its pointer at the zero of the scale, and the flag is turned to one side or the other until it

is found that, with no current going through the rings, the balance rests in its sighted position.

(7) *To measure a current* the weight is slipped along the scale until the balance rests in its sighted position. The strength of the current is then read off approximately on the fixed scale (called the inspectional scale), with aid of the finely divided scale for more minute accuracy, according to the explanations given in sect. 11 below. Each number on the inspectional scale is twice the square root of the corresponding number on the fine scale of equal divisions.

(8) The slipping of the weight into its proper position is performed by means of a self-releasing pendant, hanging from a hook carried by a sliding platform, which is pulled in the two directions by two silk threads passing through holes to the outside of the glass case.

(9) Four pairs of weights (sliding and counterpoise), of which *the sledge or carriage and its counterpoise constitute the first pair*, are supplied with the instrument. These weights are adjusted in the ratios of 1 : 4 : 16 : 64, so that each pair gives a round number of amperes, or half-amperes, or quarter-amperes, or of decimal sub-divisions or multiples of these magnitudes of current, on the inspectional scale.

(10) The useful range of each instrument is from 1 to 100 of the smallest current for which its sensibility suffices. The range of this instrument is from 1 to 100 centi-amperes. The following table shows the value per division of the inspectional scale corresponding to each of the four pairs of weights—

	Centi-amperes per division.
First pair of Weights.	0·25
Second „	0·50
Third „	1·0
Fourth „	2·0

(11) The fixed inspectional scale shows, approximately enough for most purposes, the strength of the current; the notches in the top of the aluminium scale show the precise position of the weight corresponding to each of the numbered divisions on the fixed scale, which practically annuls error of parallax due to the

position of the eye. When the pointer is not exactly below one of the notches corresponding to integral divisions of the inspectional scale, the proportion of the space on each side to the space between two divisions may be estimated inspectionally with accuracy enough for almost all practical purposes. Thus we may readily read off 34·2 or 34·7 by estimation with little chance of being wrong by 1 in the decimal place. But when the utmost accuracy is required, the reading on the fine scale of equal divisions must be taken, and the strength of current calculated by aid of the table of double square roots given at the end of this book. Thus, for example, if the reading is 292, we find 34·18, or say 34·2, as the true scale reading for strength of current; or, again, if the balancing position of the pointer be 301 on the fine scale, we find 34·70 as the true reading of the inspectional scale.

(12) The centi-ampere balance, with a thermometer to test the temperature of its ampere rings, and with platinoid resistances up to 1600 ohms, serves to measure potentials of from 10 volts to 400 volts, and up to 2000 volts with specially constructed high resistances.

TABLE X.

CONSTANT OF THE CENTI-AMPERE BALANCE WHEN USED AS A VOLTMETER.

Weight used.	Resistance in circuit.[1]	Volts per division of fixed scale.
First Pair of Weights	400	1·0
" "	800	2·0
" "	1200	3·0
" "	1600	4·0

[1] Including resistance of the instrument, which is about 50 ohms.

If the second pair of weights is used, the constants will be double of those noted above.

(13) **Instructions for the Adjustment of the Standard Balances.**—The instrument should be levelled in accordance with the indications of the attached spirit level, by means of the levelling screws on which the sole-plate of the instrument stands.

(14) In this centi-ampere balance, the beam can be lifted off its supporting ligaments by turning a handle attached to a shaft

passing under the sole-plate of the instrument. This shaft carries an eccentric, on the edge of which rests the lower end of a vertical rod, which is fixed at its upper end to a tripod lifter. When the instrument is to be packed for carriage, or when it is to be removed by hand from place to place, the lifter should be raised ; but when it is fixed up for regular use, it is advisable to keep the beam always hanging on the ligaments.

(15) The carriage is fitted with an index to point to the movable scale, and is intended to remain always on the rail. One or other of the weights is to be placed on the carriage in such a way that the small hole and slot in the weight pass over the conical pins. The weights are moved by means of a slider, which slides on a rail fixed to the sole-plate of the instrument, and carries a pendant with a vertical arm intended to pass up through the rectangular recess in the front of the weight and carriage. The slider and weight are shown in position in the figures. The slider is moved by silk cords, which pass out at the ends of the glass case. When the cords are not being pulled for shifting the weight, their ends should be left free so that the pendant may hang clear of the weight. When a weight is to be placed on or removed from the carriage, the slider should be drawn forward at the top until it is clear of the weight, and then pushed to one side until the weight is adjusted, when it may be replaced in position in a similar manner.

(16) Cylindrical counterpoise weights with a cross-bar passed through them are supplied for the purpose of balancing the sliding weights when they are placed at the zero of the scale. The sliding weight should be placed so that the index of the carriage points to the zero of the scale, and the proper counterpoise weight should be placed in the trough, fixed to the right-hand end of the beam, with its cross-bar passing through the hole in the bottom of the trough. The flag which is attached to the cross-trunnion of the beam should then be turned by means of the handle projecting from under the sole-plate, until the index on the end of the movable scale points to the middle one of the five black lines on the fixed scale opposite to it. *Care must be taken when making this adjustment that the fork which moves the flag is not left in contact with it, as this would impede the free swing of the beam.* The fork should be turned back a little after

each adjustment of the flag, and, when the flag is being adjusted, it is better to watch the flag itself, and make successive small adjustments until the beam stands at zero, than to make successive trials by pushing round the handle while watching the position of the index.

If the ligament has stretched since the instrument was standardized, the index at one end of the movable scale will be found to be below the middle line on its vertical scale, when the index at the other end is correctly pointing to the zero position. The error so introduced would be a small one, but it may be easily put right by slightly loosening the screws fixing the pillared frame, which supports the movable beam, to the base plate, and raising it by slipping one or two thicknesses of paper below it until the indices simultaneously point to their zero position.

(17) A lens is supplied with each instrument for facilitating accurate observation, either when reading the position of the weight or when adjusting the zero.

(18) The vibrations of the beam may be checked so as to facilitate reading by bringing the pendant, which moves the weight, lightly into contact with it, in such a way as to give a little friction without moving the weights.

(19) In using the centi-ampere balance as a voltmeter when great accuracy is required, care must be taken that the effect of change of temperature in changing the resistance of the coils of the instrument, and of the external resistance coils, is allowed for; and in this use of the instrument it is advisable to employ currents such as can be measured by the lightest weight on the beam. When the instrument is to be used as a voltmeter, four resistances are provided, three of which are each 400 ohms, and the fourth is less than 400 ohms by the resistance of the coils of the instrument at a certain specified temperature. The smallest resistance is intended to be included by itself in the circuit when the lowest potentials are being measured, and in series with one or more of the others when the potential is so high as to give a stronger current than can be measured with the lightest weight on the beam. The correction for temperature is, for the copper coils of the balance, about 0·38 per cent. per degree Centigrade, and for the platinoid resistances, about 0·024 per cent. per degree Centigrade.

Anti-Inductive Resistance for the Kelvin Standard Electric Balances.

When a balance of the above type, such as, for instance, the *centi-ampere* or *composite* instrument, is to be used as a voltmeter, four resistances are provided, three of which are each 400 ohms, and the fourth is less than 400 ohms by the resistance of the coils of the balance at a certain specified temperature. These resistances are doubly wound so as to be non-inductive, and are made of platinoid wire, wound on suitable frames, so as to pre-

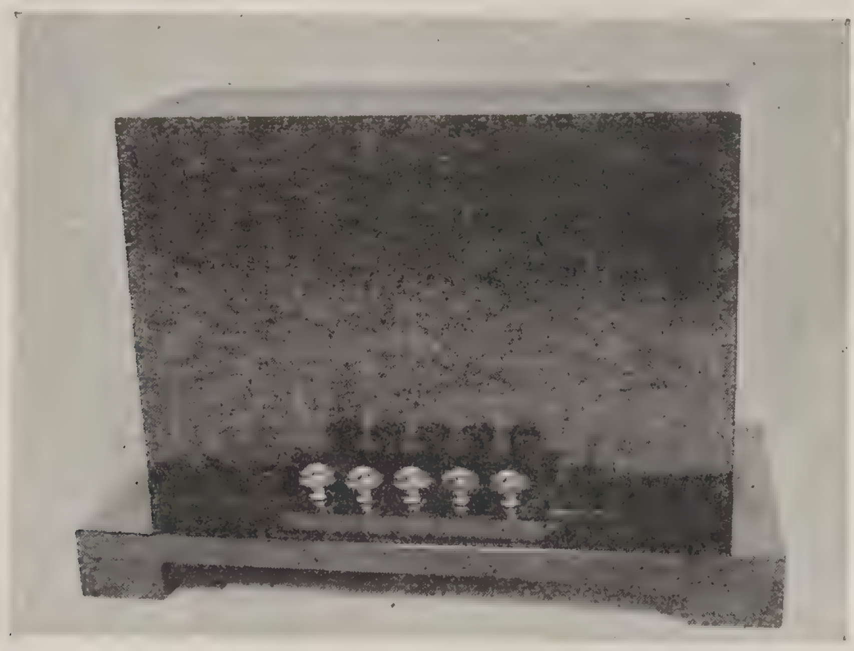


Fig. 236.

sent a maximum amount of cooling surface. The frames are enclosed in a box with apertures at the top to allow of the warm air getting out. Fig. 236 represents such a box for use with a Kelvin standard balance. The smallest resistance is intended to be included by itself in the circuit when the lowest potentials are being measured, and in series with one or more of the others, when the potential is so high as to give a stronger current than can be measured with the highest weight on the beam. The correction for temperature is, for these anti-inductive platinoid resistances, about 0·024% per degree Centigrade.

Composite Balance.

(1) This instrument is similar in form to the centi- and deci-ampere balances, but the pair of fixed coils at one end of the beam are made of a rope of insulated wires similar to that used for the coils of the hekto-ampere balance. Separate electrodes are provided for the rope coils, and for the fine wire coils. A switch which allows the movable coils either to be included in the circuit by themselves or in series with the fixed fine wire coils is attached to the under side of the sole-plate of the instrument. When the handle of the switch is turned to "Watt," the movable coils alone are in the circuit; but when the handle is turned to "Volt," both the movable and the fixed fine wire coils are in the circuit.

(2) The composite balance (Fig. 237) can be used as a hekto-ampere balance, or as a Wattmeter, or as a voltmeter, by following the instructions given below.

To enable the composite balance to be used as a direct-reading Wattmeter or voltmeter, a separate anti-inductive resistance of platinoid wire, subdivided into four coils, is usually supplied. The first coil is equal to the resistance of the fixed fine wire coils, and is intended to be included in the circuit of the movable coils when the instrument is used as a Wattmeter. The second coil is arranged to make up 200 ohms with the resistance of the fine wire movable and fixed coils. The third coil is 200 ohms and the fourth is 400 ohms.

It is not advisable that the current through these resistances should be allowed to exceed 0·5 ampere.

Instructions for the use of the Composite Balance.

(3) The balance should be levelled and the stop screws turned back out of contact with the cross trunnion and the front plate of the beam so as to leave it free to oscillate.

(4) To use the instrument as a centi-ampere meter or as a voltmeter the switch is turned to "Volt," and one or other of the weights marked VW_1, VW_2, VW_3, used. The current flowing through the instrument is then to be calculated from the constant given in the certificate sent with the instrument.

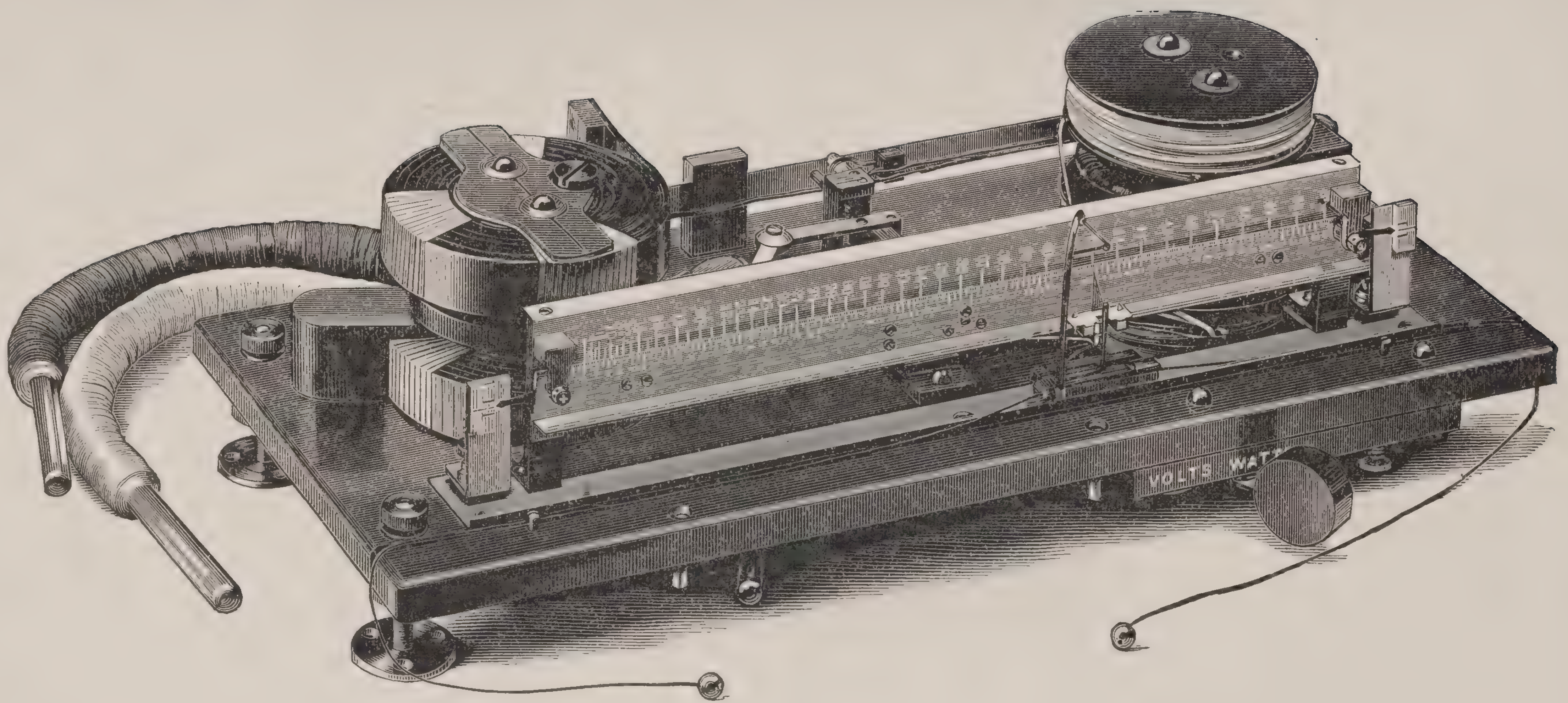


FIG. 237.

Constant of Composite Balance when used as a Centi-ampere Balance.

Weight used.	Centi-amperes per Division of Fixed Scale.
Sledge + VW_1,	0·5
,, + VW_2,	1·0
,, + VW_3,	2·0

The volts on the terminals are calculated from the current in amperes and the resistance in ohms (including the anti-inductive resistance, if any) in circuit. If V be volts, C current, and R resistance,

$$V = CR.$$

The anti-inductive resistance is arranged so that the instrument reads a round number of volts per division.

Table XI.

Constant of Composite Balance when used as a Voltmeter.

Weight used.	Resistance in Circuit.[1]	Volts per Division of Fixed Scale.
Sledge + VW_1	200	1·0
,, ,,	400	2·0
,, ,,	800	4·0

[1] Including the resistance of the instrument, which is about 30 ohms.

If the second pair of weights (Sledge + VW_2) be used the constants will be double of those noted above.

(5) To use the instrument as a hekto-ampere meter the switch is turned to "Watt" and the thick wire coils inserted in the current circuit in such a way that the right-hand end of the beam is repelled up. Either the sledge alone or the weight marked WW is to be used in this case. A measured current is then passed through the suspended coils, and the constants given in the certificate for the balance used in this way are calculated on the assumption that this current is, as there stated, 0·25 ampere, but any other current which is convenient in the circumstances may be used. The current through the suspended coils may be measured by means of the instrument itself arranged for the measurement of volts. This may be done by first mea-

suring the current which the difference of potential between the supply conductors of an electrical installation, or between the poles of a battery, causes to flow through the coils of the instrument and its external resistance, and then turning the switch to "Watt," and at the same time introducing a resistance into the circuit equal to the resistance of the fixed coils.

Constant of Composite Balance when used as a Hekto-ampere Balance.

Weight used.	Amperes per Division of Movable Scale.[1]
Sledge + WW_1,	0·250
„ + WW_2,	0·500
„ + WW_3,	1·000

[1] With 0·25 ampere through movable coils.

N.B.—The constants vary inversely as the current through the fine wire coils.

(6) To use the instrument as a Wattmeter, one terminal of the fine wire coils is joined to one end of the anti-inductive resistance and the other terminal to one of the leads; the other end of the resistance being joined to the other lead. The thick wire coils are inserted in the main circuit as described in sect. 5 above. With the instrument thus joined up, the current through the suspended coils and the E.M.F. between the leads may be obtained by the operations described in sect. 4 above, since the presence of the thick wire coil in the circuit causes no appreciable error: or the E.M.F. may be taken from the electrostatic voltmeter used on the circuit, and from its indications the current in the suspended coil circuit calculated. The watts are then to be calculated from the E.M.F. on the leads and the current through the thick wire coils by the formula

$$W = VC = cCR,$$

where c is the current in the suspended coil circuit, C the current in the thick wire coils, and R the resistance in the circuit.

The weights sent out with the instrument are arranged to give a round number of Watts per division of the scale with a known anti-inductive resistance in series with the fine wire movable coils.

TABLE XII.

CONSTANT OF COMPOSITE BALANCE WHEN USED AS A WATTMETER.

Weight used.	Resistance in Circuit with Movable Coils.[1]	Watts per Division of Movable Scale.
Sledge + WW_1,	200	12·5
,, ,,	400	25·0
,, ,,	800	50·0
,, + WW_2,	200	25·0
,, ,,	400	50·0
,, ,,	800	100·0
,, + WW_3,	200	50·0
,, ,,	400	100·0
,, ,,	800	200·0

[1] Including resistance of movable coils, which is about 12 ohms.

Adjustable Magneto-static Current Meter.

(1) The magneto-static current meter (Fig. 238) consists essentially of a small steel magnet or system of magnets suspended in the centre of a uniform field of force due to two coils, each having one or more turns of copper ribbon or wire, and also under the directive influence of two systems of powerful steel magnets.

(2) The suspended system of magnets is attached to one end of a vertical shaft passing down centrally through an opening in the sole-plate of the instrument from an indicating needle, which is supported by a jewelled cap resting upon an iridium point.

(3) The two systems of directive magnets are circular in form, and each ring is composed of two semicircular magnets placed in a brass cylindrical frame with their similar poles together. Each system is securely fixed to a circular brass frame, which fits on to the cylindrical case of the instrument in such a manner that the systems are capable of being turned round, together or separately, as explained below.

(4) The instrument has a "tangent scale," which is adjusted in its position before the instrument is sent out, so that the needle indicates equal differences of readings for equal differences of current. The scale consists of a hundred divisions, and for most purposes it is convenient to set the field magnets in such a position

that the needle points to 0, and to use the scale from that point upwards towards 100. Sometimes, however, it may be found convenient to measure currents, whose direction is being occasionally reversed, without being at the trouble of reversing the electrodes in the contact clip; in that case the zero should be set to the division 50 at the middle of the scale, and readings taken on each side of it. It must be remembered that when the point taken as zero is changed, the *constant*, by which the indications of the instrument have to be multiplied to give the current in amperes, is changed in proportion to the cosine of the angle between the zero point and the middle of the scale; and as this

FIG. 238.

angle is 60°, the *constant* with the zero at 50 on the scale is exactly double the *constant* with the zero at 0 on the scale.

(5) The instrument is provided with a "lifter," which serves to raise the needle off the iridium point when it is being moved about from place to place. This lifter is in the form of a ring placed below the needle, and may be raised or lowered by turning the handle attached to an eccentric passing through the side of the instrument on a level with the scale. It also serves as a checker, by bringing it lightly into contact with the pointer, so as to stop its vibrations.

(6) The instrument has an advantage, important for some practical purposes, of being available as an accurate direct-reading

current meter, through a continuous range of from 1 to 100 times its smallest current, which may be anything from half a milli-ampere to 4 amps., according to the number of turns in the coils supplied with the instrument. It is not, however, available as an alternate current instrument, and it must be remembered that the magnetism of the steel directing magnet does not remain *absolutely* constant. With good quality of steel, a proper preliminary *ageing* of the magnet (by heating it several times in boiling water and cooling it again, and subjecting it to somewhat varied rough usage) brings it to a condition in which its magnetism is found to remain exceedingly nearly constant month after month and year after year. Still, it should never be relied upon as *absolutely* constant, and for accurate laboratory work it is therefore necessary to occasionally standardize it.

(7) Another advantage which the instrument has is that, when a standard instrument is available, its constant is capable of being varied to any desired value down to one-tenth of that which it has with its directive magnets in their strongest position. Thus if the constant should be 3 amps. per division of the scale, with the similar poles of the magnets coinciding, it may be adjusted to any value down to 0·3 amp. per division.

(8) **Instructions for Use of the Magneto-static Current Meters.**—The instrument should be levelled, in accordance with the attached spirit-level, by means of the levelling screws.

(9) *To Adjust the Pointer to Zero.*—(*a*) Loosen the two lower milled-headed screws clamping the magnet frame, and turn the frame round till the pointer stands at zero. (*b*) Reclamp the frame by tightening the two screws.

(10) *Adjustment of the Scale.*—The scale, as stated above (sect. 4), is firmly clamped in its place before sending the instrument out, and this position is marked by two lines on the outside of the case, one horizontal and the other vertical, just below the 0 of the scale. The horizontal line is engraved below the movable top of the instrument, and the vertical one on the side of the case. Should the top of the instrument have been inadvertently moved, and the scale thus put out of adjustment, it may be set right by slightly loosening the two slotted screws and turning the top round till the extremities of the two lines coincide.

(11) If the needle should by accident be slightly bent,[1] and so render a new adjustment of the scale necessary, this may readily be made in the following manner :—Set the zero, by the field magnets, to the division 50 at the middle of the scale, then join the instrument in series with another current instrument of convenient form, and pass a current through both sufficient to give a deflection of about 40 divisions on the magneto-static instrument; reverse the current on the magneto-static instrument only, and set the scale so that equal deflections, read in divisions, are given on each side of the zero for equal currents, as indicated on the auxiliary instrument. The zero must, of course, be reset by the magnets every time the scale is moved. When the scale has been adjusted to this position, firmly clamp the top of the instrument by the two slotted screws, and again mark the position of the horizontal line on the outside of the case.

(12) *Adjustment of Constant.*—The constant may be quickly varied as follows :—Join the instrument in series with any reliable current instrument of known accuracy, such as the deci-ampere balance, and pass a convenient current through both instruments, observing the readings. Break the current, loosen the two upper pairs of milled-headed screws, and turn the top system of magnets relatively to the lower, so that the similar poles of the two systems are brought closer together or moved further apart, according as it is desired to make the instrument respectively less or more sensitive. Reclamp the screws and adjust the zero as described in sect. 10. Again make the current, and note the reading on the two instruments. The desired reading on the magneto-static may be obtained quickly after one or two approximations, care being always taken to readjust the zero after each movement of the top magnets.

(13) When convenient it is always best to standardize the instrument in the place where it is to be used; but when it is intended to move it from place to place, it should be standardized in such a position that when the needle is pointing to zero it is in a direction approximately east and west.

[1] If it is bent so largely as to be perceptible to the eye, it ought to be straightened by hand as nearly as may be.

Electrostatic Voltmeters.

These voltmeters have the great advantage of being available as accurate measurers of potential on direct and alternating systems, and, being electrostatic, they use no current, and consequently require no temperature correction. They are therefore free from the causes of error so prevalent in instruments of the electro-magnetic type, whose accuracy is impaired by variations of temperature, and which when used on alternating systems are affected by errors due to self-induction ranging with the period of alternation.

The instruments are made on the principle of an air condenser, having one of its parts movable about an axis, so as to increase or diminish the capacity. The condenser is enclosed in a metal case, for the double purpose of protecting the movable part from air currents, and from the disturbing influence of any electrified body, other than the fixed portion, differing from it in potential. In these instruments, the fixed portions consist of two sets of quadrant-shaped cells in metallic connection with each other, and formed by a number of parallel brass plates. These cells are fixed by an insulating support to the case of the instrument, and a terminal passes from them to an insulated binding screw on the outside of the case.

The movable portion in all the instruments is in metallic connection with the surrounding case. In the multicellular voltmeters this connection is made through the suspending wire. The movable portion carries the pointer, which indicates by direct readings the difference of potential between the two parts of the condenser.

The action of the instrument, shortly stated, is as follows:—When the fixed and movable plates are connected respectively to two points of an electric circuit, between which there exists a difference of potential, the movable plate tends to move so as to augment the electrostatic capacity of the instrument, and the magnitude of the force concerned in any case is proportional to the square of the difference of potential by which it is produced. In the multicellular voltmeters this force of attraction is balanced by the torsion of the suspending wire.

The Kelvin Multicellular Electrostatic Voltmeter.

The arrangement of the parts of this instrument is shown in Figs. 239, 240, and 241. These figures apply to an early form of the

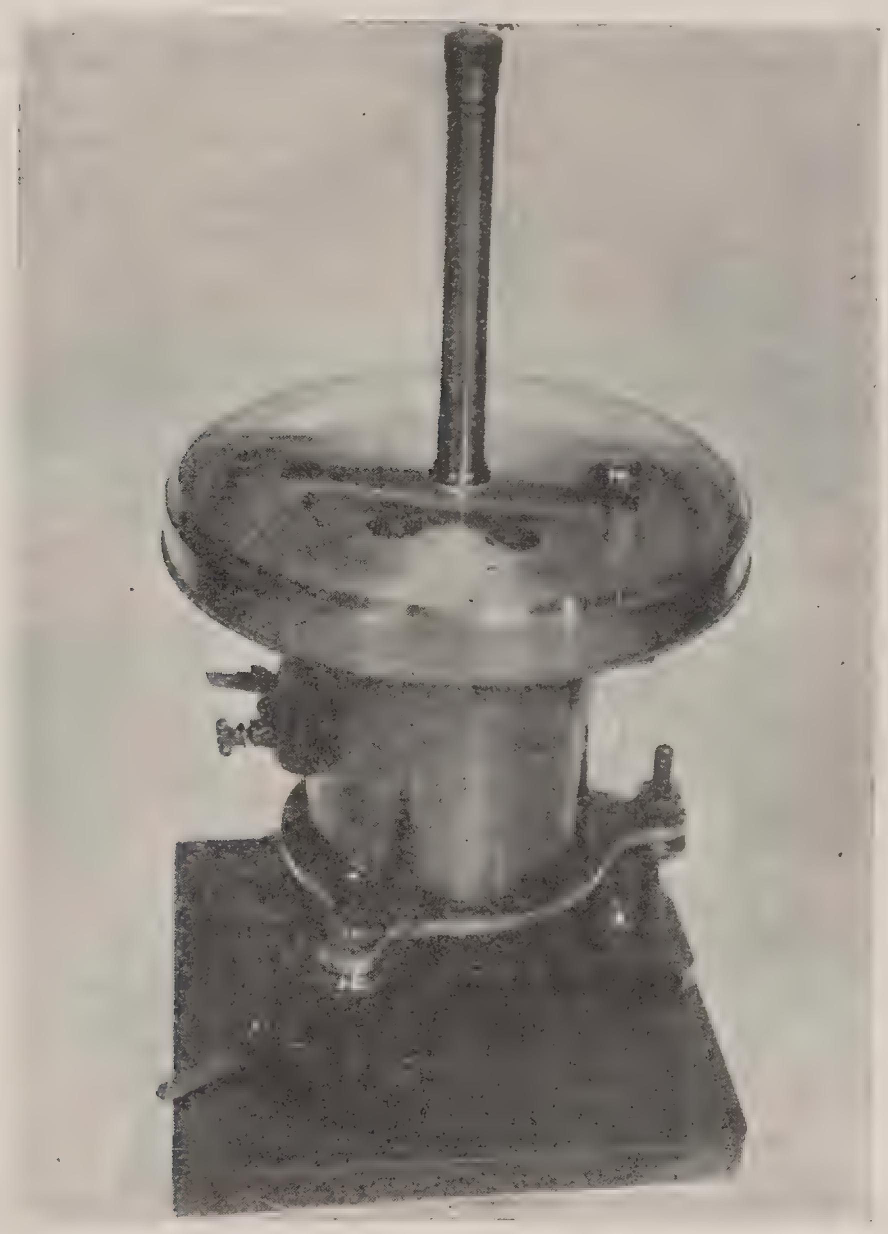


FIG. 239.

voltmeter, and differ in two matters of detail from the voltmeter as now made. For simplicity in manufacture the cells are now made with straight backs, and the plates looked at in plan are, therefore, triangular instead of square, as shown in Fig. 241. A coach-spring has now been interposed between the suspending wire and the spindle carrying the vanes, as explained below.

The insulated cells are formed of triangular brass plates fixed into saw cuts in a brass back piece so as to be equal distances apart and accurately parallel to each other. Two sets of those cells *C* are fixed relatively to each other, as shown in Fig. 240, by a vulcanite support to the sole-plate, so that their plates are

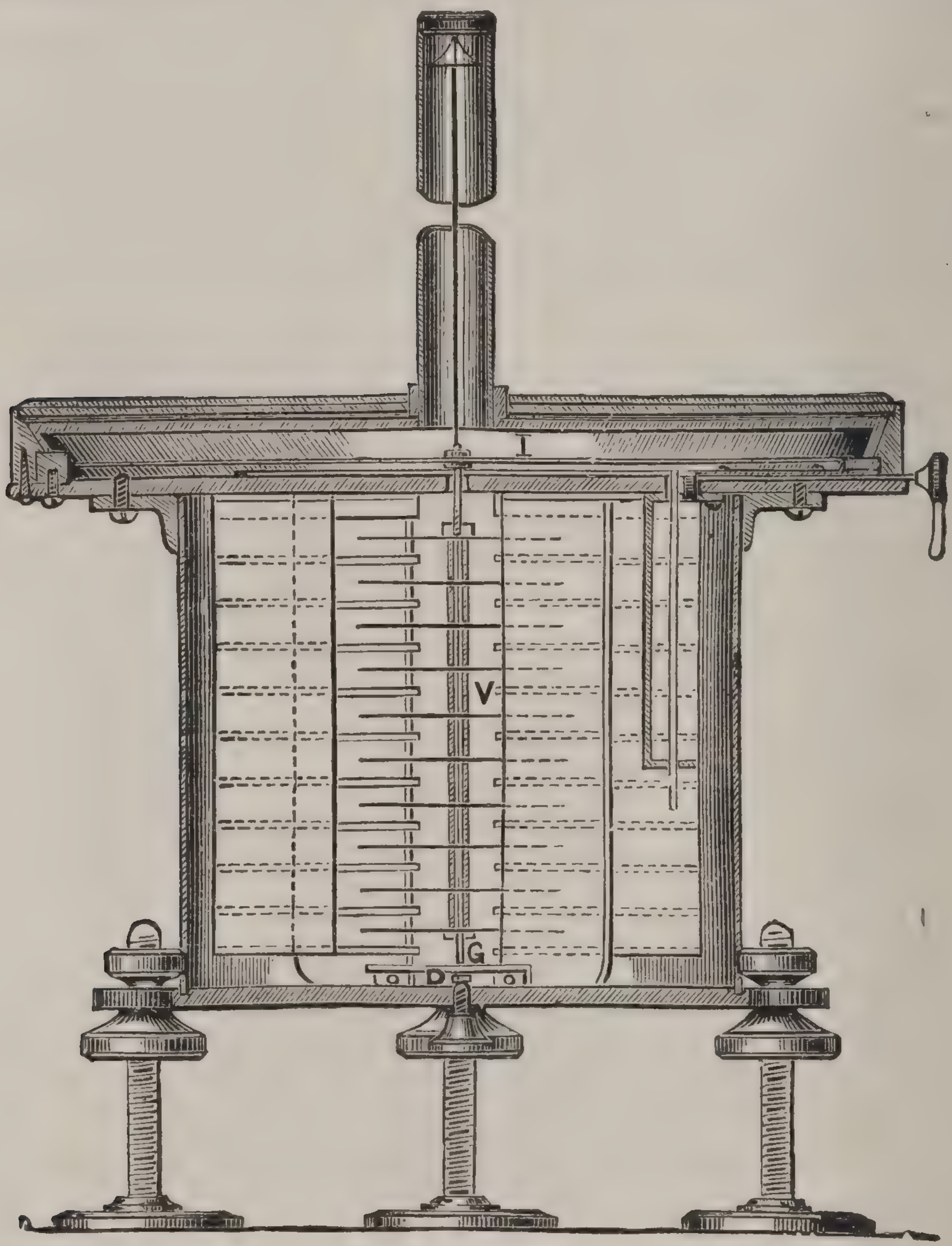


FIG. 240.

horizontal, and are completely enclosed within the brass cylindrical case of the instrument.

On the top of this cylinder is a shallow horizontal circular scale-box containing the scale of the instrument, and having a glass cover, which serves to protect from currents of air the movable indicator *I*, and the scale and interior parts from dust.

For the movable part a number of vanes, *V*, similar in form to those of the quadrant electrometer are used. These vanes are placed parallel to each other on a spindle with distant pieces between them. The top end of this spindle passes through a small hole in the sole-plate of the instrument, which forms the bottom of the scale-box, and is attached to a small coach-spring, which in turn is secured to one end of a fine iridio-platinum wire suspended from a torsion head at the top of a vertical brass tube.

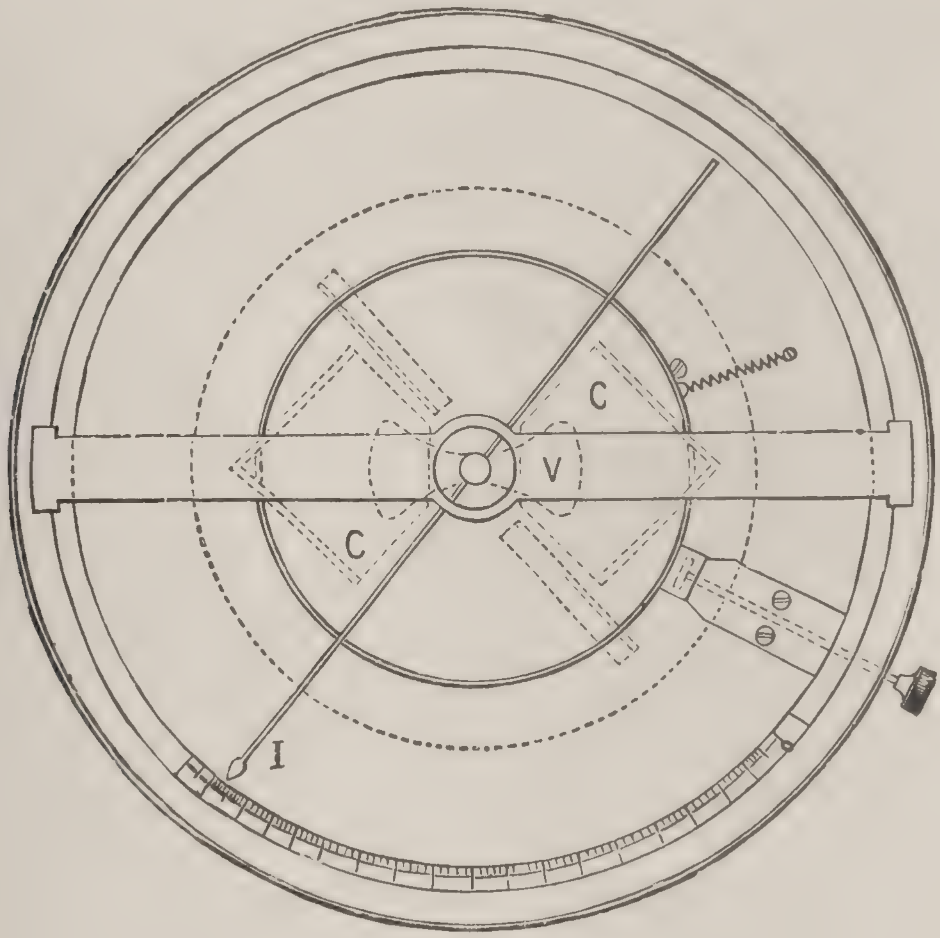


Fig. 241.

The torsion head may be turned by means of a forked key provided for the purpose, and is clamped, to protect it from accidental displacement, by a cap which screws on to the end of the tube. The coach-spring has sufficient resilience to allow the spindle to touch a guard stop, and so saves the suspension from injury in event of the instrument being roughly set down.

Two vertical brass repelling plates, which also act as guard plates to prevent the movable part from turning beyond its prescribed limits, are fixed to the bottom of the sole-plate. These

two plates carry a guide plate, *G*, with a circular opening in it, through which the lower end of the spindle passes. A little brass disc, or head, *D*, is attached to the end of the spindle, sufficiently large to prevent its passing back through the hole in the guide plate. Thus the movable part is effectually secured from swinging about so as to be injured, and by no possibility can it come into contact with the insulated quadrants. When the instrument is level the spindle hangs free by the suspending wire, so that the

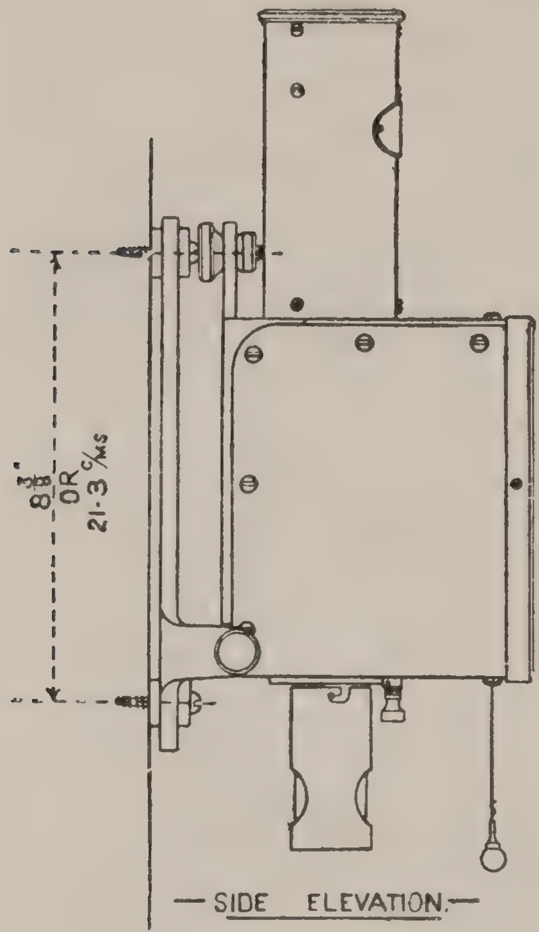


Fig. 242*a*.

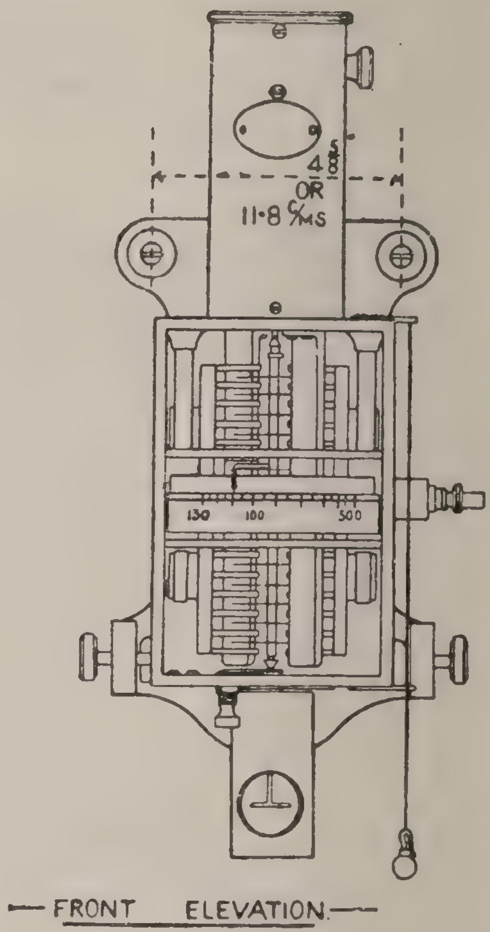


Fig. 242*b*.

vanes are horizontal, and each is in a plane exactly midway between those of two contiguous condenser plates.

An aluminium needle attached to the top of the spindle indicates, on the horizontal circular scale fixed to the upper side of the sole-plate, the difference of potential between the movable and fixed portions of the condenser by direct readings in volts.

Engine-room Pattern Multicellular.—The description of the instrument given above refers to the horizontal scale or laboratory pattern. In the new engine-room pattern (Fig. 242 *a* and *b*), the

parts are in every way similar, but the instrument has a vertical scale. A vane attached to the spindle turns in an oil dash-pot and gives the instrument a dead-beat action.

Portability.—A small thumb-screw is placed in the centre of the base plate below the instrument, which can be screwed in so as to lift the weight of the spindle and vanes from the suspending wire and clamp the disc on the end of the spindle against the guide plate. A lifter or checker is also provided similar to that used in the magneto-static instruments.

A switch is attached to the insulated terminal of the instrument by which the voltmeter can be taken out of circuit when desired. The switch, after breaking circuit, puts the case and the insulated cells in metallic connection.

Instructions for the Use of the Multicellular Electrostatic Voltmeter.

When received from the maker the indicator needle with attached vanes will be found supported by means of the thumb-screw below the instrument, and also by the circular lifter, or checker, turned up so that the weight of the needle and vanes is taken off the suspending wire.

The scale is graduated to read directly in volts.

To set the instrument up for use.—(*a*) Unscrew the thumb-screw, and turn down the checker, so that the needle swings clear; (*b*) level the instrument so that the spindle of the vanes passes down centrally through the intersection of the two black cross-lines on the sole-plate.

To adjust the zero, if necessary.—Unscrew the cap on the top of the tube, remove the washer, turn the torsion head by means of the forked key until the pointer stands at 0 on the scale. Replace the washer and screw on the cap again. Before adjusting the zero turn the switch so that the insulated cells are in metallic connection with the case.

Arrangement for portability.—When the instrument is to be removed from place to place, see that the needle is lifted by turning up the checker, and when it is packed for use as a portable instrument, always screw up the thumb-screw as mentioned above.

As aluminium is electro-positive to brass, the instrument reads

about $\frac{1}{5}$ of a volt too low when the positive pole of a battery or dynamo is attached to the upper or insulated terminal of the instrument; and about $\frac{1}{5}$ volt too high if connected in the opposite direction. With alternating currents it is correct.

Fig. 243.

Crompton D'Arsonval Galvanometer.

A convenient form of sensitive galvanometer, designed for laboratory use, with a large range of adjustment, and made by

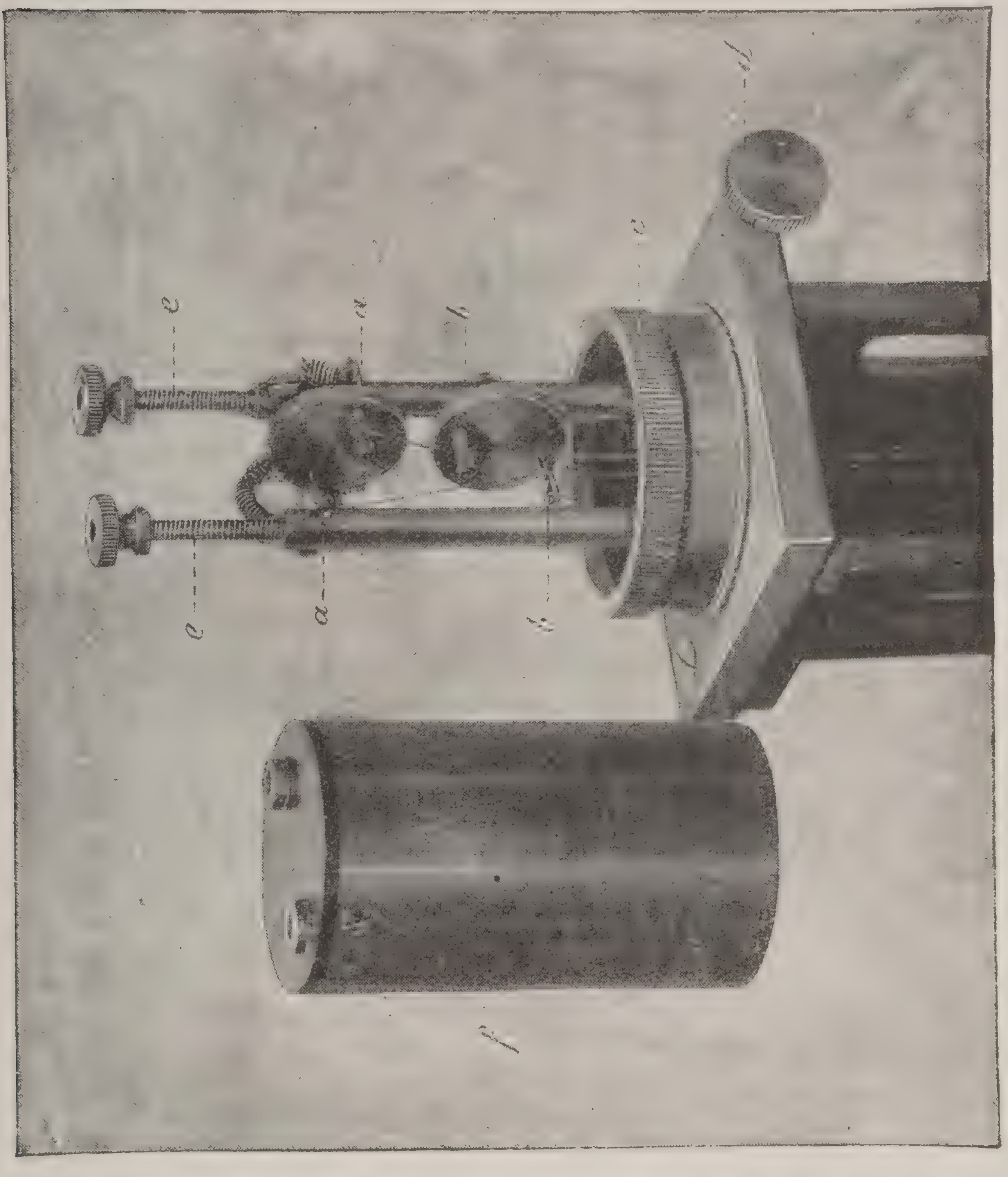


FIG. 244.

Messrs. Crompton and Co., Chelmsford, is shown in Fig. 243, and the details of construction in Figs. 244 and 245.

The moving part of the instrument is shown in Fig. 245. A

circular coil of wire hangs by a bifilar suspension between the poles of a permanent magnet, an iron core being fixed in the centre. The suspension ligaments are of very thin copper strips, and are connected to the coil by means of a silver clip, which allows the coil to be easily disconnected.

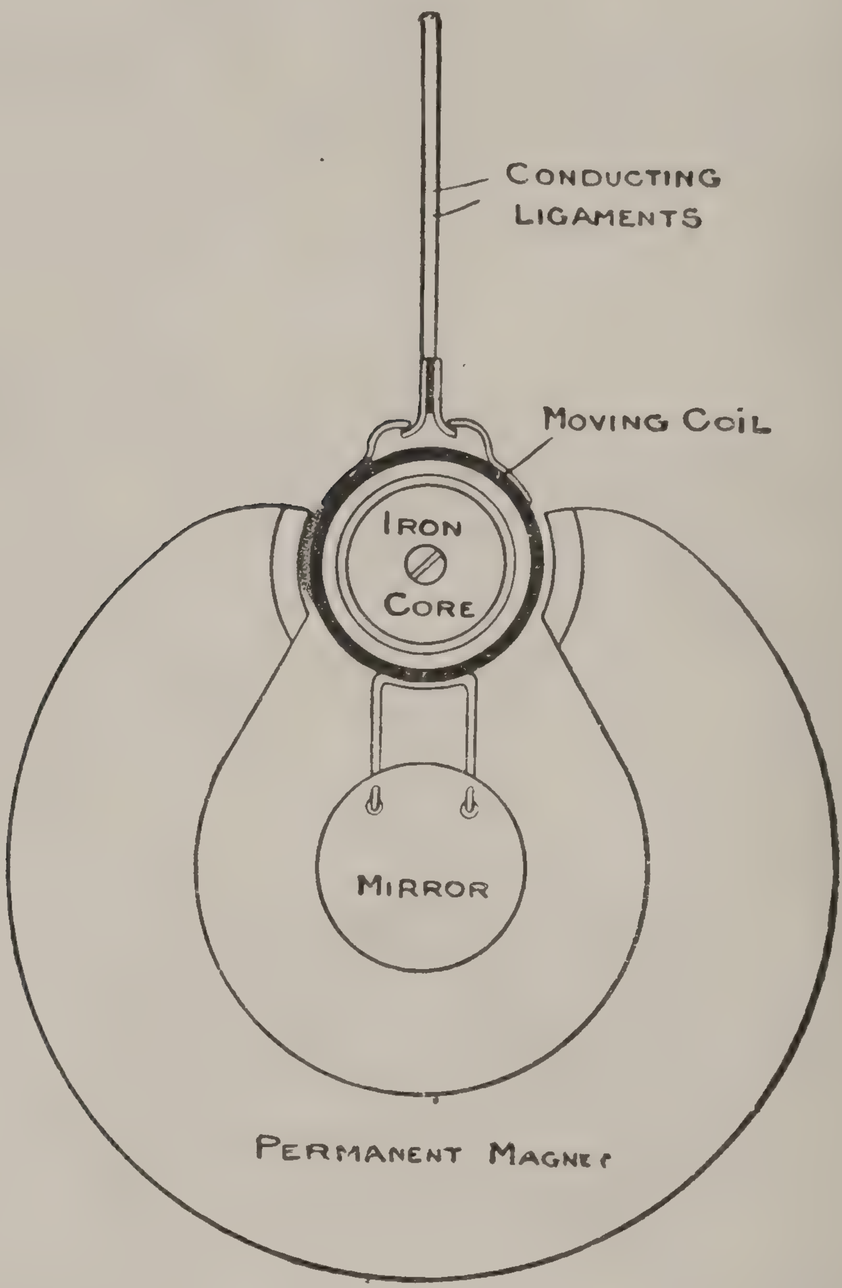


FIG. 245.

The mirror is hung from the coil by fine aluminium hooks passing through holes pierced in the mirror, so that this is easily detached, and is not distorted by the setting of cement or the pressure of a clip.

Fig. 245 shows the construction of the bifilar suspension head. The ligaments are attached to two pins *aa* fixed in a disc, by turning which the tensions of the two may be made equal. They pass over two pins *bb* placed on another disc, by turning which the distance between the ligaments may be adjusted, and the sensitiveness of the instrument increased or diminished.

The whole head is raised or lowered by turning the milled edge *C*, and is rotated slowly by turning the worm spindle *d*.

The two pillars by which the cover *f* is secured serve as the terminals of the instrument.

Coils are made having different numbers of turns from 100 upwards, and the sensitiveness of the instrument when adjusted to give a complete period of oscillation of from eight to ten seconds is nearly as follows—

TABLE XIII.

No. of Turns on Coil.	Resistance, including Ligaments.	Deflection of beam in minutes of Arc	
		For 1 Micro-volt.	For 1 Micro-ampere.
100	2 ohms	6	35
300	30 ,,	3·5	105
1000	1000 ,,	0·6	350

The coils can be fitted with small closed rings of copper, which damp their movements to any desired degree.

Without these the coils are for practical purposes perfectly ballistic.

An electric lamp used without a lens is the most convenient light for the above galvanometer, the filament being focussed on the screen by the mirror. This latter is large (25 m.m. diameter) and ground to a radius of one metre.

Sensitive Portable Galvanometer.

When a Wheatstone Bridge has to be used for outside work, other than in a test-room or laboratory, or when fairly delicate tests have to be made on the "line," a portable type of galvanometer or detector which is as sensitive as possible must be

available. One of the best forms of such an instrument is illustrated in Fig. 246. It consists of a fairly flat-shaped coil of fine insulated wire, having a resistance usually of between 1000 and 2000 ohms, placed on its side in a brass contain-case provided with a glass top and glass window in the side just opposite the scale. A magnetic needle, to which a long light pointer is attached, is pivoted between jewelled bearings in the middle of a flat rectangular brass tube which is capable of being slipped inside the similarly shaped aperture in the coil of wire.

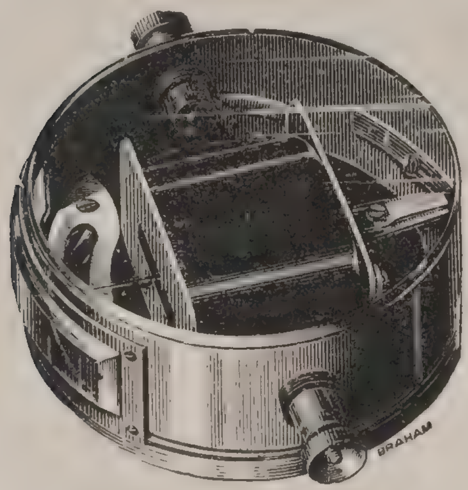


Fig. 246.

The pointer protrudes outside one end of the coil and moves over a suitable scale, part of which is seen to the left of Fig. 246. A strip of mirror is let in under the scale and shows through an aperture in it, thus enabling errors due to parallax to be avoided. A needle clamper, actuated by a button on the edge of the case, enables the needle to be clamped during transport and damage to the pivoting thus avoided.

When no current flows the magnetic axes of coil and needle are perpendicular, when the pointer is at 0 at the middle of the scale. Then the effect of a current is to cause the needle to set itself parallel to the axis of the coil, so giving a deflection to one side or the other of zero. This form of instrument is a very sensitive one and very suitable for portable work with a Wheatstone Bridge.

Parr's Direct-Reading Dynamometer Measuring Instruments.

These instruments depend for their action on the mutual force of repulsion between two circuits or coils carrying either the same or different currents, one circuit being fixed and the other movable. The instruments, which have been devised and perfected by the author, possess some very important properties

that it may be well to note here. They contain *no iron* whatever and very few metal parts, consequently they measure either the *true watts, volts,* or *amperes,* as the case may be, *in any alternating current circuit,* and are quite independent of the periodicity of the circuit. They are of the switchboard type, *direct-reading,* and have extremely open scales, extending over $\frac{9}{10}$ths of the circular dial.

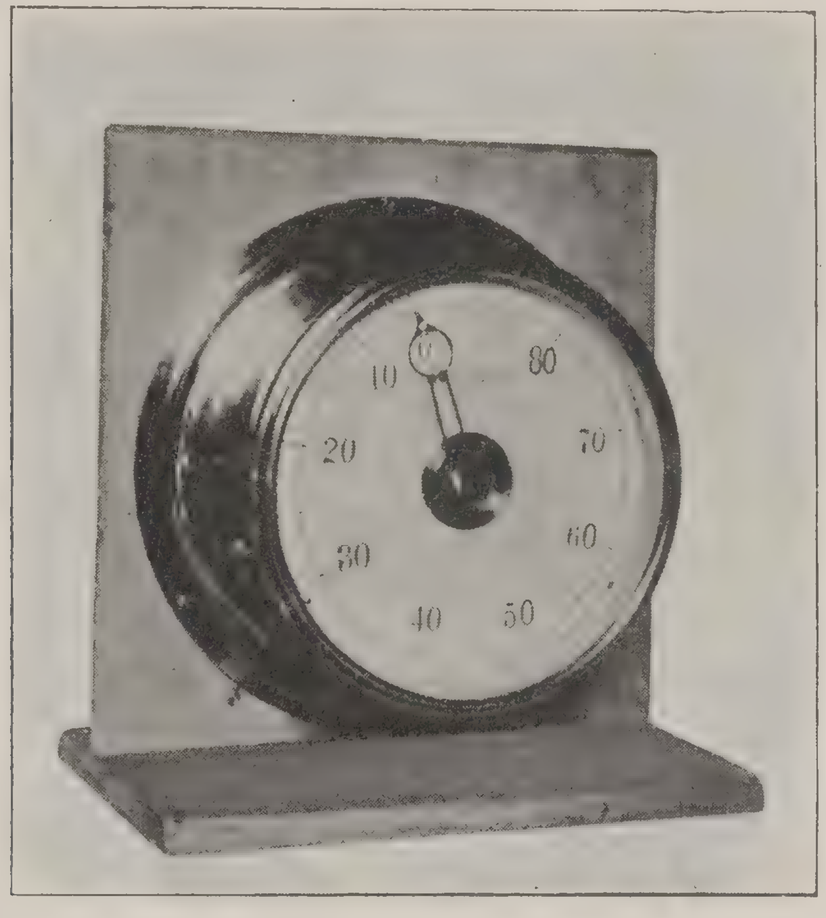


Fig. 247.

Fig. 247 shows an ammeter for 84 amperes, the scale graduations commencing at 3 amps. and continuing *nearly uniformly* to the end. Fig. 248 shows an internal view of this same 7-inch ammeter. As seen it consists of two fixed coils and two moving coils, carried at the end of a horizontal arm capable of rotating on a vertical spindle pivoted in jewelled centres. This spindle has rigidly attached to it a horizontal arm, to the end of which is fixed a flexible metallic strip that passes almost once round a special pulley carried by a horizontal spindle moving in jewelled centres and carrying the pointer at the front end, a

hair-spring, and balance-arm for the pointer. The moving coils and pointer are controlled by the hair-spring, the tension of which can be adjusted by a moving arm. Current is let into and out of the moving coils through non-spillable mercury cups. A damping vane and trough is added, also unspillable, by means of which the instruments are made *dead beat* to any desired

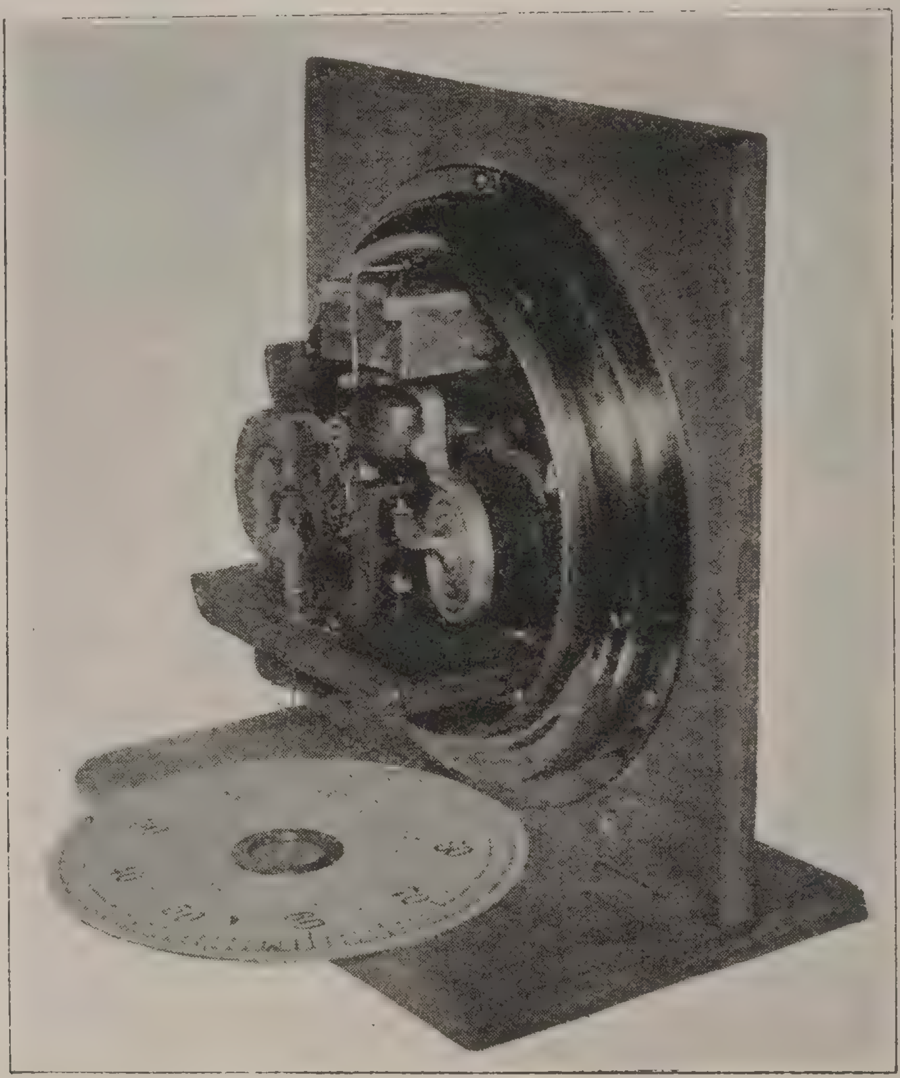


Fig. 248.

extent. The moving coils are clamped by a suitable arrangement during transport and seen to the right of Fig. 248. The moving coils are in contact with their respective fixed coils when the pointer is at 0 and no current flows. Repulsion ensues when a current is sent through the instrument, and according to whether it is an ammeter, voltmeter, or Wattmeter, so the pointer deflects and indicates directly the quantity to be measured. The instruments of course read equally accurately

on direct-current circuits, and having such very wide, open and uniform scales, a 6-inch instrument can be read with certainty many yards away.

Siemens Torsional Voltmeter.

This instrument may be used either as an ammeter for very small currents or as a voltmeter with a very extended range,

FIG. 249.

and provides a good example of the method of converting one into the other.

It consists of two coils, wound with fine insulated wire, and of elongated oval section, with their axes collinear and horizontal. They are connected in series and wound so that the north pole

of one faces the south pole of the other, *i. e.* the two coils may be regarded as one coil with a gap in the centre.

Between the coils is placed a horse-shoe or bell magnet suspended by a silk fibre. A spiral spring is attached to the magnet and to a torsion head at the top of the case, so that by turning the head, a twist is applied to the magnet, proportional, of course, to the angle of turning of the torsion head. To the magnet is attached a pointer, for the zero position of which the magnetic axes of coils and magnet are at right angles.

The magnet also carries an aluminium vane, moving between two brass cheeks which act as stops, the vane assisting in stopping the vibrations.

On passing a current through the coils so as to induce the polarity indicated by the small letters *ns*, *ns* (Fig. 250), which represents a symbolical sectional plan of the coils C and needle NS in a horizontal plane passing through their centres, the magnet NS tends to turn *counter-clockwise* in the direction of the arrow, so that its magnetic axis would coincide with that of *ns*, *ns*. Then the angle through which the torsion head has to be turned (in a *clockwise* direction) in order to bring the magnet pointer back to zero measures the moment of the couple exerted by the coils on the magnet, *i. e.* the current flowing through those coils.

The coils C together have a resistance of 100 ohms in the instrument illustrated, and it is wound with such a number of turns that when 1·7 volts are placed across the terminals of the instrument itself, the torsion head makes one complete turn or 170 divisions to bring the magnet pointer back to zero. Hence 1 scale division = 0·01 volt. To make it read 0·1 and 1 volt per division, 900 and 9900 ohms respectively must be put in series with the instrument, when the extremities of the combination will now form the voltmeter terminals. These extra anti-inductive resistances are contained in the receptacle to the left in Fig. 249, which is provided with a plug top for inserting these resistances at pleasure. They should be wound with a

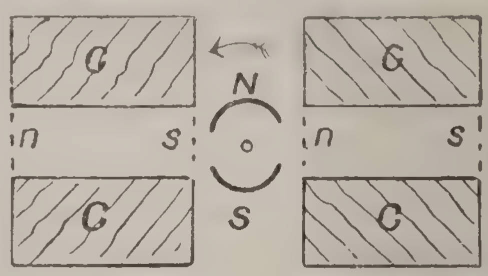


Fig. 250.

material having a high specific resistance and *low temperature coefficient of variation of resistance* for reasons already given.

This instrument can be used as a low-reading ammeter, for since 1 division $= 0{\cdot}01$ volt and the whole resistance of its coils $= 100$ ohms, $\therefore$ 1 division $= \frac{0{\cdot}01}{100} = 0{\cdot}0001$ ampere.

Adjustment of voltmeter.—If the magnet has been clamped for transport, release it by turning the milled-headed rod which passes through the edge of the base at the back.

Very carefully level the instrument by turning the levelling screws so that the pointed pin, attached to the moving magnet, is just over the cross marked on the fixed stud under it. The moving system should now be quite free, at all events laterally. See therefore that this is the case.

The height of the moving magnet can be adjusted to give freedom of motion by carefully turning the milled-headed pin which passes into the torsion head at the top of the instrument. Set the torsion head with its pointer to zero, and then bring the magnet pointer to its zero by turning the wooden base which carries the coils. Fix the instrument in this position by turning the milled-headed pin which projects from under the base. In using the instrument thus adjusted to measure current, place it directly across the low resistance provided, and to measure higher voltages connect it in series with the separate anti-inductive resistance provided with it, when the extremities of the combination will then be the terminals of the voltmeter.

Caution.—Make quite sure that the correct resistance is plugged in, otherwise the instrument may be fused up. The plug may be used as a make and break key. Being a + and − instrument, it must be connected up in circuit that the magnet pointer tends to move in the opposite direction to that of the torsion head

Siemens Electro-Dynamometer.

This instrument depends on the electro-dynamical action of one circuit which carries a current on another circuit carrying either the same or a different current, and is illustrated in Fig. 251. It consists of a base supported on three levelling screws and carrying

an upright standard, to which latter is fixed two distinct stationary coils usually wound with two different gauges of wire and number of turns so as to obtain a wider range of sensitiveness and measurement than would be possible with only one fixed coil.

A movable coil, the plane of which is perpendicular to that of the fixed coils in the normal position of the former when actually

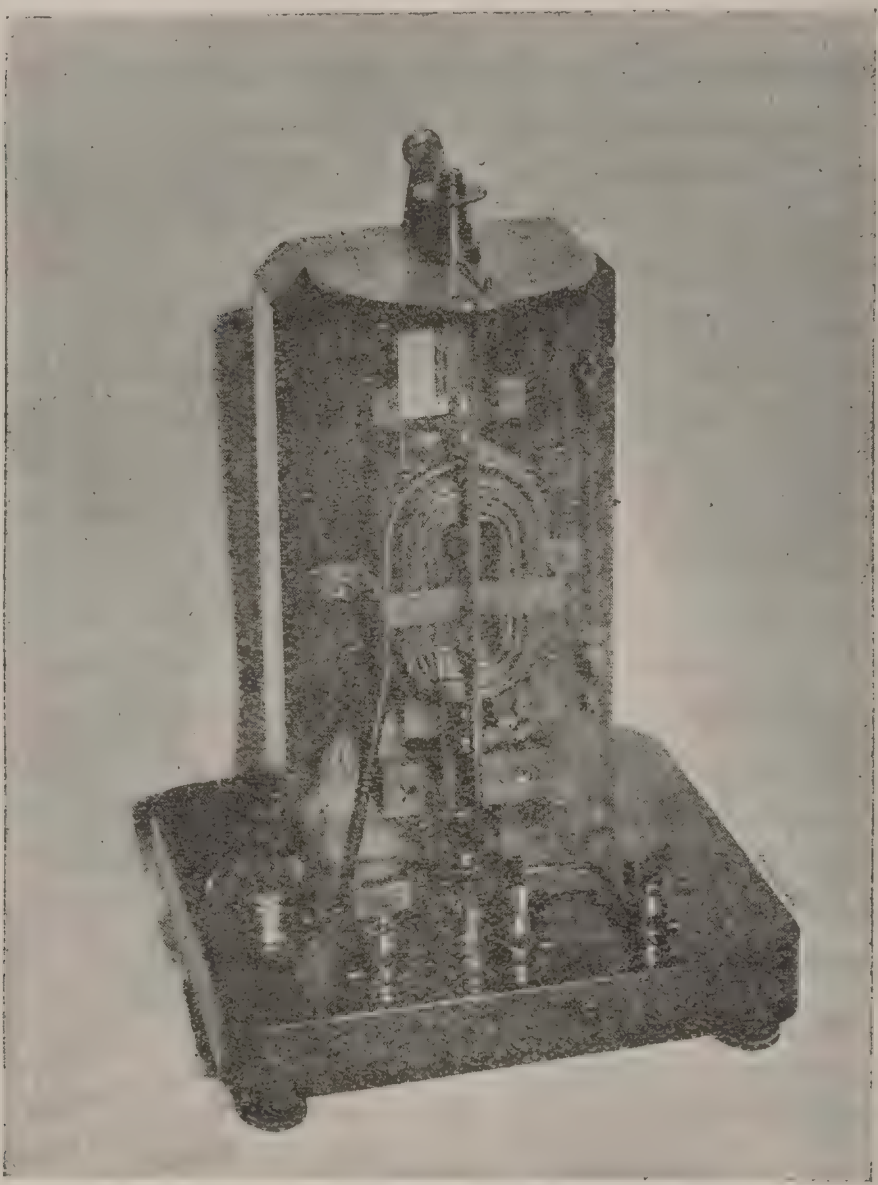


Fig. 251.

measuring a current, is suspended by means of a silk fibre from a torsion head, at the top of the instrument, carried at the centre of a graduated sale, which is itself screwed to the top of the standard.

A rather long helical spring, composed of a sufficient number of turns, has one end fixed to the under side of the torsion head and

the other to the top of the moving coil, which is thus controlled by the turning of the head. Electrical connection is made with the moving coil through two mercury cups into which its ends dip and which are directly in a line, *under* the point of suspension. A milled-headed pin or rod, carried by a light support, seen at the back of the scale, passes through the hollow torsion head, and has attached to it the silk thread that suspends the movable coil and which passes down through the torsion head. Hence, by turning the rod round the swing coil can be raised or lowered so as to clear the other fixed fittings. The moving coil, which may consist of more than one turn, can be raised and clamped during transport by a spring clamp (not seen in Fig. 251) at the back, controlled by a milled nut. The instrument requires to be carefully levelled before using to ensure perfect freedom with the moving coil. The levelling screws and spirit-level are added for this purpose, though sometimes a plumb line is used in place of the latter. The fixed and moving coils are in simple series; one end of each of the fixed coils goes to the outside terminals, the other ends both to the top mercury cup and the lower cup to the centre or common terminal. Thus there are two sensibilities, viz. the moving coil in series with either fixed one, according to whether the centre and left or centre and right pair of terminals are in use. Since at the actual moment of measuring a current by the dynamometer, the fixed and movable coils are always in the same position (*i. e.* their axes perpendicular) relatively to one another, due to the index pointer on the moving coil always being brought back to zero by turning the torsion head, the couple or force, whether of attraction or repulsion, exerted by one coil on the other is $F \propto C_1 \times C_2 \propto C_1 \times C_1 \propto C_1^2$ where C_1 and C_2 are the currents in the two coils which are equal or the same. But this force is just balanced by the force of torsion exerted by the spring $\propto$ angle of torsion or the deflection D of the torsion head. Hence, $D \propto C^2$,

$$\therefore\ C \propto \sqrt{D},$$

$$\text{or } C = K\sqrt{D} \text{ amperes},$$

where K is the constant for the particular fixed coil used which gives an equation of equality.

This is the *law of the Siemens electro-dynamometer.*

Some of these instruments have scales divided into numbers $\propto$ to the square roots of the usual divisions, and in such cases the

current $C = K \times$ scale reading simply. In using these instruments care must be taken to either twist the "leading in and out" leads together, or run them very close so that the swing coil may not be affected by the current in these leads.

In calibrating or using the instrument with direct currents, it must be so placed that the plane of the suspended coil when in its zero position is perpendicular to the plane of the magnetic meridian of the earth. The reason for this is, that when the swing coil carries a direct current it is acted on by the earth's magnetism independently of the action due to the current in the fixed coil, and the position of rest for the first cause is when the planes of the magnetic meridian and swing coil are at right angles. For alternating currents there is no such action.

Siemens Dynamometer-Wattmeter.

Except for the swing or movable coil, this instrument is precisely similar to the preceding dynamometer. It is illustrated in Fig. 252, which indicates two or three alterations to the general form of the Wattmeter, which the author has thought it beneficial to make. The one illustrated is provided with two thick fixed coils, as in the dynamometer, Fig. 251, connected directly to the three large terminals in the middle, so that two distinct sensibilities can be obtained instead of usually only one.

The swing coil now consists of many turns of fine insulated wire wound on a light rectangular frame of ebonite or boxwood. Only a few of the turns are wound inductively, the rest being doubly wound and therefore non-inductive. The total resistance of the swing coil is, however, that due to the sum of all the turns, which may amount to 5000 ohms or more. Current is led into and out of the swing coil through thicker wires soldered to the fine wire and which dip into the two mercury cups. These last-named are directly connected to a separate pair of small terminals seen on the extreme right and left, having no electrical connection whatever with the thick coils.

The scale is provided with a mirror for the purpose of avoiding errors due to parallax in reading the position of the torsion head pointer. The mirror glass covers the scale, but a circular strip of

silvering is removed just over the scale, enabling it to be seen but preventing it getting dusty and dirty. In all other respects this Wattmeter is the same as the dynamometer shown in Fig. 251. When the swing coil is returned to its zero, by turning the torsion head, we have as before its deflection $D \propto C_1 \times C_2$, but if $C_1 =$ the

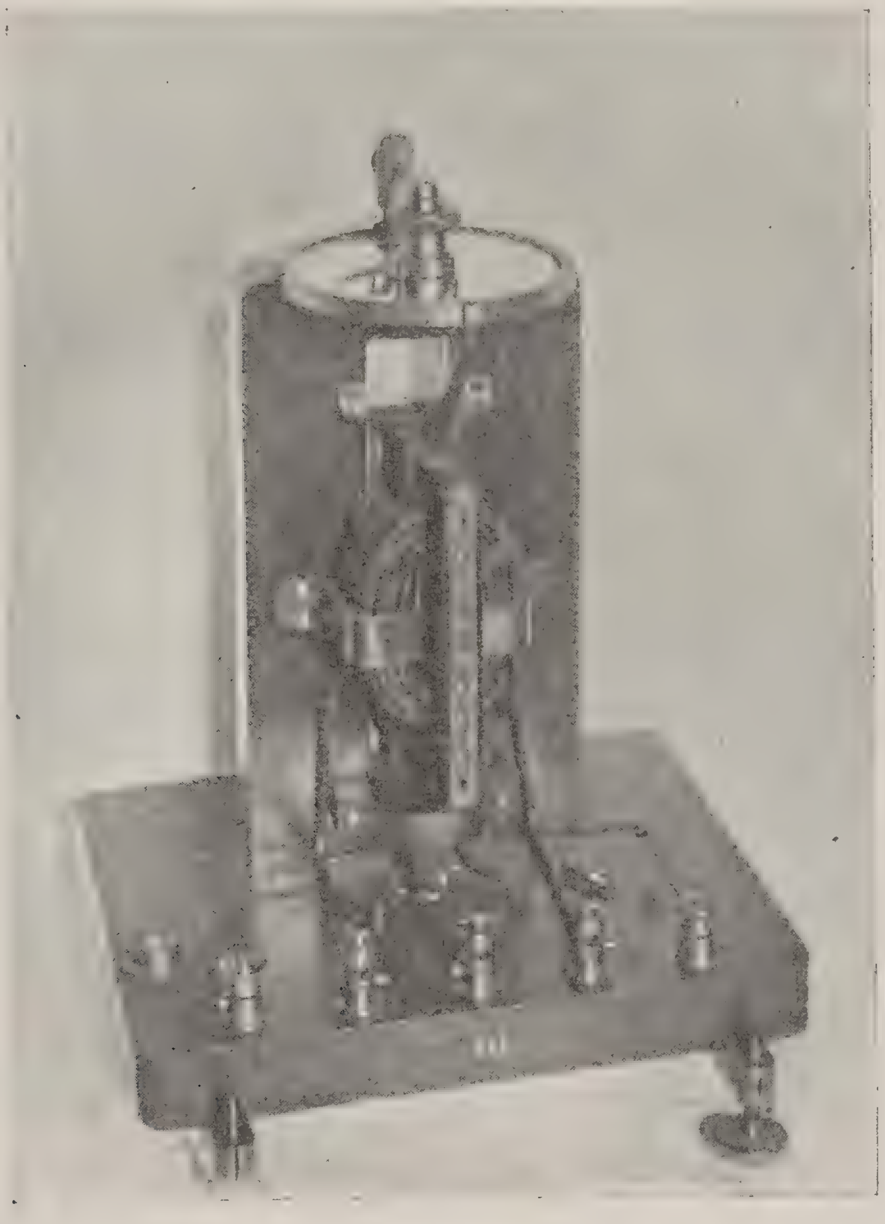


FIG. 252.

main current and C_2 the current in the fine coil, which is placed across the mains, and therefore is $\propto$ to the voltage (V), we have $D \propto C_1 V \propto$ Watts,

$$\text{or Watts } (W) = KD,$$

where K is the constant of the instrument for the thick coil used. Thus by combining the voltmeter and ammeter in one and the

same instrument, the deflection of the new instrument so formed measures the power in Watts absorbed by any circuit. Though the Wattmeter is of no great use in direct current work, since we usually require both the amperes and volts separately and can always multiply them and so obtain the power when desired, the instrument is of incalculable value in alternate current work, since, if nearly non-inductive, it is the best known means of obtaining the *true power* in such a circuit, the product = amps. × volts not giving this quantity.

The same precautions are necessary in using the Wattmeter as in the dynamometer, and in addition errors may arise through the warming up of the swing coil and consequent alteration of its resistance, as in the case of electro-magnetic voltmeters. The error that may be introduced by the earth's field is explained on p. 580.

Change-over Switch.

Fig. 253 represents a form of switch suitable for large currents and which can be used in one or other of four ways as follows—

(1) As a single-way, single-pole switch by connecting to the centre and either of the end terminals on the same side.

(2) As a single-way, double-pole switch by connecting the centre and an end terminal on the same side in one main, while the other centre and corresponding terminal at the same end is put in the other main.

(3) As a two-way double-pole switch by connecting the common circuit to the two centres and each branch circuit to the pair of terminals at one end.

(4) As a reversing switch by connecting the circuit or portion to be reversed to the centre pair and the main current to the pair at either end, cross connecting the corner terminals at the ends by temporary wires.

The figure shows the construction fairly clearly, and it consists of two metal blades carried by smaller extensions at their lower extremities and capable of turning about a horizontal axis in the upright standards which form part of the base carrying the middle terminals and inner wedge blocks. Four similar wedge blocks

(two at each end) are carried by metal bases on which the four end terminals are fixed. When the blade levers are up (as seen), the metal parts, electrically connected to the six terminals, are insulated from each other, as the lever blades are also insulated from each other, their upper extremities being fixed by an insulator cross-piece to which the handle is fixed. When the blades are

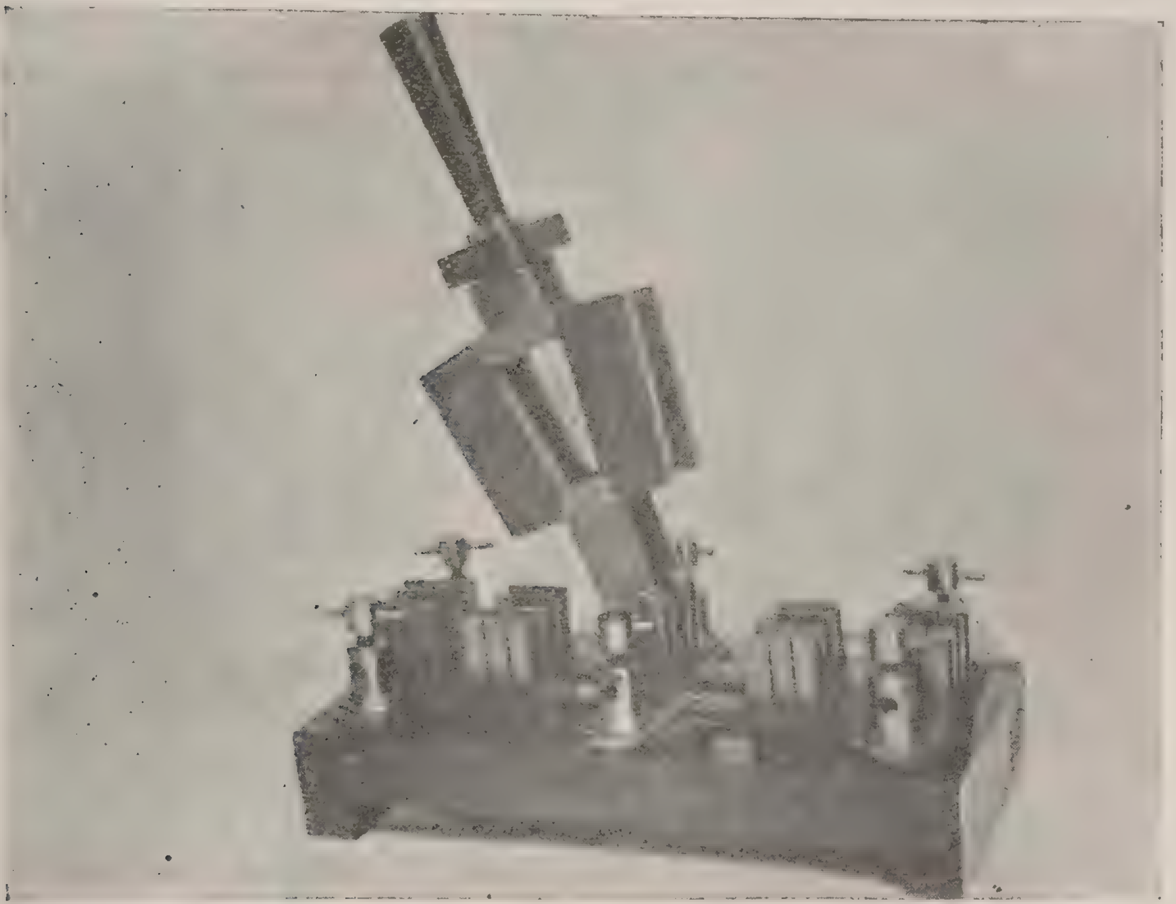


Fig. 253.

pushed down one side or the other into their respective pairs of wedge contact blocks they short circuit these, thus joining the centre and end terminals on one side together, and likewise those on the other side.

Keys.

A form of key, which, though not very portable, is extremely useful in a test room, is shown in Fig. 254, and is otherwise known as a Pohl's commutator. It consists of a wooden, though preferably polished ebonite, base *M*, containing 6 small pure-copper

mercury cups *ABCDEF*, each suitably connected to a terminal (not shown).

A polished, and preferably corrugated, ebonite rod or bar G is supported on two pure-copper pillars S_1 S_2, the lower ends of which dip into and are securely pinned to the cups B and E respectively, so as to be capable of rocking backwards and forwards in B and E. G carries two curved pure-copper rods r_1 r_2, connected electrically to S_1 and S_2 by the copper strips p_1 p_2 respectively. Thus as G is rocked to one side or the other, so r_1 and r_2 are dipping simultaneously into cups C and F respectively, or A and D, Fig. 254 showing the latter position. Consequently in this position A and B will be in electrical connection and also D and E with

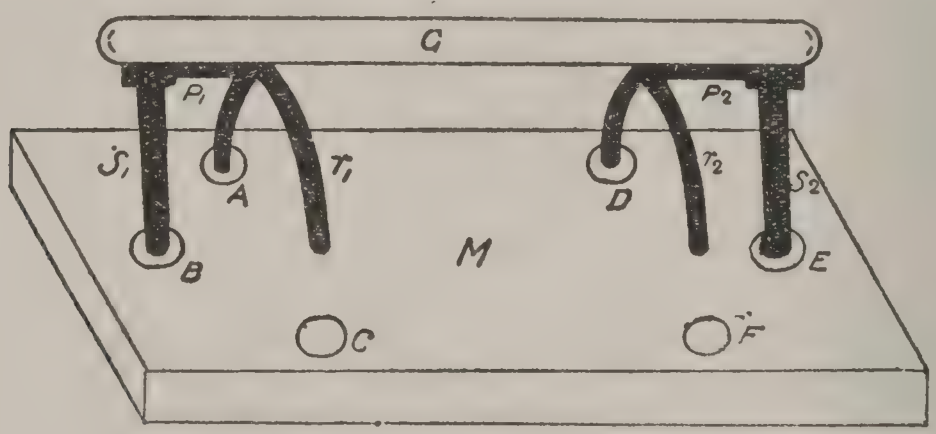


FIG. 254.

G, turned over then B and C are connected instead and also E and F.

It will now be evident that such a rocking key can be used in one of four ways—

(1) As a single-way, single-pole key by connecting to B and A or B and C, and likewise at the other end.

(2) As a single-way, double-pole key by connecting one wire to B and A and the other of the pair to E and D.

(3) As a 2-way double-pole key by connecting the common circuit to B and E and the branches to AD and CF respectively.

(4) As a reversing key by connecting the circuit to be reversed to B and E and the main circuit to either A and D or C and F, cross connecting A and F and also C and D.

This kind of key or Pohl's commutator is sometimes convenient

to use in insulation resistance work, and for such, the base M should be of well-polished ebonite, quite clean and free from dust, while the cups and terminals should not be too close together. It is an advantage also for G to be corrugated and well polished, so that leakage to the hand on turning G over is reduced to a minimum. Owing to the key having mercury cups, it cannot be said to be portable in the ordinary sense of the word

A less elaborate though equally useful key is shown in Fig. 255. If anything it is a little more portable than the Pohl's key, but cannot be operated quite so quickly. It will perform the same four offices as the above key, but has the advantage that no cross connection is necessary in order to be able to employ it as a reversing key. The principle of construction is shown in Fig. 255, which represents a plan of the key. On a wooden or preferably a polished ebonite base B are four terminals T_1 T_2 T_3 T_4 connected by pure-copper strips (shown black) to mercury cups a, b, c, and d.

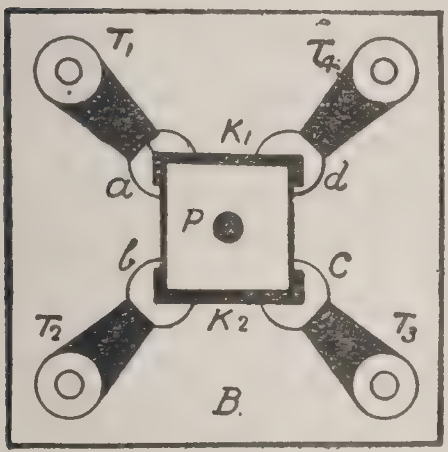


FIG. 255.

A cubical block of polished ebonite P capable of sliding up and down on, and also of turning on, a central metal pin, carries two pure-copper strips K_1 K_2, the ends of which terminate in legs which dip into the cups a . . . d.

In the position shown, terminals T_1 and T_4 are in electrical connection through K_1, and also T_2 and T_3 through K_2. If, however, P is raised out of the cups, turned round 90°, and again slipped into the cups, then T_1 and T_2 will now be in connection through K_2 (if P was turned clockwise), and T_3 T_4 through K_1. The key can obviously be used in the following ways—

(1) As a single way, single pole key by joining to any pair of adjoining terminals, *e. g.* T_1 and T_2.

(2) As a single-way, double-pole key by connecting the pair of incoming leads to any adjacent pair of terminals such as T_1 T_2, and the out-going to the opposite pair T_3 and T_4.

(3) As a reversing key by joining the main leads to either pair of diagonally opposite terminals, *e. g.* T_1 and T_3, and

the circuit to be reversel to the other pair T_2 and T_4 or *vice versâ.*

(4) As a 2-way key by joining one main to any one terminal, *e. g.* T_1 and the branches to T_2 and T_4.

A Highly-Insulated 2-Way Spring Tapping Key

is shown in Fig. 256, and consists of a well-polished ebonite base B, supporting at one end a well-polished ebonite standard S, to which is fixed two brass contact terminal blocks t_1 t_2.

Let into and carried by the top of the standard S is an ebonite rod R, which at its other end supports an ebonite

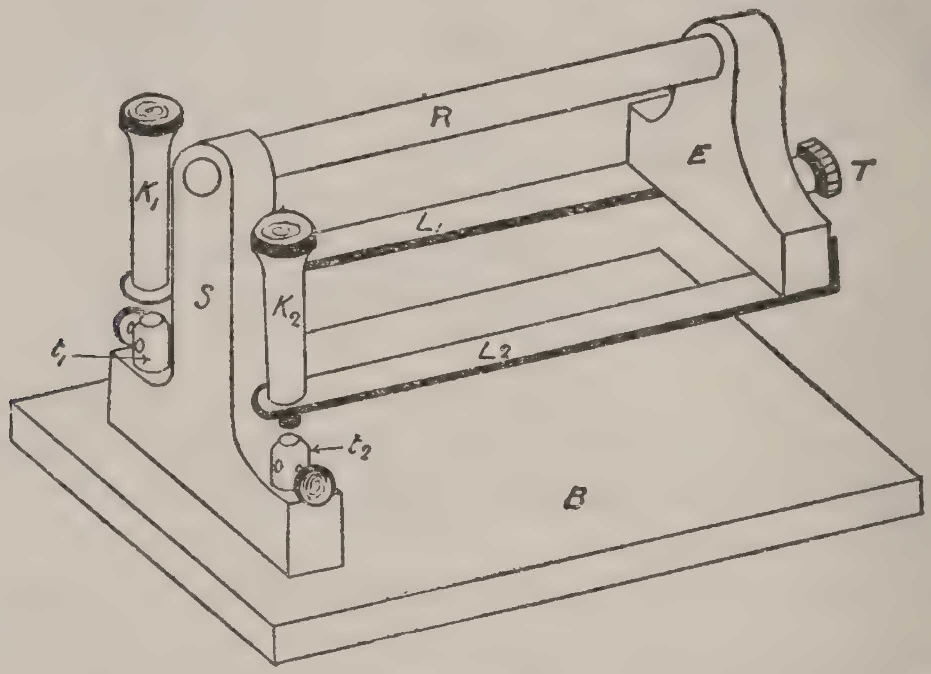


FIG. 256.

block E. To E is fixed two springy brass strip levers L_1 L_2, electrically connected together and to the common terminal T at that end.

The free ends of the spring strips are provided with rather long ebonite knobs K_1 K_2 for the finger of the operator to tap.

In the use of this key, if the high-potential wire is connected to the common terminal T_1 the path of any leakage lies from T across E, then along R and down S to the base B, and thence to earth. This being long gives the key a high-insulation resistance,

and to still further increase this, all the ebonite parts should be well polished and quite clean and free from dust.

It will be noticed, that since there is a lever to each *way*, it would be possible to press both at once. Unless otherwise directed this must be absolutely avoided, as serious damage may be done in consequence.

Fig. 257 shows, in plan, a convenient form of 2-way *sliding* switch. It consists of a wooden or ebonite base B, to which is fixed three terminals T, T_1 and T_2. The two latter, T_1 and T_2, make permanent contact with the *contact blocks* b_1 b_2 respectively, while T acts also as a centre for the contact lever L to turn on. The knob K is merely for the purpose of conveniently turning the lever L.

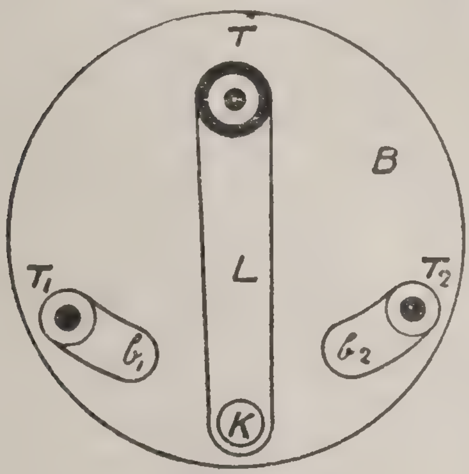


Fig. 257.

In the position shown in the Figure, there is no connection between T and either T_1 or T_2, but by turning L so as to rest on b_1 or b_2, then connection is made between T and T_1 or between T and T_2 respectively.

It will be noticed that only one contact can be made at one and the same time. This is an advantage in some cases, where the simultaneous making of both ways might cause an accident.

Arc Lamp Photometer Cradle.

In the photometrical testing of electric arc lamps it is necessary to be able to have the means of measuring the candle-power of the arc in several directions, making various angles with the horizontal line passing through it. In some cases this is done by raising or lowering the lamp vertically, the beam from it being reflected along the bench by a fixed plane mirror suitably placed, but capable of rotating on an axis.

The author has devised the cradle shown in Fig. 258, in which the lamp to be tested is placed. The lower part is a rigid frame-

FIG. 258.

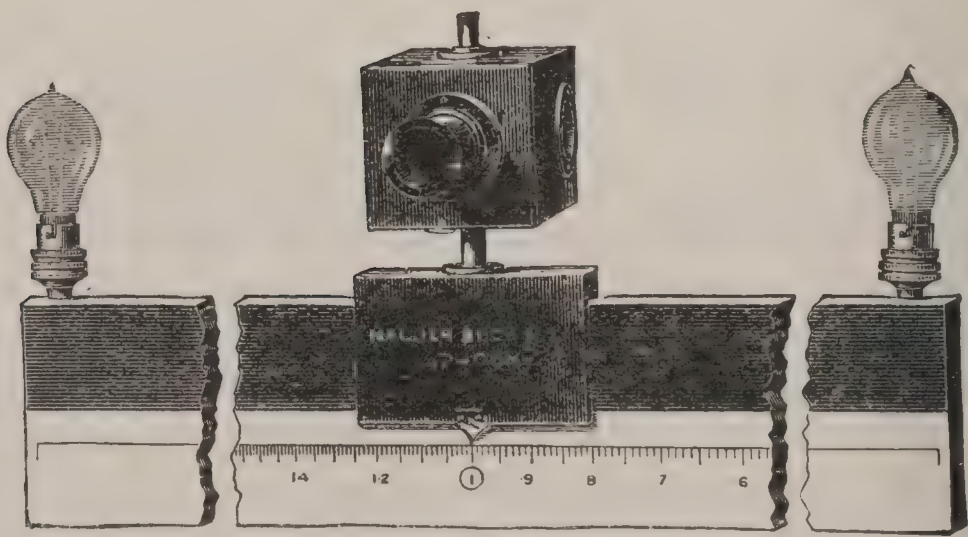


FIG. 259.

work which is placed on the photometer bench and carries an upper frame capable of moving round a hollow tubular horizontal axis, which latter is itself capable of moving in the fixed frame and has attached to it a pointer, seen in Fig. 258, just above the two large terminals. In this upper frame is placed the arc lamp, which can therefore be inclined at any angle to the vertical, as read off by the pointer on its scale.

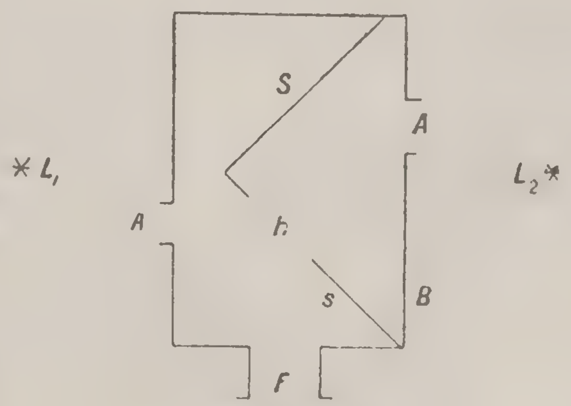


FIG. 260.

The carbons of the lamp are first adjusted so as to touch at a point in the axis of rotation, as seen through the tubular axis. Thus the arc in turning as the cradle is turned, always maintains the same position relatively to the photometer bench, and the author has found that next to no difficulty in the regulation of the arc by its auto-mechanism occurs up to 50° or 60° from the vertical. After this the carbons have to be partly regulated by hand.

Direct-Reading Bar Photometer.

This form, due to Mr. Trotter, has a direct-reading scale on its bank which shows without calculation the ratio between the standard and lamp under test when the sight-box seen in the middle of the bank (Fig. 259) is moved so as to obtain an equal illumination of the screens. The general arrangement is very suitable for making rapid tests on electric glow lamps, as slight variations of E.M.F. affect both lamps equally and do not cause appreciable errors. Fig. 260 shows a sectional plan of the sight-box *BB*, in which *AA* are the apertures at the sides to admit the beams of light from the two lamps L_1L_2 to be compared. *SS* are two screens, the illuminations of which are compared. One of these contains a star-shaped hole for the purpose of enabling the further screen to be seen through the funnel or window *F*, through which the observer looks.

Illumination Photometer.

This is a portable direct-reading instrument devised by Mr. A. P. Trotter, by which the illumination at any spot in a street or building can be at once measured in terms of the illumination given by an amyl-acetate standard of light, this being found more reliable and less troublesome than ordinary standards of light. A general view of the instrument is shown in Fig. 261, and a sectional elevation in Fig. 262.

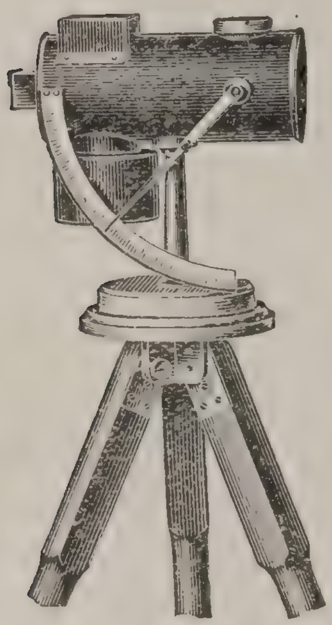


FIG. 261.

INSTRUCTIONS FOR USE.

Remove the end cap with the mirror *M* on it by the bayonet joint; remove the cover from the lamp and light it, then replace the end cap. The flame can now be seen in the mirror. The top of the flame should just touch the point of the bent arm. The flame is raised or lowered by screwing the lampholder in or out by its lower end. By unscrewing the holder completely it can be drawn out for refilling.

Having adjusted the flame to the right height the cap on the top is removed and the photometer is set so that the paper screen *S* is horizontal.

The general illumination to be measured falls on this screen *S*. Across the centre of it is a small slit *T* through which an inside screen *R* illuminated by the standard lamp can be seen. The observer's eye must be vertically over the hole in the screen. The inside screen *R* is then adjusted by the outside arm *P*. When the illumination of the outside screen *S* is greater than that of the inside, the slit *T* will appear dark; when less the slit will appear bright. With a little practice the slit can be made to nearly, if not quite, vanish; the illumination is then shown on the scale *F* by the pointer *P*. The unit used is the illumination due to one standard candle at one foot; that is, if a balance is obtained when the pointer is at "1," the illumination is equal

to that of 1 standard candle at 1 ft. distance, if at "2" the illumination is twice this, and if at "·1," one-tenth of that which would be given by a standard candle a foot away.

The inside screen *R* has a slight blue tint which to a great extent removes the colour difficulty.

A slightly yellow upper screen is provided for measuring with very blue lights.

Be careful the eye is vertically over the slit. Holding one

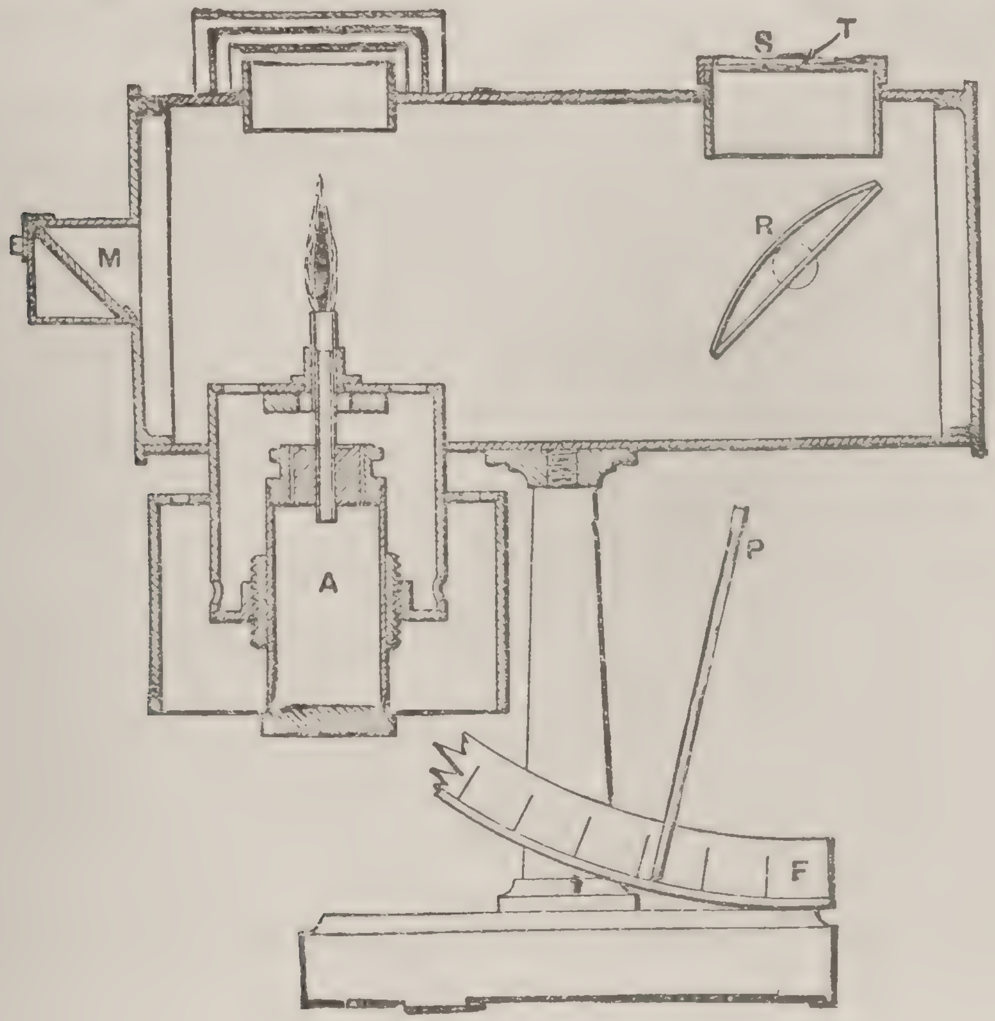


FIG. 262.

finger near the eye and between it and the hole will make it easier to get the vertical line.

Be careful to stand so that the body is not between the screen and any source of light.

The slit must be square across the instrument.

See that the height of the flame is right before and after each reading; the flame sometimes increases a little, especially just after being lighted.

The flame is not affected by a gentle breeze, and on windy nights the instrument can easily be shielded by a piece of cardboard or brown paper.

Only pure amyl-acetate must be used, and the wick must be cut square across without ragged points.

Keep the cap on over the screen when not in use, and do not let the screen get dirty.

There is a small pin at the centre of the pivot of the arm; when the instrument stands so that its shadow thrown by any lamp falls on the scale, it gives the cosine of the angle of incidence, having which the actual C.P. can be calculated. The height of the lamp is equal to the distance from the post when the shadow falls at 45°.

Photometer Screens.

Of these there are many different types, all of course effecting the same purpose, namely, that of enabling equality of illumination due to two different sources of light to be visibly determined, and hence their relative intensities. Fig. 263 shows a form of Bunsen screen arranged inside a "sight-box" seen with its top or lid open to enable the inside to be seen. It consists of two discs of plain paper having its centre portion greased in the form of a star. These discs are seen one near each end of the "sight-box," which is dull black inside and prevents stray light due to external or internal reflection from getting to the Bunsen star discs or screens. Two vertical plane mirrors are placed as shown, symmetrically with regard to the discs, and each at 45° to the back of the box.

The images of the two discs can be seen in the two mirrors through two rectangular open windows in the box shown in front of Fig. 263, and thus the intensities of their illuminations can be compared by the eye. It should be noticed that the sight-box illustrated in Fig. 263 cannot give true results, since the ratio of the squares of the distances from the light sources to the star discs and again to the centre of the box are not the same. The box is given, however, to indicate this fact and to act as a warning to users of it. Another form of this screen is shown

resting on the top of the "sight-box," illustrated in Fig. 264. Here there is but one disc with a plane mirror on either side of it, making equal angles with it, and enabling an image of each side of the screen to be seen without looking directly at the disc itself.

The lower part of Fig. 264 illustrates a form of balancing screen due to Jolly, and consisting of two rectangular blocks of clear paraffin wax, cemented together but separated by a film of silver paper so as to prevent direct transmission of light right

FIG. 263.

through the two blocks. They are placed inside a sight-box to prevent stray light getting to them, hence the two blocks will appear equally bright when the two sources of light to be compared, placed on either side of the sight-box, illuminate them equally.

One of the best photometer screens is that due to Messrs. Lummer and Brodhun, and is shown symbolically in Fig. 265. In this type the light from the two sources to be compared falls on a screen SS, having equally light surfaces. The rays are then

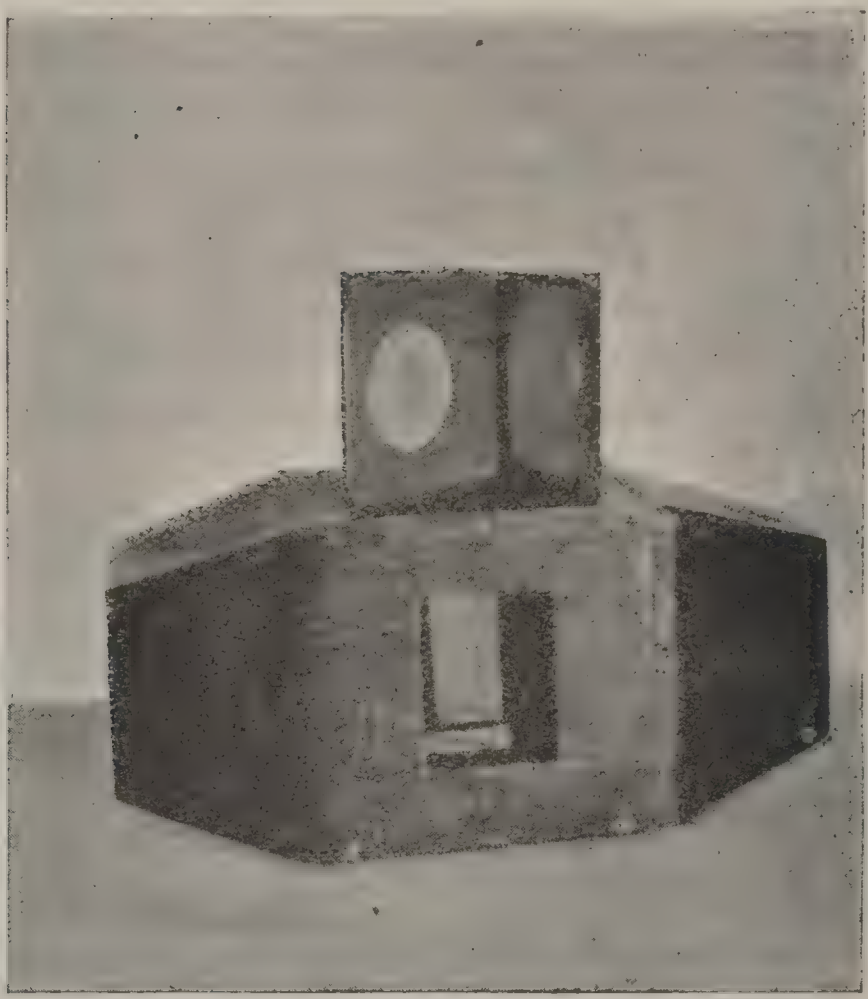


Fig. 264.

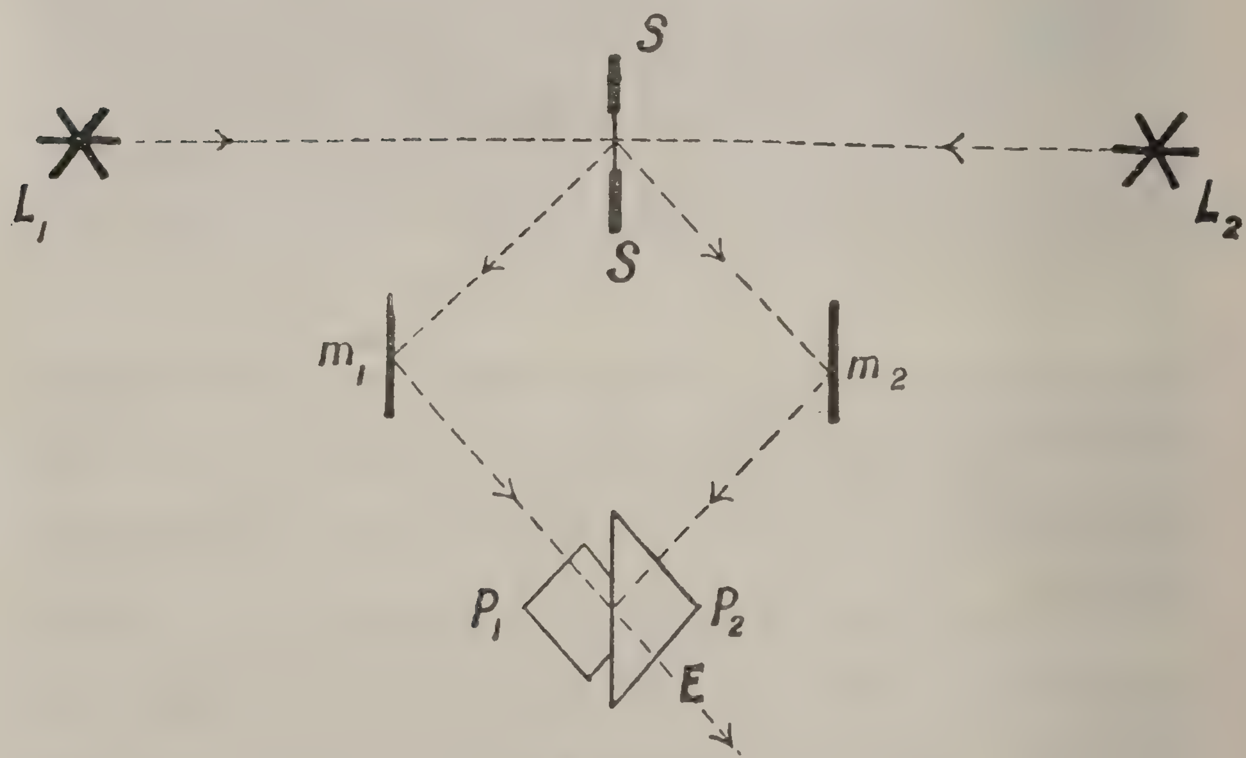


Fig. 265.

reflected by two similar plane mirrors m_1 and m_2 to two prisms P_1 and P_2, of which P_2 is the ordinary form, while P_1 has a curved surface one side which touches P_2.

The reflected rays from m_1 are able to pass through the combination to the eye placed at E, while those from m_2 are deviated to E also. Thus the screen SS when equally illuminated both sides cannot be seen at E.

"Methven Screen" Photometric Standard of Light.

When used with just an ordinary amount of care this standard of light is one of the most convenient and accurate, requiring no elaborate preparation, as in the case of some other standards, before being used. A standard two-slot "Methven screen," together with a carburetter, is shown in Fig. 266. The former consists of a vertical brass plate or screen, bent round at right angles at the bottom, and to the under side of which is fixed a tubular metal foot which fits into the hollow standard supporting the whole screen.

To the upper side of this angle-piece is fixed a London Argand burner provided with a stettite cone and a cylindrical glass funnel. Two pairs of brass bars are screwed into the screen at respectively $2\frac{1}{2}''$ and $3''$ above the top of the burner. The screen has a hole at its centre across which slides a silver plate containing two rectangular windows, the size of which are determined by the height of the flame and the C.P. of the light emitted from them. This in the Methven screen, shown in Fig. 266, is 2 C.P. either when the mean height of the peaks of the flame are on a level with the two top bars and the long narrow slot in use with ordinary coal-gas, or with the flame on a level with the two lower bars, the short broad slot and the coal-gas carburetted. The carburettor is shown on the left of Fig. 266, and is merely a metal reservoir containing pentane liquid, which is highly volatile, the vapour mixing freely with the coal-gas and enriching it as this latter is made to pass through the receptacle by manipulating the three stop taps seen

on the branch tubes. For a more detailed description of the Methven screen, see Slingo and Brooker's *Electrical Engineering*.

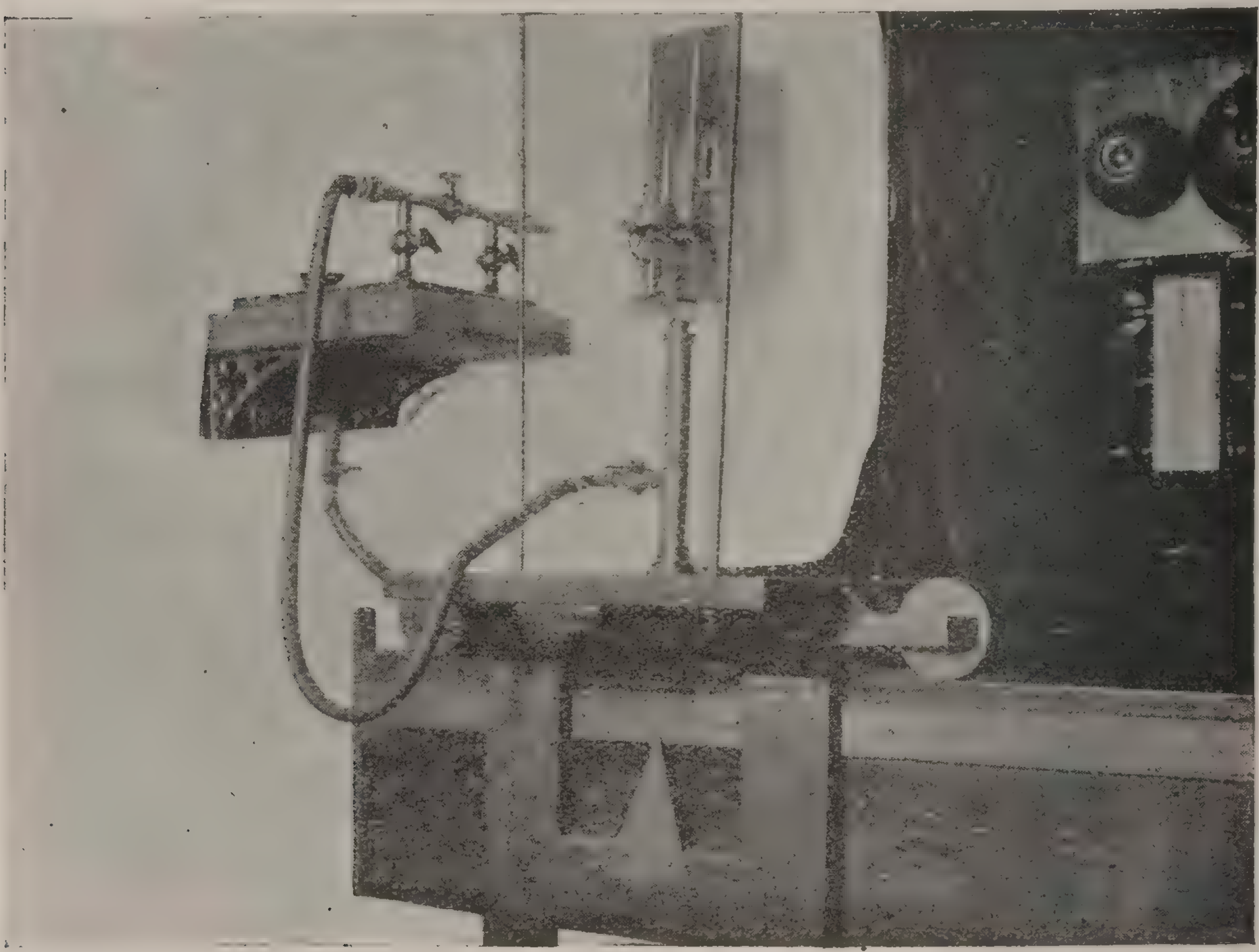


FIG. 266.

It should be noticed that the position of the screen is rather misleading, arising from the fact that it is shown turned round out of its normal position to show more clearly the burner, etc.

Adjustable Carbon Rheostat.

An extremely useful form of continuously adjustable rheostat, suitable for large currents as well as small ones, is illustrated in Fig. 267, this particular one being capable of carrying about 25 amperes continuously or 30 to 40 amperes for short periods of some minutes' duration.

It consists of a row of square flat plates of hard gas-retort carbon, resting on a fairly broad ledge of slate or some other suitable insulating material screwed to an iron bar underneath

it which is fixed to the ends of the framework of the rheostat. These ends are also fixed to tie rods at the sides, to the top of each of which is screwed an over-lapping strip of vulcanized fibre to guide the plates, and at the same time not to short-circuit them. The row of carbon plates are terminated by thick cast-iron plates, extended at the top and carrying the terminals shown. The left-hand terminal plate is insulated from that end by a plate of vulcanized fibre, while the row of plates is com-

Fig. 267.

pressed against this by a screw working against the right-hand end. In case at any time the right-hand terminal plate is removed and inserted in some intermediate position in the row, an extra cast-iron plate the same size as the carbons is provided this end to guard against the friction of the carbons should the screw press on them by mistake. One valuable advantage of this rheostat is that, being non-inductive, it can be used with alternating currents in cases where many other forms of rheostats could not be. The resistance varies from a minimum when the plates are tightly compressed, to a maximum when they are loose.

Incandescent Lamp-Box Resistance.

In many tests on alternating current appliances, such as transformers, alternators, etc., difficulties arise in obtaining a non-inductive resistance in which to take up the output of the appliance, for, as is well known, the product of the volts and amperes only represents the *actual* or *true power* in Watts absorbed, *providing* the load-absorbing *rheostats* are truly *non-inductive.*

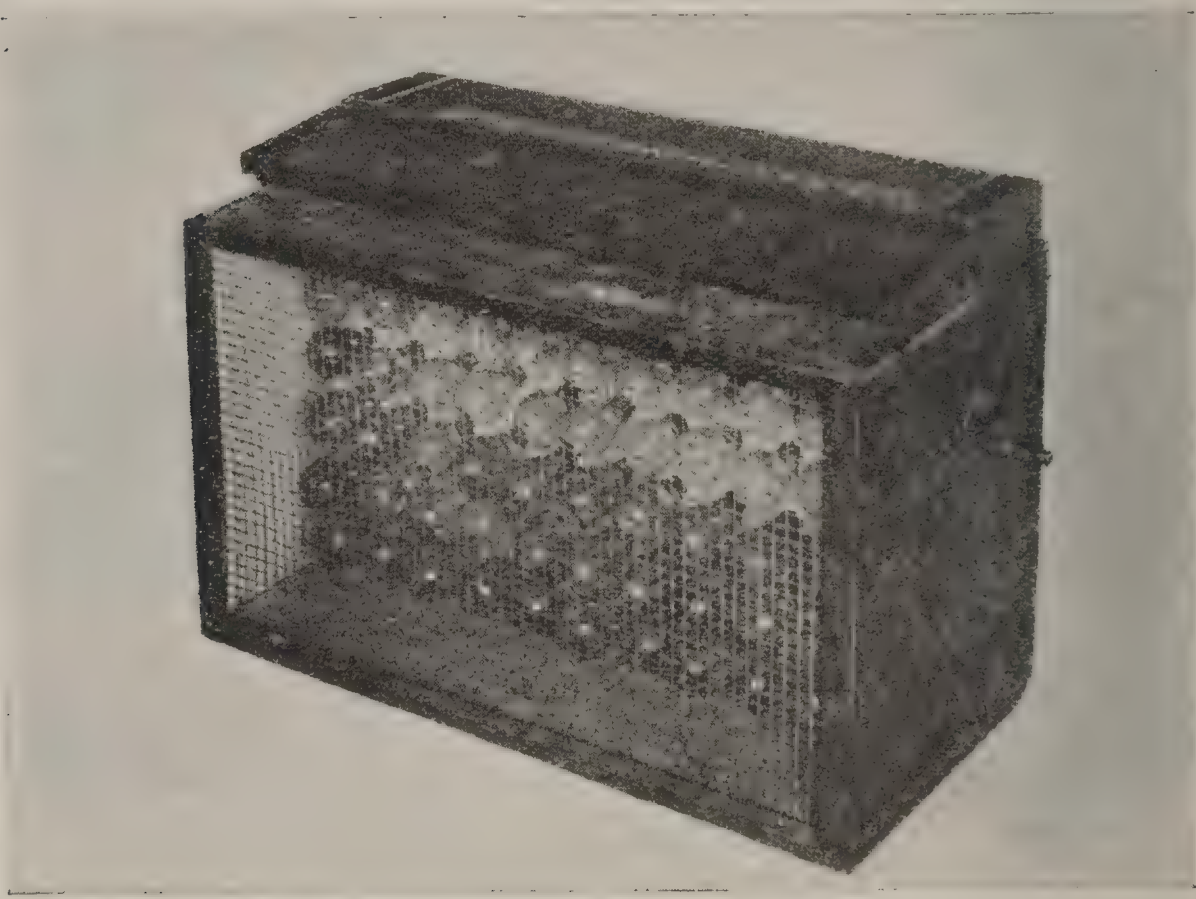


Fig. 268.

This can be obtained with specially wound rheostats made of ordinary iron wire or other special alloy. A water rheostat is, however, simpler, though not perhaps so convenient to manipulate, while Fig. 268 illustrates a still more convenient form of non-inductive (to all practical purposes) rheostat, which the author has designed for his own purposes, and one that is fairly portable with care. The particular one shown consists of a containing case and box, with an internal partition which supports some 60

or 70 glow lamps, composed of 8, 16 and 32 C.P. lamps, capable of absorbing some 7 or 8 E.H.P. All the woodwork the lamp side of the partition is covered with a double layer of asbestos cloth, and the front of the box is protected only by a grating to allow of free circulation of cold and hot air. Each lamp is connected to its own pair of mercury cups in a special switch-board, seen on the top of the box towards the back, by means of which the lamps can at once be connected *all in parallel, all in series,* or

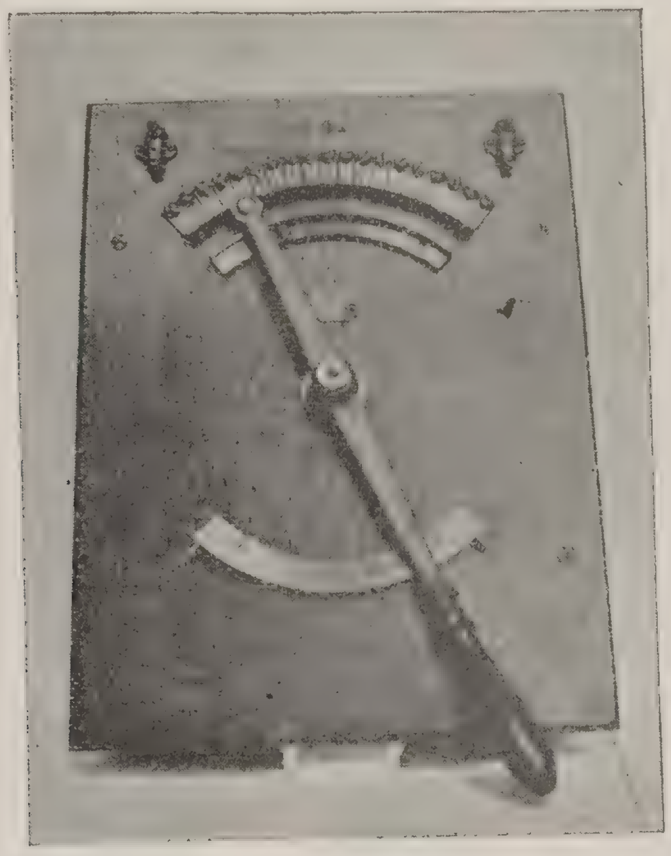


FIG. 269.

in any other intermediate combination, to suit requirements. The side opposite the grating hinges down, giving free access to the lamp-holders and connections to the mercury switch-board. This lamp-box resistance requires care in carrying about, as the mercury tends to come out of the two long slots if the box is much tilted. It is, however, extremely convenient, and when the lamps are used in parallel in alternating current work, their effective self-induction is extremely small, so that when absorbing the load from the secondary side of a transformer (say), the amps. × volts will be the true power absorbed, *i. e.* developed by the transformer.

Adjustable Rheostat.

Fig. 269 shows a convenient wire-wound rheostat, capable of step-by-step variations between 0 and the maximum resistance, which is about 40 ohms, and of carrying continuously 6 or 7 amperes. It is a form of rheostat eminently suitable for regulating the shunt circuit of either a dynamo or motor of that type. As will be seen with reference to the figure, it consists of a box or case containing the coils (not seen), and which is fitted with a top carrying the multiple way lever switch of the form shown. The circular row of studs seen in the upper part of Fig. 269 are connected to the coils inside in such a way that the right-hand end block gives the full resistance of all the coils in series between the terminals at the top, when the lever is on this block. The rheostat, being intended to be used in a vertical position, has a wire grating at the top and bottom in order to obtain a cooling circulation of cold air through the interior. The coils inside are strung between insulators, so that even if they do get very hot, the inside of the box will not be much harmed.

Continuously Variable Rheostat.

This rheostat is of the same type and make as that illustrated and described relative to Fig. 271, consequently a further description is unnecessary. By winding the rheostat with, say, the same gauge in a higher specific resistance material than platinoid, such as eureka, manganin, or reostene, it is easy to obtain a resistance which can be varied perfectly continuously from something like 30 or 40 ohms down to 0, and that will carry a maximum current of about 4 or 5 amperes. This in a test-room or laboratory is extremely useful, enabling very fine adjustments of resistance, and consequently of current or pressure, to be obtained when desired.

The reader is referred to the description of the same make of rheostat shown in Fig. 271.

Improved Rheostat.

The object of the rheostat, invented over forty years ago by Wheatstone, is to provide an electric resistance which can be varied continuously.

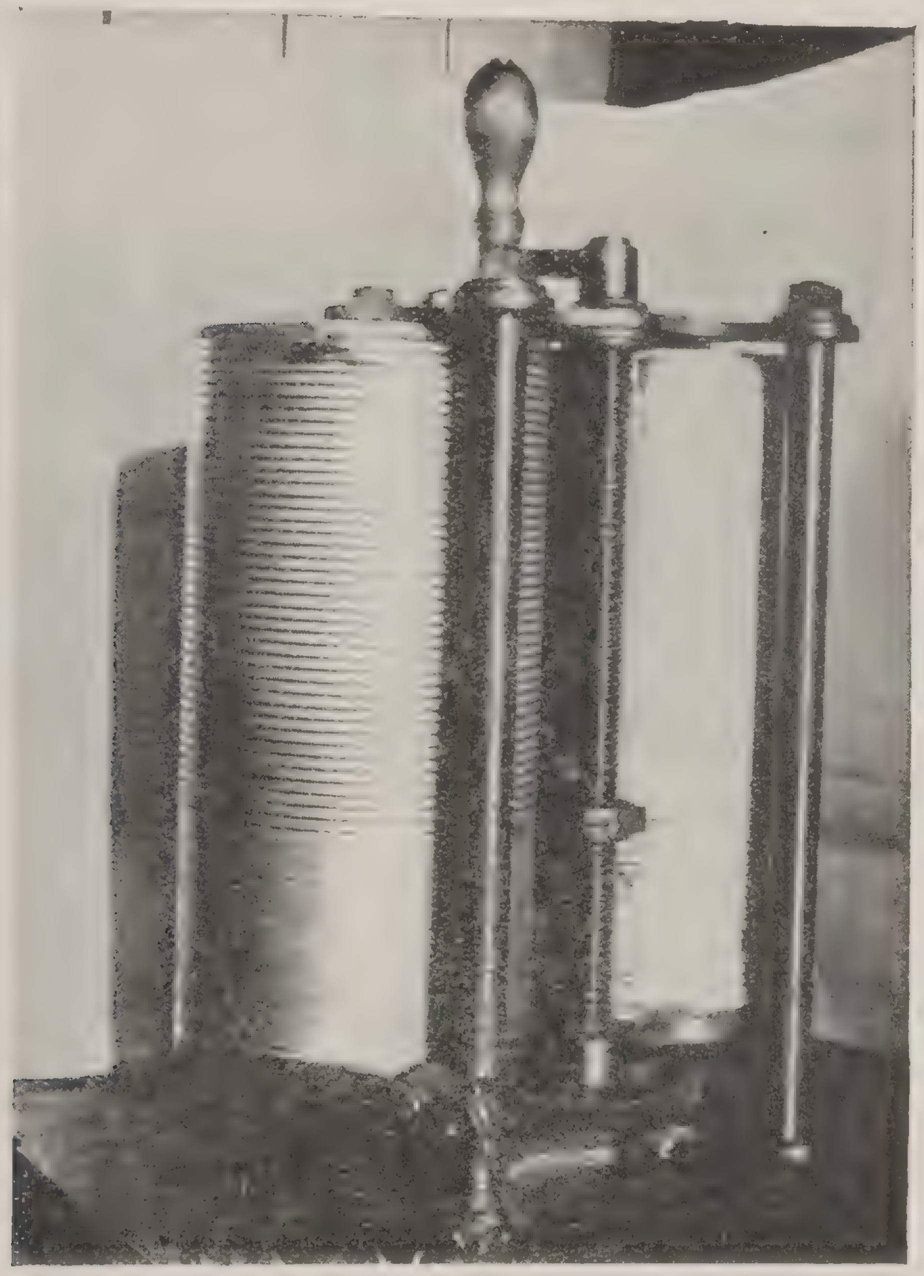


FIG. 270.

The instrument shown in Fig. 271 is an improved form (due to Lord Kelvin) of Wheatstone's rheostat, in which the wire is guided from one cylinder to the other by a fork carried along through the requisite range by a nut travelling on a long screw-

shaft. This screw-shaft carries a toothed wheel which turns the two cylinders by means of toothed wheels attached to their shafts. A watch-spring, as in Jolin's improvement of Wheatstone's rheostat, keeps the wire always tightened to the proper degree. A leather buffer at each end of the range of the nut acts as a guard against overwinding in either direction.

In a high resistance rheostat the conducting cylinder and the wire are both of platinoid, a metallic alloy having properties which make it specially suitable for the purpose. It has very high electric resistance, very small temperature variation of resistance,

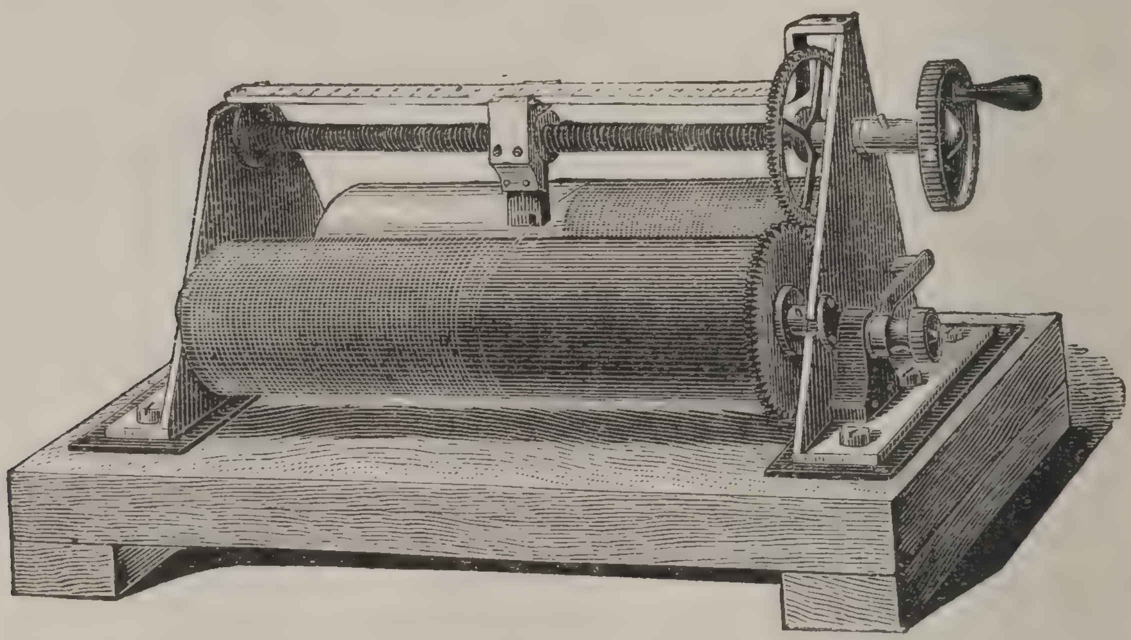


FIG. 271.

and its surface remains almost or altogether untarnished in the air. On account of the last-named property the contact between the wire and the conducting cylinder, and continuity in action, which was a great difficulty in the old form of apparatus, is very complete.

In a low resistance rheostat the conducting cylinder in this instrument is made of brass, nickel-plated so as to avoid tarnishing, and the wire used is copper, also nickel-plated. The rheostat can be supplied to carry currents as high as 30 amperes. The relation between material resistance and current for these rheostats is as follows, viz.—

TABLE XIV.

Wire.	Approximate Resistance.	Maximum Current.
Platinoid.	600 ohms.	0·25 amperes.
"	100 "	2·0 "
"	20 "	10·0 "
Copper.	0·4 "	30·0 "

Continuously Variable Rheostat.

Fig. 272 illustrates another slightly different form of Kelvin's improvement on the original Wheatstone rheostat. The actual

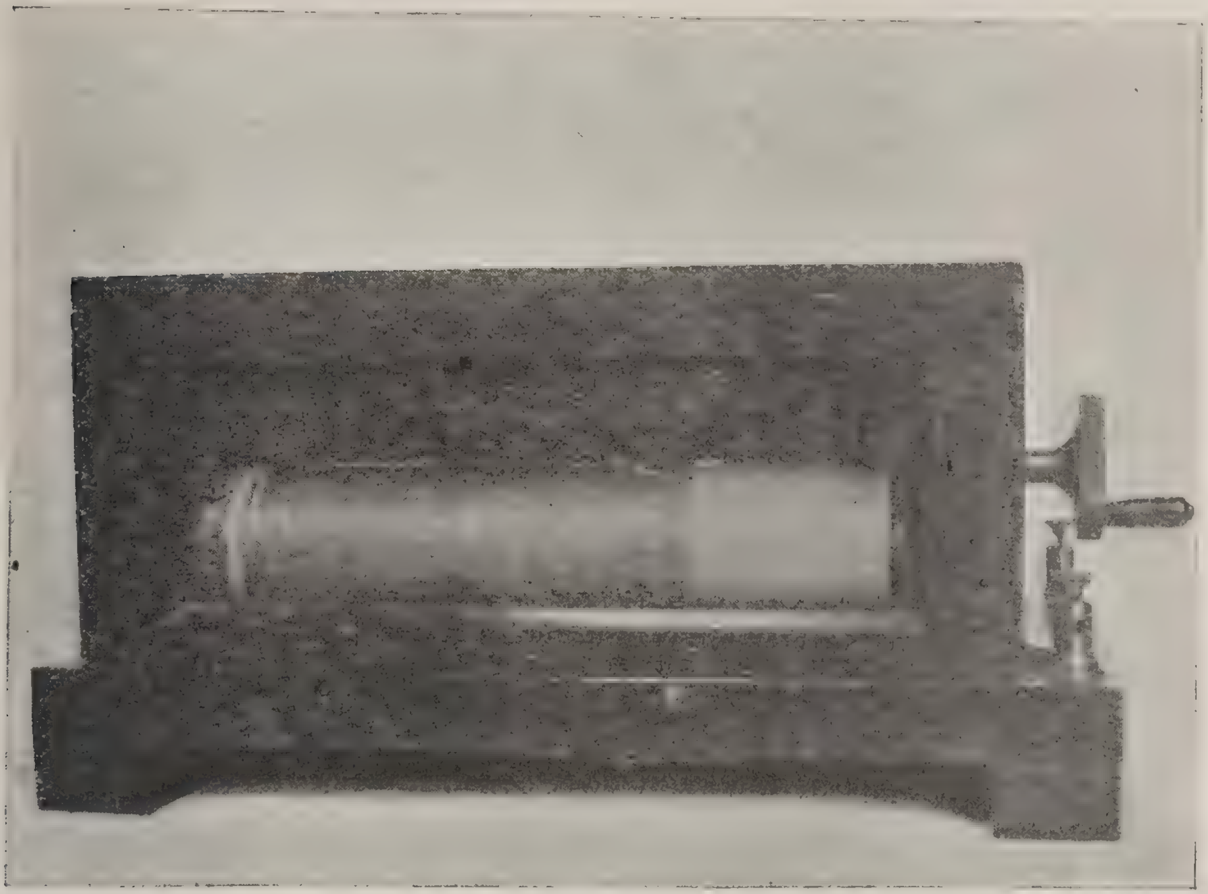


FIG. 272.

construction is the same as that described in the two preceding, Figs. 270 and 271, except that the one here shown is intended for finer and lighter work, as seen from the smallness of the parts and gauge of wire employed. The second cylinder is just behind the one shown in the figure. Fairly fine wire is used, and

the rheostat so wound may have a resistance of about 100 ohms as a maximum, capable of carrying one or two amperes. Like the "Wirt" form of resistance, described in *Practical Electrical Testing* by the author, it forms a most useful type of resistance for small work, such as calibrating low resistance voltmeters, enabling a series of different readings to be taken without actually altering the number of cells in the E.M.F. used.

The reader should refer to the description of the principle of this form of rheostat which is given relative to Fig. 271.

Fixed Standard Low Resistances.

There are many different forms of these depending to a great extent on the value of the resistance, and also on the particular make. Fig. 273 shows a set of five different forms of standard low resistances made by Messrs. Crompton and Co., and primarily intended for use with the potentiometers made by them also. The resistances can, of course, be used for any other purpose than this which requires a standard resistance of accurately known value, capable of carrying large currents without sensible heating or alteration.

The one shown standing on end at the top is of a slightly different form to the rest, being of the tubular water-cooled type.

These resistances consist of a sheet or strip of metal, or a coil of wire, each provided with four terminals, two for connection to the circuit and two for connection to the potential leads.

The smaller resistances take the form of a coil or spiral fixed in a mahogany frame, and also of flat strips either bent or straight, the largest size—300 amperes and over are of sheet or water-cooled type.

They are constructed of manganin, an alloy which has been thoroughly tested, and which has the great advantage that within ordinary limits of accuracy (say one part in 1000) no temperature correction whatever is necessary: but for measurements with the potentiometer requiring an accuracy exceeding this, a curve, giving the temperature value for the whole range of

current that the instrument is capable of carrying, is supplied with each resistance.

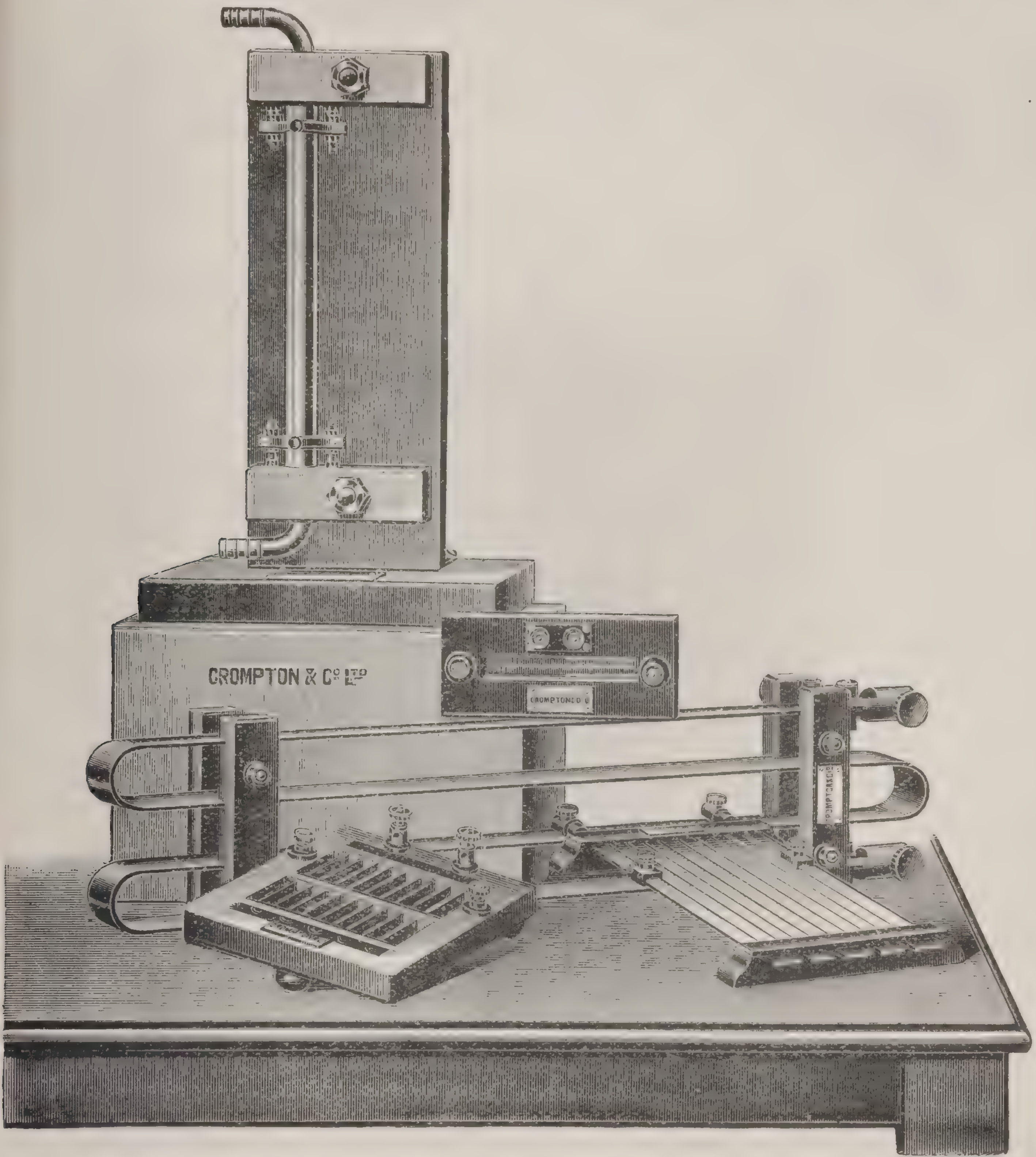


FIG. 273.

Such a curve, together with other forms of adjustable and fixed standards of low resistance, are given in the author's work entitled *Practical Electrical Testing.*

Stand Coil Rheostat.

A most convenient form of current rheostat of a portable nature, at all events one that can be moved comfortably about any testing-room, is illustrated in Fig. 274. It consists of an

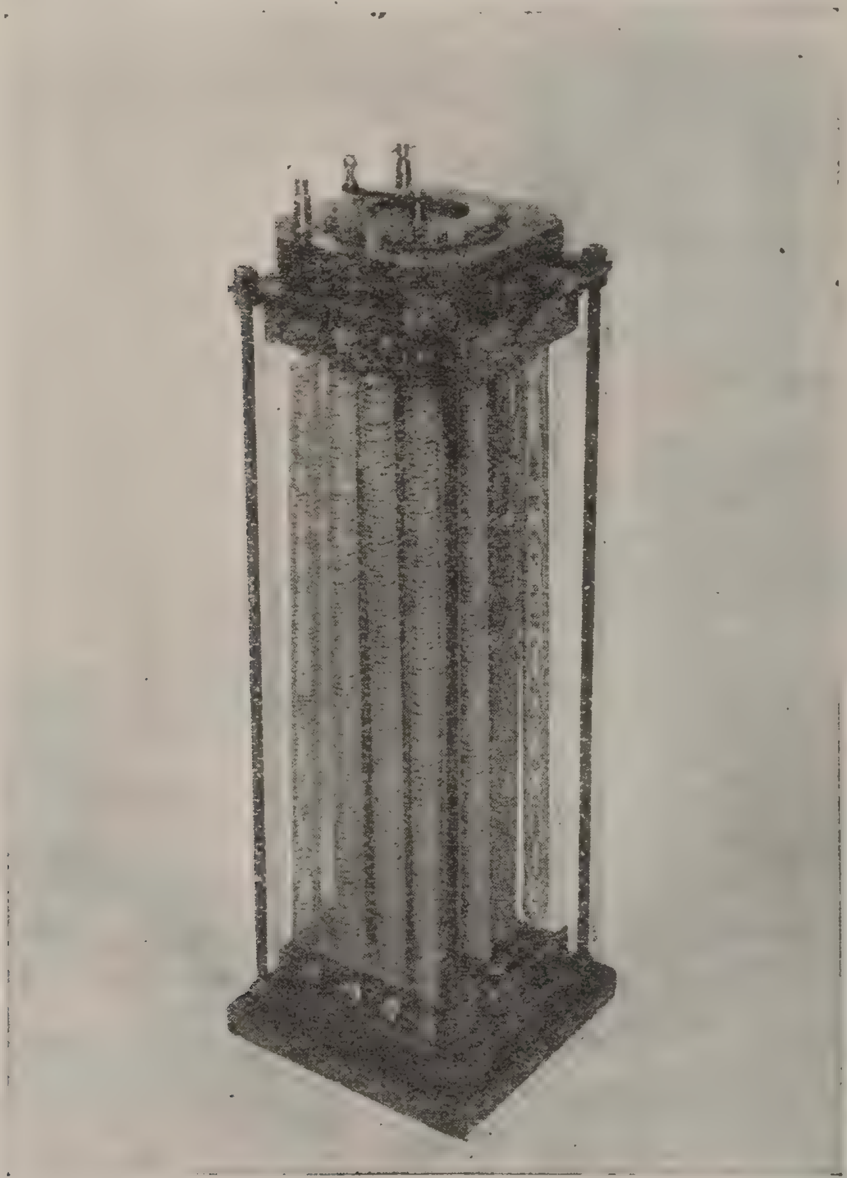


Fig. 274.

iron frame or stand, of as light a construction as possible, in order to be light, between the top and bottom of which are stretched bare wire spirals of either iron or some high-resistance alloy. These coils are spaced sufficiently far apart to prevent them easily touching should the rheostat receive a slight knock, and are all connected in series, their junctions being connected

to a multiple way switch seen on the top. This latter consists of several studs or blocks arranged in a circular form and having their upper surfaces turned up in the lathe so as to be quite level. A suitable spring lever pivoted in the centre of the ring of blocks is capable of turning almost once round between two stops and of making contact with each block as it passes over it.

One of the two terminals of this stand coil rheostat (seen on the top) is connected to the lever centre, and the other to one end of the series of coils. Thus the resistance between the terminals can be varied from nothing (*i. e.* short circuit) to the maximum by as many steps as there are contact blocks on the switch. A great mistake, which is usually made by rheostat makers, is to use the same gauge of wire throughout the rheostat, for clearly the gauge should increase as we begin to cut out, since the current is thereby increased also. The above rheostat, made to the author's designs, contains about four or five gauges of wire.

Three-Phase Liquid Rheostat.

For three-phase alternating current work there are two distinctive forms of rheostats needed, differing merely in the terminal arrangements. One form may be termed a "*through*" rheostat, which would be required for regulating the current supplied by a generator to, say, a motor; the other form may be termed a "*closed*" rheostat, which would be required for absorbing the load from a generator.

In all forms the rheostat must operate equally on each of the leads of a three-phase system, otherwise the balance and symmetry of the currents will be thrown out and will cause considerable trouble. Three-phase rheostats may be either metallic or liquid in nature, but in any case the moving contacts must move simultaneously by equal amounts when manipulating the rheostat as a whole. Fig. 275 shows a three-phase liquid rheostat designed by the author, and which will negotiate currents up to about 30—40 amps. It can be employed either as a "through" or "closed" rheostat, and consists of three exactly similar flat semicircular-shaped iron troughs or boxes placed side by side in

alignment on a wooden base board, but not in contact with one another. Each is secured to the base board through its flat metal foot, carrying a terminal which therefore makes electrical connection with the box as a whole.

These terminals are clearly seen in Fig. 275. Each box has a semicircular-shaped iron plate, carried by a brass spindle, which passes through bearings of insulating material let into the opposite sides of each box. The ends of each spindle terminate just outside the box in enlarged metal bosses, the successive adjoining pairs being direct coupled mechanically through couplings of insulating material, but are discontinuous electrically.

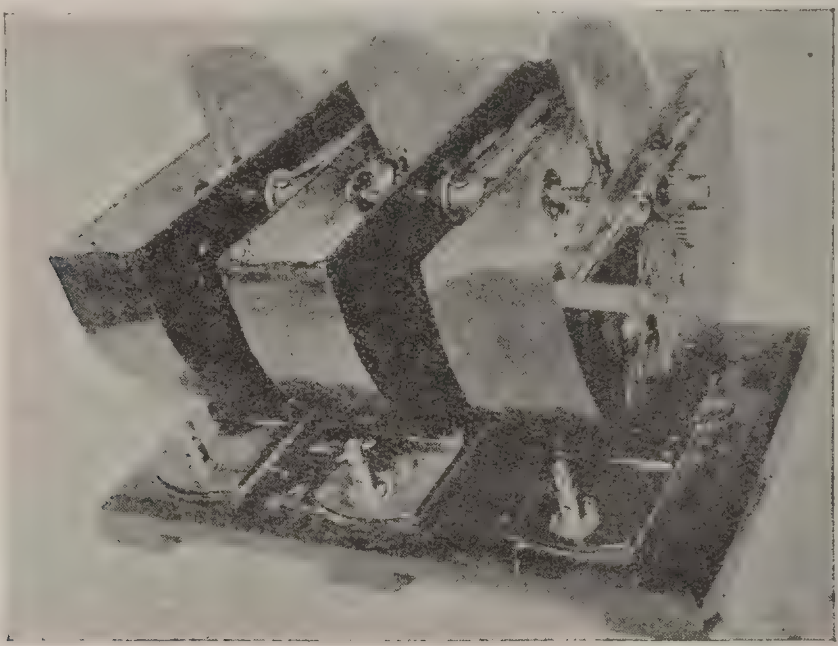


FIG. 275.

This compound shaft or spindle is rotated with the plates by means of a worm and worm-wheel gearing. Seen to the right-hand end of Fig. 275.

Three other terminals (not seen in the figure) at the back of the rheostat make electrical connection to each section of the spindle, and therefore to each plate through spring strips rubbing on the proper bosses as the spindle is turned.

A solution of washing soda and water of the *same* density is used in each trough, and must not, of course, reach up so high as to make contact with the spindles. Thus when none of the terminals are cross-connected, the rheostat becomes a "*through* type," but when the three at one side are all joined together, we

have a closed rheostat, suitable for absorbing the load from a generator connected to the remaining three terminals.

It should be remembered that as each of the three sections of this water rheostat must operate equally on turning the shaft, the level of liquid must be the same in each trough in addition to it being of the same density.

Magnetic Curve Tracer.

This instrument, devised by Prof. Ewing, shows the magnetic quality of iron, steel, or other magnetic metal, by exhibiting the curve which connects the magnetization B with the magnetizing

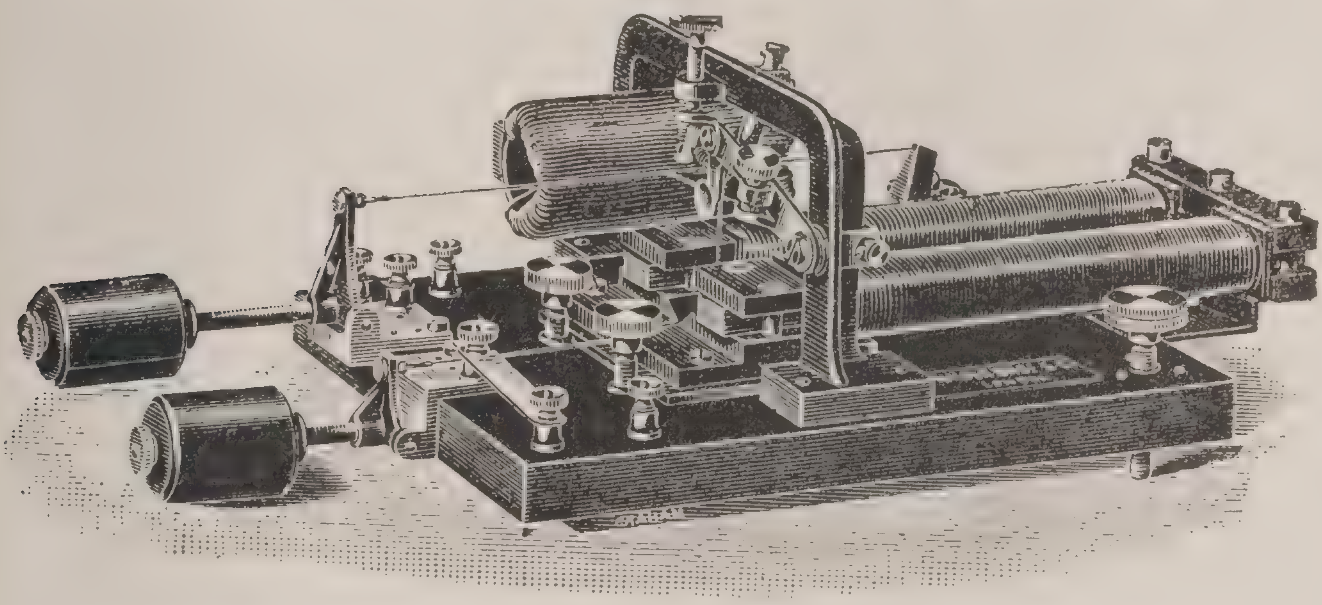


FIG. 276.

force H in any magnetizing process. The curve is exhibited upon a screen by the spot of light reflected from a mirror, which receives two components of motion. The vertical component is proportional to the magnetization, and the horizontal component to the magnetizing force. The instrument is shown in Fig. 276, and Fig. 277 is a diagram showing the functions of the various parts. The mirror is pivoted upon a single needle-point, which leaves it free to turn both ways, and it is connected by threads to the middle of two stretched wires, AA and BB, in such a manner that when either of the wires sags the mirror suffers a corresponding deflection. The threads

are kept taut by light springs, the tension of which is adjustable. The wires are stretched in narrow slots, forming gaps in two magnetic circuits, *DD* and *C*. One of these circuits, *DD*, is made up of the iron or steel to be examined, along with suitable pole-pieces and yoke, and the current which passes through the magnetizing coils of this circuit passes also through the stretched wire, *BB*, in the gap of the other magnet. The other magnet is constantly magnetized by a steady current, and a steady current also flows through the stretched wire *AA*. Hence,

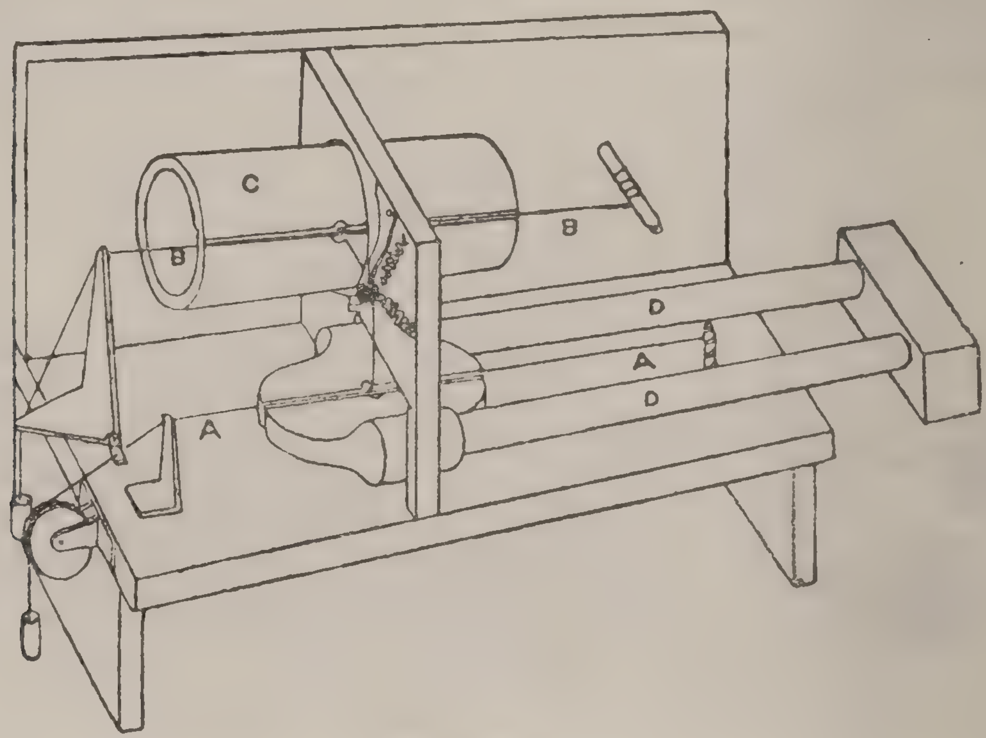


FIG. 277.

when the magnetizing current of *DD* is altered, the wire *BB* sags out or in, and gives horizontal motion to the mirror proportional to the magnetizing force acting on *DD*. And when the magnetism of *DD* is altered, the wire *AA* sags up or down, giving vertical motions to the mirror proportional to the changes of magnetism.

The samples to be tested form the arms of *DD*. They may be solid rods, or rods built up of thin strips, or of wire. The rods supplied with the instrument are of soft sheet-iron, built up of insulated strips, with a net cross-section about 1 in. by ½ in., and about 18 in. long. In preparing other rods for comparison of magnetic quality, the same dimensions are to

be chosen as those of the standard rods. Clamps are provided at the pole-pieces to allow the rods to be readily inserted and removed.

The same constant current will serve for the wire *AA*, and the magnetizing coil of the tubular magnet *C*. A current of about four amperes will serve well, but more or less may be used, according to the amount of movement which it is desired to give to the mirror. The amplitude of the movements can also be regulated by shifting in or out the weights on the bell-crank levers which keep the stretched wires tight. For high-speed work the wires should be kept very tight, and a small mirror should be used. The magnetizing current must be made to vary in a continuous manner; not by sudden makes and breaks. When these precautions are taken magnetic cycles may be performed so rapidly that the reflected light appears on the screen as a continuous curve. A special commutator is supplied, to allow of rapid but gradual variations and reversal of the magnetizing current. It consists, essentially, of two fixed and two revolving plates of zinc, immersed in a solution of zinc-sulphate.

In ordinary testing it is more convenient to make the magnetic changes occur slowly, and to mark with a pencil the successive positions of the spot on the screen. A sheet of paper on a small drawing board, set up against a wall or other vertical support, is a suitable screen. The source of light may be an ordinary galvanometer lamp, furnished with a pair of cross wires instead of the usual single wire. For high-speed work a small spot of light is necessary, which is obtained by placing a screen with a small hole in it just in front of the lamp. Horizontal and vertical datum lines are marked by moving (by hand) the wires *BB* and *AA* respectively, and marking the path of the spot. A variable resistance is to be inserted in the magnetizing circuit of *DD*, to allow successive points of the magnetizing curve to be obtained: two zinc plates suspended near together in a weak solution of zinc-sulphate, in such a way that they can be more or less deeply immersed, will serve well for this purpose. A rapid commutator is also to be put in this circuit, to allow the specimens under test to be demagnetized by rapid reversals of continuously diminishing magnetizing force, if it is wished to determine the curve of *initial* magnetization. In comparing other samples

with the standard bars, care must be taken to preserve the same scale of B and of H, by not changing the constant current in the wire AA and magnet C, nor the tension of the stretched wires.

The absolute scale of H may be calculated, if required, from a knowledge of the number of turns in the winding of the magnet limbs, and that of B may be found by means of an

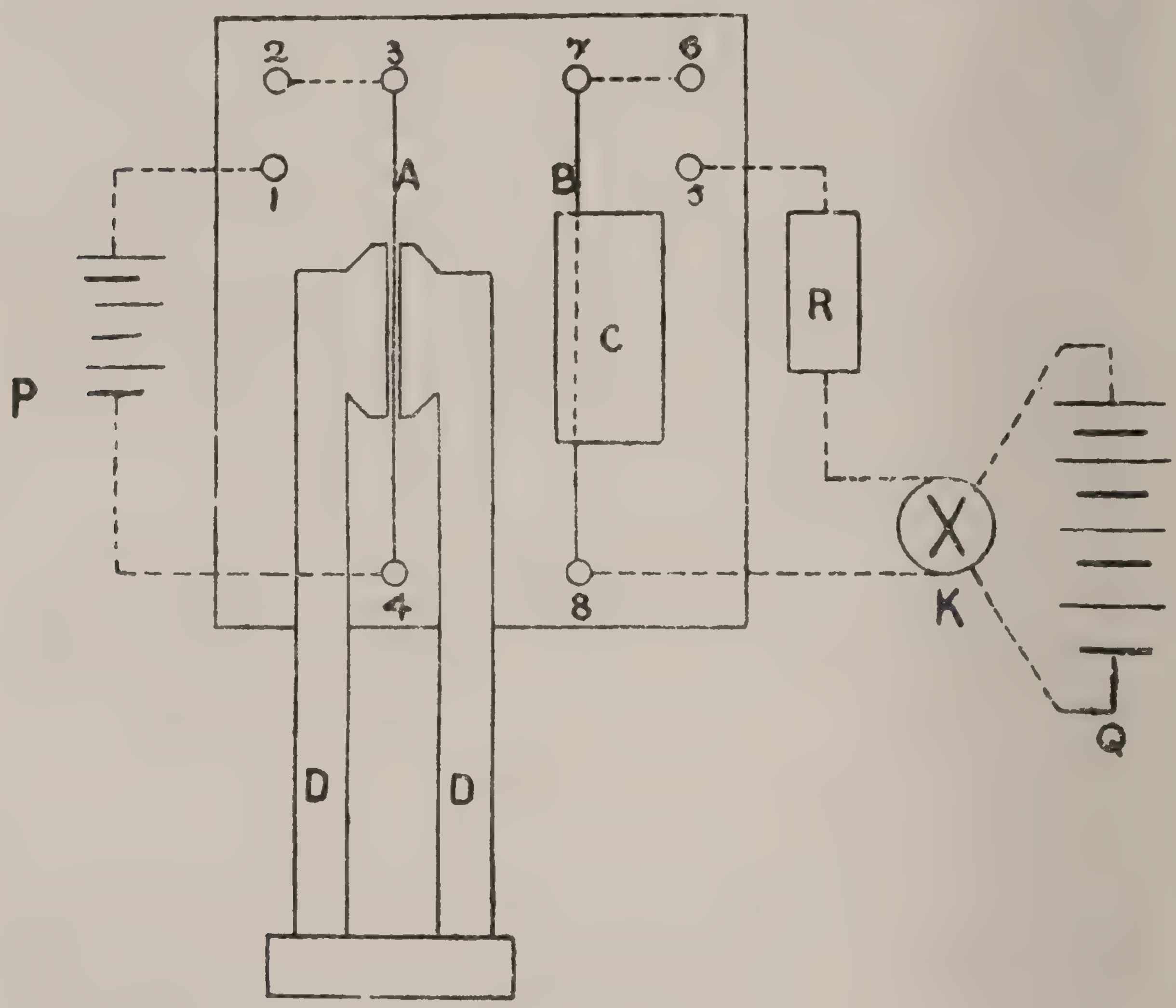


FIG. 278.

auxiliary ballistic galvanometer, by winding an induction coil of a few turns round one or both of the limbs.

Fig. 278 shows the electrical connections with the terminals as they are placed on the slate base of the instrument.

Terminals 1 and 2 are those of the coil which magnetize the tubular magnet C; terminals 5 and 6 are those of the main magnet coils on DD. The constant current is supplied by P,

and taking the course 1, 2, 3, 4, passes in series through the magnetizing coil of C and the stretched wire A. The variable current is supplied by Q, through the commutator K and adjustable resistance R. Taking the course 5, 6, 7, 8, it passes through the main magnetizing coil and the stretched wire B. The copper strip which is used to connect 2 with 3 may be put between 1 and 3 instead, 2 and 4 being then the battery terminals in that circuit. The effect of this is to change the general slope of the figure on the screen from right to left or *vice versâ*. One of the two arrangements is right when an ordinary screen is used; the other is right when the screen is a piece of tracing paper or ground glass, viewed from behind.

Care should be taken to adjust the instrument so that when a complete cycle of magnetic reversal is performed, the figure on the screen will be symmetrical, with the extremities equidistant from the zero point or origin, which corresponds to the condition of no current in B and no magnetism in DD. To secure this, see that the stretched wires are as nearly as may be judged in the middle of their respective slots, both vertically and horizontally. Set the mean position of the mirror perpendicular to the pivot needle, by adjusting the needle's position by the screws provided for that purpose. The light springs which keep the connecting threads taut are to be set so that they remain considerably stretched even when the mirror moves to its furthest limit in the direction tending to slacken them.

Fig. 279 is an example of the curves obtained by the instrument shown in Fig. 276. It is the copy of one half of a cyclic curve of reversal, along with the initial curve taken after demagnetizing the specimen by reversals, and also the curve obtained by reapplying the magnetizing current after it had been reduced from its maximum to zero. Owing to the existence of an air gap in the magnetic circuit under test, the diagram is sheared over to the right, and true values of the magnetic force would be found by measuring horizontal distances from some such line as YY, instead of from the vertical line. This shearing does not affect the *area* enclosed by the cyclic curves of reversal, and need not therefore be taken account of in measuring the comparative amounts of energy dissipated by

magnetic reversals in different specimens or in the same specimens with different values of the magnetism.

In addition to its use for determining these areas, for comparing the magnetic qualities of different samples of iron, and for investigating the properties of magnetic curves generally, the instrument may be used as a galvanometer by making the

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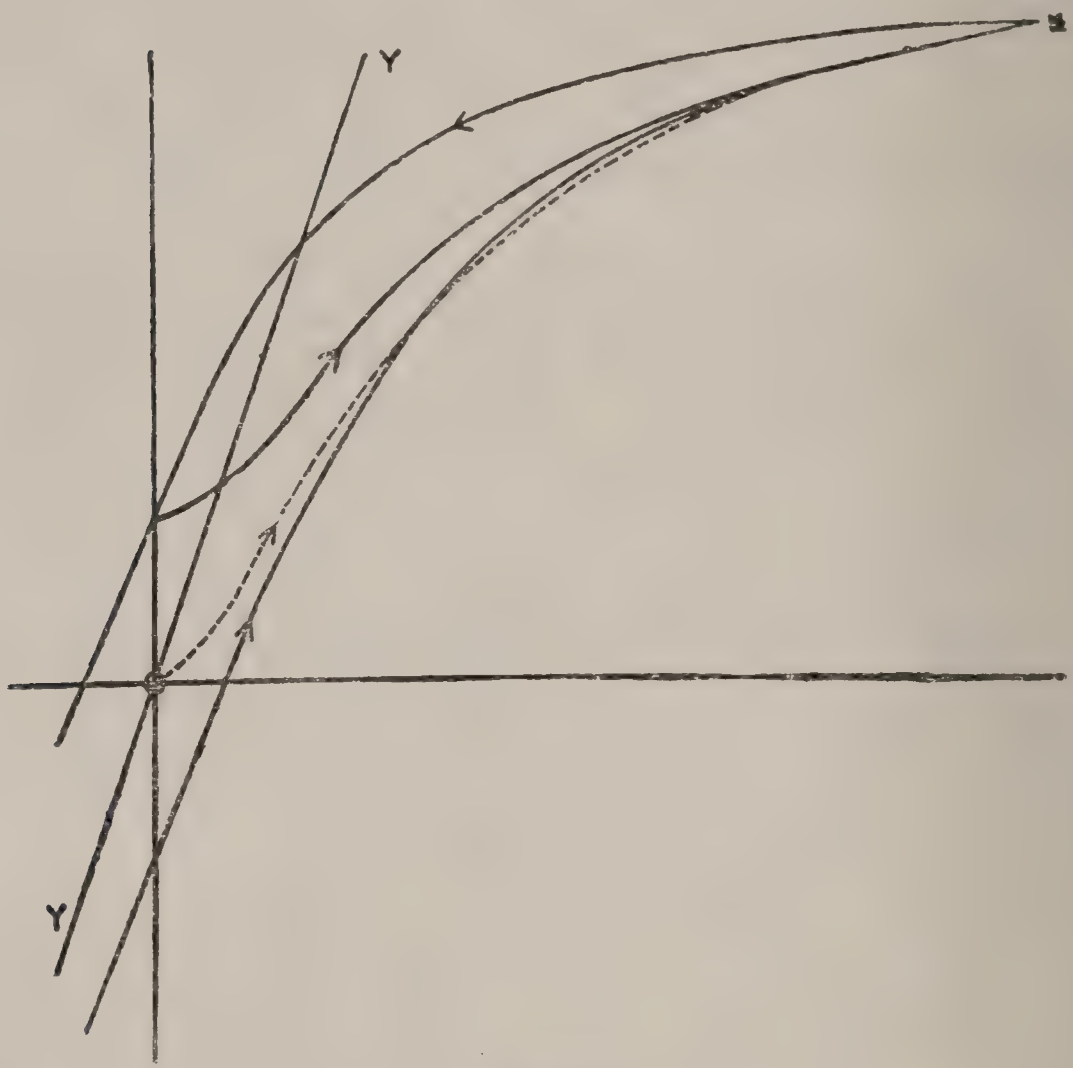

Fig. 279.

current to be measured pass through either of the stretched wires, while the magnet, in the slot of which the wire is stretched, is kept in a constant state of magnetization. This will be found useful in cases where an extremely dead-beat indication is wanted, and by making the spot of light register its position photographically on a moving plate or paper satisfactory records of rapidly fluctuating currents may be obtained.

The Permeameter.

A useful piece of apparatus, known as the "Permeameter," is illustrated in Fig. 280, by means of which the magnetic quality of different materials can easily and rapidly be found. The method is essentially a workshop one, and the principle of it is due to Professor S. P. Thompson.

FIG. 280.

The arrangement consists of a somewhat massive hollow rectangular-shaped block of good soft wrought-iron forged to the shape shown. Inside this is a magnetizing solenoid wound on a thin brass tube with thin flanges or ends, its length being just that between the insides of the block ends. The sufficiently long rod of magnetic material to be tested, having its lower end faced quite true, passes freely but closely through the top end of the block, down through the solenoid, and beds on the carefully "faced" inside of the bottom end of the block. The protruding end of the rod has a metal pin through it, which is caught by a double hook on the lower end of an ordinary spring balance, the top end of which is suspended by a gut cord passing over a fixed pulley and attached to the lever shown. This permeameter is also fitted with an arrangement for testing the specimen ballistically. It consists of a small flat coil fixed to a brass plate, which slides backwards and forwards between guides. The rod passes through this coil and beds as before on the block, at the same time keeping the coil back against the force of two spring strips on the outside of the

block. Immediately the rod is suddenly pulled up the coil flies out, and a circuit joined to its terminals will receive an electromagnetic impulse proportional to the field just broken. In this way the ballistic and traction methods can be made to check one another in the final results obtained.

Condensers.

Fig. 281 shows a general view of the Kelvin standard air Leyden condenser, and Figs. 282 and 283 a plan and sectional

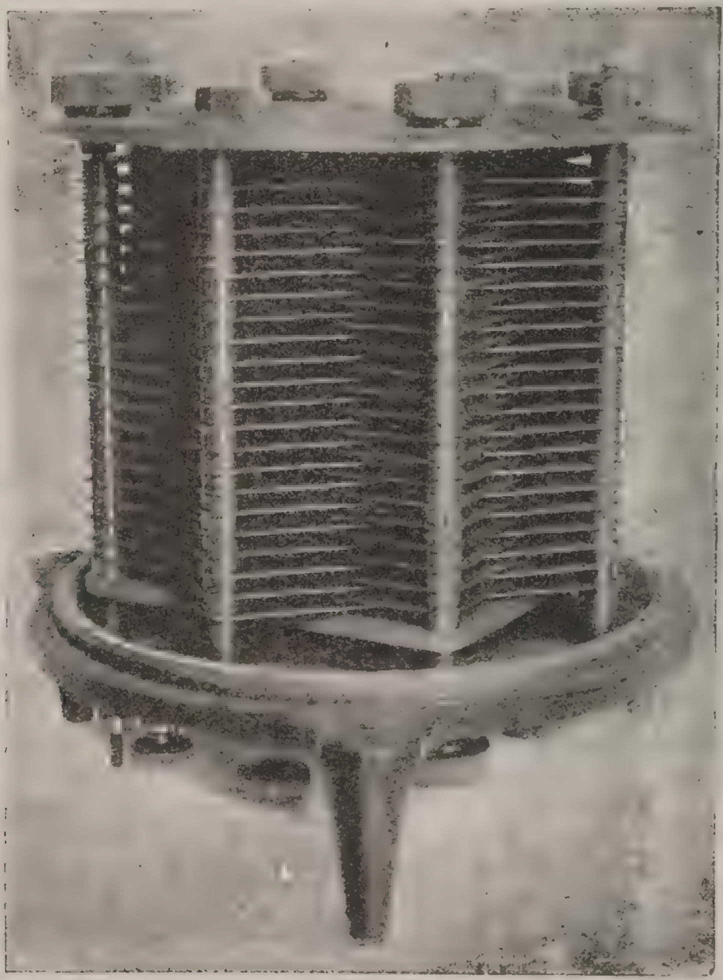

FIG. 281.

elevation of the same. The instrument is formed by two mutually insulated metallic pieces, which we shall call A and B, constituting the two systems of the air condenser or Leyden. The systems are composed of parallel plates, each set bound together by four long metal bolts. The two extreme plates of set A are circles of much thicker metal than the rest, which are all squares of thin

sheet brass. The set *B* are all squares, the bottom one of which is of much thicker metal than the others, and the plates of this system are one less in number than the plates of system *A*. The four bolts binding together the plates of each system pass through well-fitted holes in the corners of the squares; and the distance from plate to plate of the same set is regulated by annular distance pieces, which are carefully made to fit the bolt, and are made exactly the same in all respects. Each system is bound

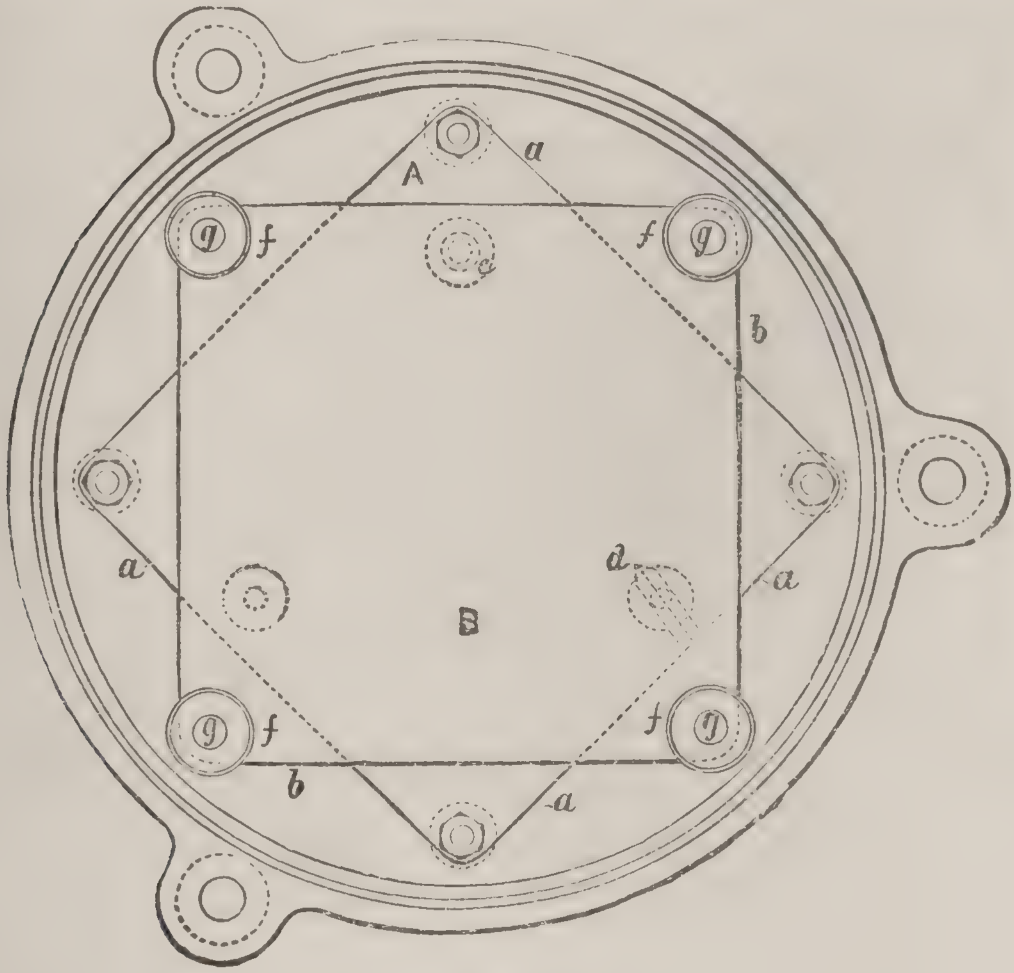

Fig. 282.

firmly together by screwing home nuts on the ends of the bolts, and thus the parallelism and rigidity of the entire set is secured.

The two systems are made up together, so that every plate of *B* is between two plates of *A*, and every plate of *A*, except the two end ones, which only present one face to those of the opposite set, is between two plates of *B*. When the instrument is set up for use, the system *B* rests by means of the well-known "hole,

slot, and plane arrangement,"[1] engraved on the under side of its bottom plate, on three glass columns, which are attached to three metal screws working through the sole-plate of system *A*. These screws can be raised or lowered at pleasure, and by means of a gauge the plates of system *B* can be adjusted to exactly midway between, and parallel to, the plates of system *A*. The complete Leyden stands upon three vulcanite feet attached to the lower side of the sole-plate of system *A*.

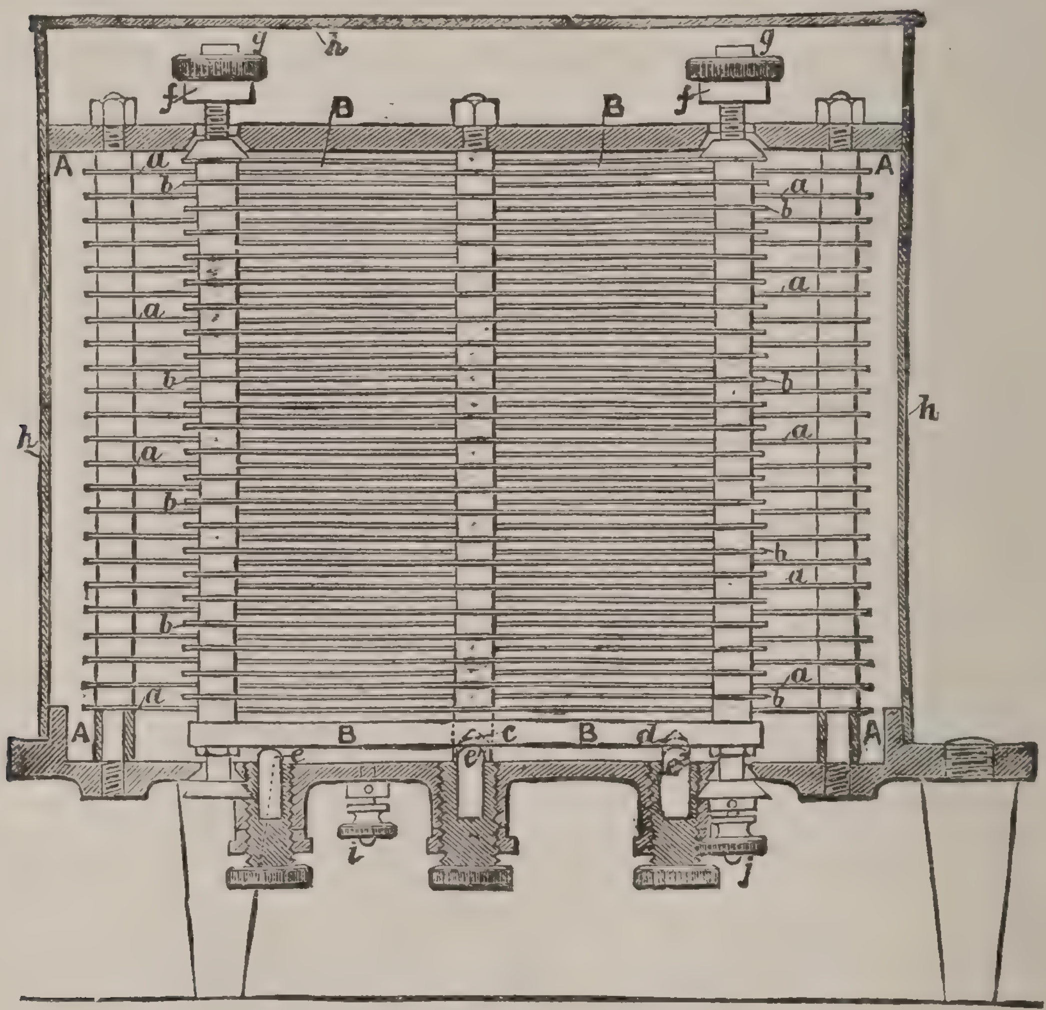

FIG. 283.

In order that the instrument may not be injured in carriage, an arrangement, described as follows, is provided, by which system *B* can be lifted from off the three glass columns and firmly clamped to the top and bottom plates of system *A*.

The bolts fixing the corners of the plates of system *B* are made long enough to pass through wide conical holes cut in the top and

[1] Thomson and Tait's *Natural Philosophy*, § 198, example 3.

bottom plates of system *A*, and the nuts at the top end of the bolts are also conical in form, while conical nuts are also fixed to their lower ends below the base-plate of system *A*. Thumbscrew nuts, *f*, are placed upon the upper ends of the bolts after they pass through the holes in the top plate of system *A*.

When the instrument is set up ready for use, these thumbscrews are turned up against fixed stops, *g*, so as to be well clear of the top plate of system *A* ; but when the instrument is packed for carriage, they are screwed down against the plate until the conical nuts mentioned above are drawn up into the conical holes in the top and bottom plates of system *A* ; system *B* is thus raised off the glass pillars, and the two systems are securely locked together so as to prevent damage to the instrument.

A dust-tight cylindrical metal case, *h*, which can be easily taken off for inspection, covers the two systems, and fits on to a flange on system *A*. The whole instrument rests on three vulcanite legs attached to the base-plate on system *A* ; and two terminals are provided, one, *i*, on the base of system *A*, and the other, *j*, on the end of one of the corner bolts of system *B*.

Revolving Contact Maker.

This is an arrangement of two contact levers or spring strips, and a revolving ring, whereby electrical contact is made between the two strips *once every revolution* of the ring at a certain particular and definite instant, and place on the circle of revolution depending on the position in which the levers touch the ring. Such an arrangement is necessary when it is desired to take the periodic E.M.F. and current curves of an alternator, and it may be fitted either to the alternator, or to a small single-phase synchronous alternating current motor, to be driven off the particular supply to be sampled. Fig. 284 represents a simple and convenient form of revolving contact maker, designed by the author and fitted to the rotating inductors of an inductor alternator. It consists of a brass frame ring screwed to the alternator portion, and carrying an ebonite ring screwed to it securely by set screws through the inner edge of the frame ring. Half the total width of this ebonite ring is turned nearly away and a brass ring slipped on and fixed to the ebonite ring by

screws passing through it sideways. A thin slip of brass (seen just in front of the spring brushes in Fig. 284) is neatly let into the ebonite ring and makes electrical contact with the insulated brass ring only. Two spring strips, insulated from one another, and pressing against the two springs in a line, are carried by a holder capable of being slid along the curve bar seen in the figure

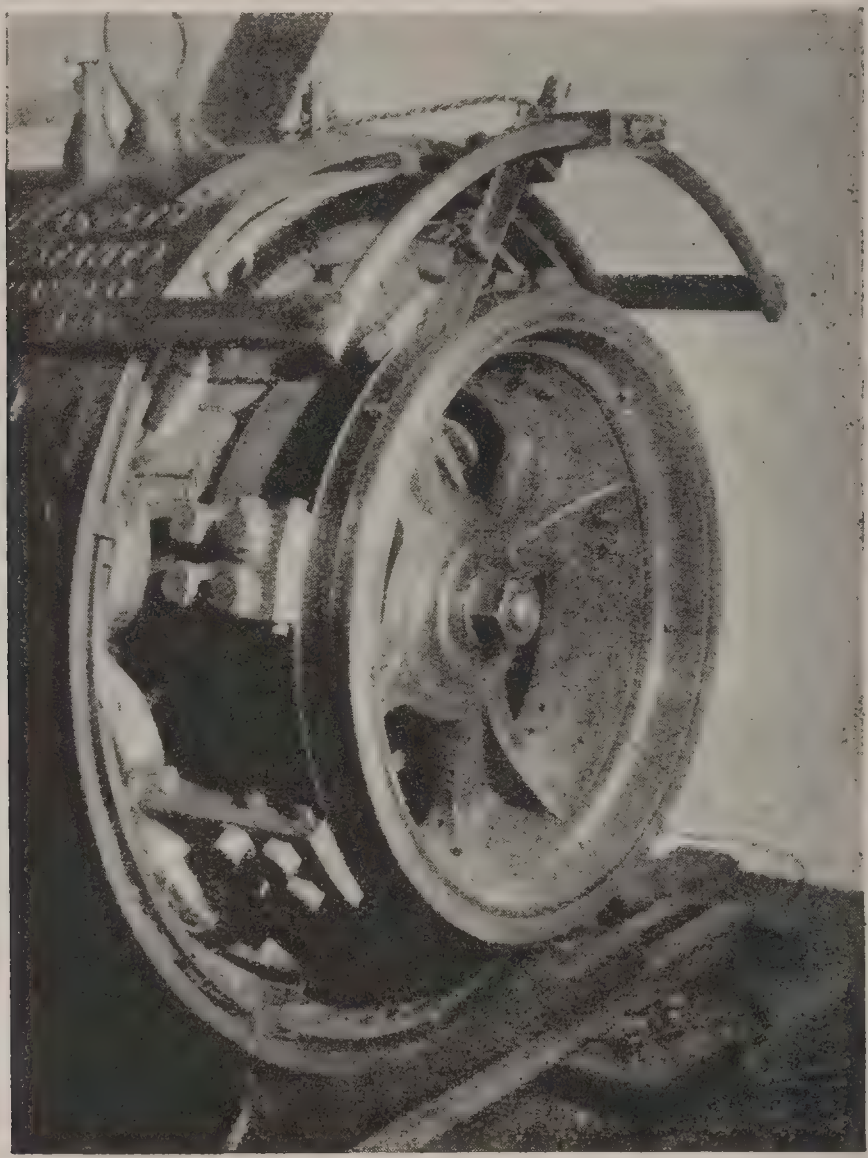

Fig. 284.

and provided with a pointer moving over a scale. Hence any circuit connected to these strips or brushes will be closed once every revolution at an instant in the *period* which depends on the position of them on the curved rod.

If the toes of the spring brushes are set, one in front of the other, the contact will be more instantaneous; otherwise it will last during the whole time required for the slip contact to pass under the brushes when set level.

Cradle Absorption Dynamometer.

In Fig. 285 is shown the principle of a typical form of *cradle* absorption dynamometer, suitable for testing the horse-power developed by small motors, but which can also be employed for larger powers up to a certain limit determined by the weight and

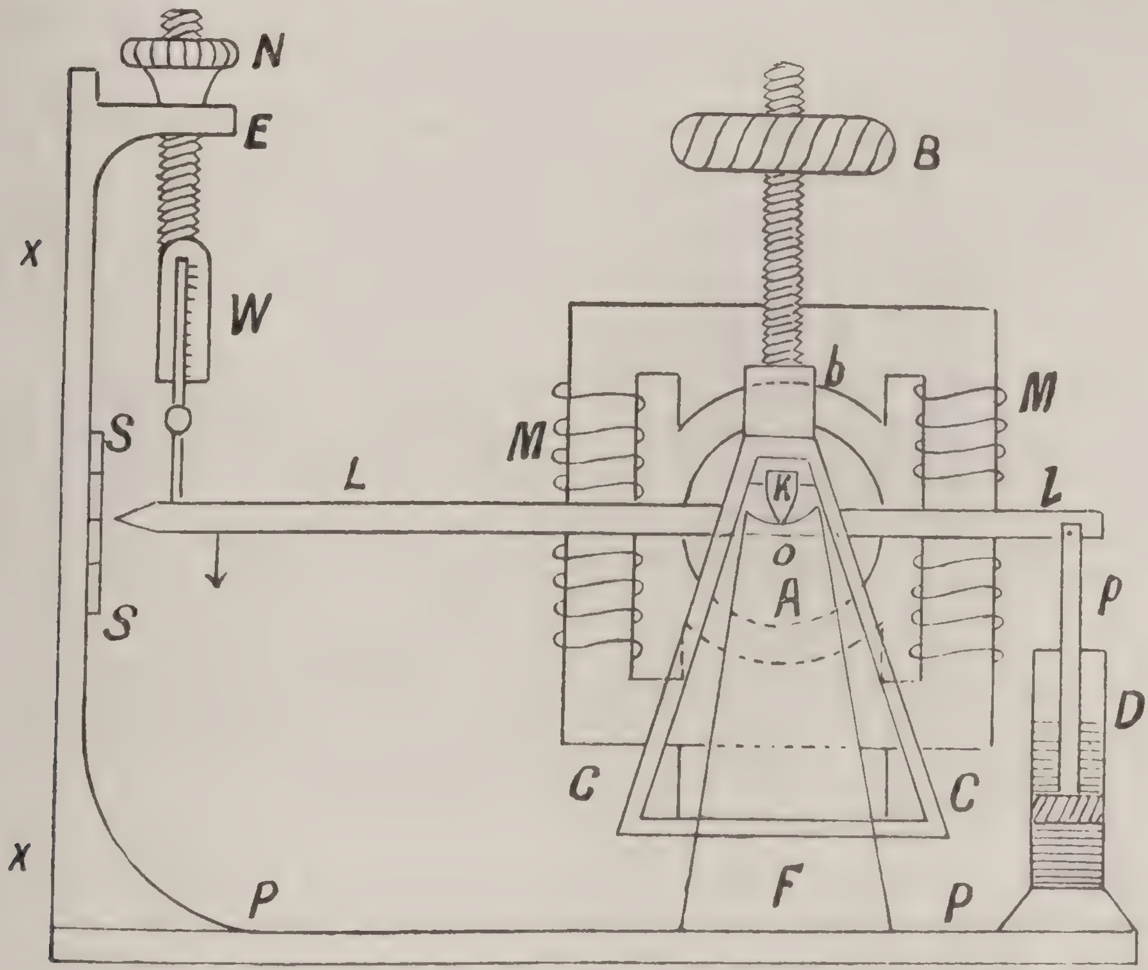

FIG. 285.

size of the motor, and the consequent difficulty in constructing the dynamometer.

It consists of a light framework (*CC*) carrying a small floor or platform at its lower extremities to which the motor *M*, to be tested, is bolted after being packed up so that the centre of its shaft is a line with the points of bearing *O* of the knife edges *K*. The steel planes on which *K* work are at the top of standards *F* carried on a light bed-plate *PP*. A block (*b*) is attached to the

top of the frame *cc*, which carries a screwed bolt on which can travel a heavy balance weight B. By raising this weight, therefore, the *centre of gravity* of M, its bed-plate and the cradle CC can be *raised* to the level of O. A light lever Ll is fixed to and moves with CC, one end being attached to the piston (p) of a dash-pot D containing some viscous fluid for damping the motions of the cradle *cc*.

The other end is attached to the lower portion of a suitable spring balance W, itself supported from a bracket E, carried by a standard (xx).

The end of the lever L moves in front of an index scale *ss*, also carried by xx.

The tail rod l, together with the piston p, are made to, once for all, balance the other part of the beam L, so that after B is properly adjusted the cradle with its motor would rest in equilibrium in any position if W was disconnected from L.

The method of procedure is therefore as follows—When the *cradle* is *exactly balanced* in the manner indicated above, the spring balance W is attached to L and the nut N screwed up or down so as to bring the lever L to the zero of the index scale SS. The motor M is now connected up so that the field magnets and consequently the cradle tend to turn counter-clockwise. This condition must be arranged for beforehand, and the motor placed in the cradle accordingly, to suit the orthodox direction of rotation of the armature A.

A cord is now wrapped *once* round the motor pulley, and its two ends stretched out horizontally. The motor will thus be made to do work against the friction between the cord and pulley, and the result will be a depression of the end L of the beam. Then bring L back to zero on the index scale by turning N and so raising the balance W.

If now W = reading of the spring balance in lbs. (say), and L = the distance in feet of its point of attachment to the lever, from the centre O, then the moment of the force resisting rotation, *i.e.* the torque $T = WL$ (pound feet), and if the speed of the armature $= n$ revs. per sec., the angular velocity $\omega = 2\pi n$.

Hence the work done per sec. $= \omega T$, and the horse-power developed $= \frac{\omega T}{550}$, since 1 H.P. = 550 ft. lbs. per sec.

Horse-Power Transmission Dynamometer.

When the mechanical power required to drive some particular machine, as for instance a dynamo, is desired to be known, it can be obtained by means of a *transmission* dynamometer. This is an appliance for measuring mechanical power without absorbing any of it, as distinguished from the *absorption* dynamometer which measures the power by wasting it all. There are two main classes of the transmission instrument, namely, those for measuring the power transmitted directly through a shaft, and secondly, those for measuring the power transmitted by a belt.

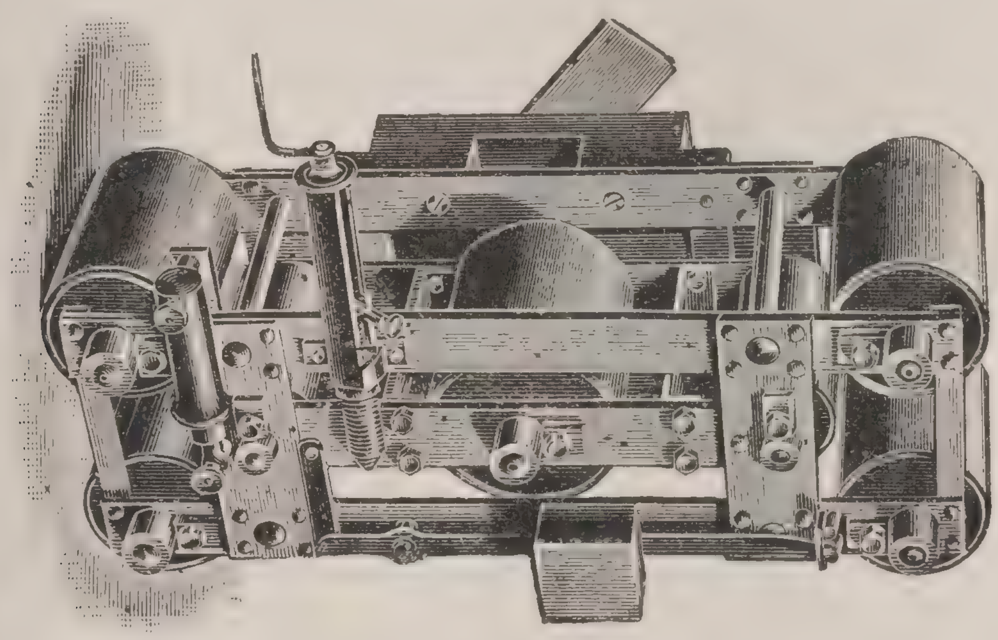

FIG. 286.

Fig. 286 represents the general view, and Fig. 287 the symbolical side elevation, of one belonging to the latter class and made by Messrs. Siemens Bros. and Co. of London. By means of it the difference in tension (T—t) between the driving (*i. e.* the tight) and slack sides of the belt can be read off directiy in pounds, and which is the only one troublesome factor required of the horse-power to be measured.

Referring to Fig. 287, this Siemens transmission dynamometer consists of four similar roller pulleys, running in bearings carried at the four corners of a light but strong iron framework (KK). Three other roller pulleys run in bearings carried by the arm or

frame (A), which is capable of oscillating about a fulcrum F on part of the main frame. The centre pulley P is really the actuating part of the dynamometer, the remaining six, namely p and g, merely acting in a sense as guide pulleys for the belt, the "tight" or driving side of which is TT and the slack side tt. The left-hand end of the arm or frame A has attached to it a link (l), which actuates a lever pointer L capable of moving on a fulcrum (f) over an index scale C, and carrying a balance weight W for the purpose of balancing the arm A with its fittings, etc. D is a dash-pot to steady, what would otherwise be, the jerky

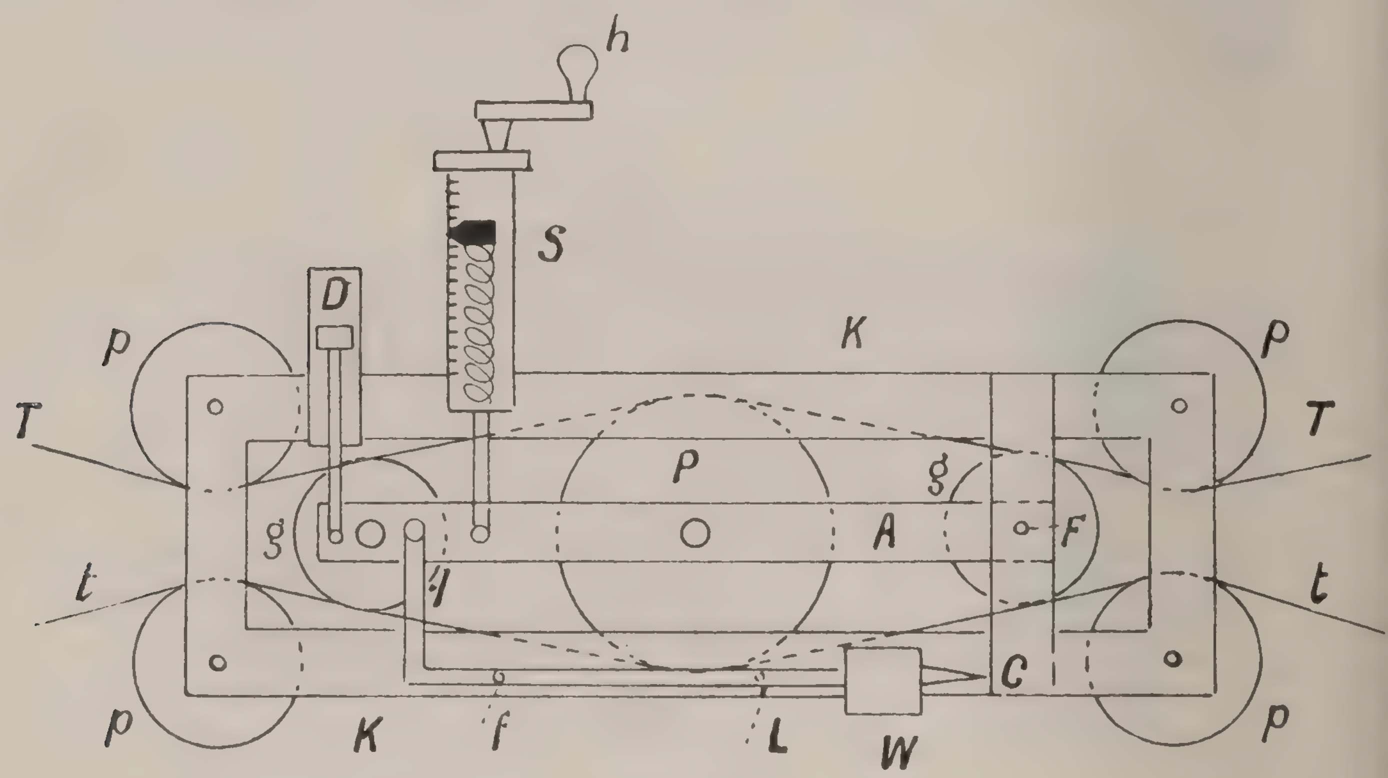

FIG. 287.

movements of A. A strong, spiral spring S, capable of being extended by turning a handle (h), is attached to the left-hand part of A. The action of the dynamometer will now be fairly obvious, and is as follows—

The tight side of the belt TT tends to force down the left-hand end of A against the smaller upward force of the slack side tt, this causes L to turn about f in a counter-clockwise direction; but L is now brought back to zero by turning (h), and when this is the case, the force exerted by S just balances and is equal to the difference in tension (T—t), and is read off on the scale over

which the pointer, attached to the top end of S, moves. Thus according as to whether S is calibrated in lbs. or kilograms so the quantity $(T—t)$ is read off in these units. If now $V=$ velocity of the belt in ft. per min., as obtained from the noted speed of the driven pulley (N) revs. per min. and its diameter d (ft.), then $V=\pi dN$, and the H.P. transmitted $= \dfrac{\pi dN\ (T—t)}{33,000.}$

The Measurement of Power transmitted by Belts.

In the case where it is desired to measure the mechanical power absorbed by some particular machine, such as, for instance, a dynamo feeding a lamp circuit, or any other kind of machine driven by means of belting, the use of a form of Prony brake or other kind of *absorption* dynamometer is inadmissible, owing to the well-known fact that such an appliance wastes all the power which it measures. In this case a transmission dynamometer has to be resorted to, of which there are several different forms, some remaining permanently in position, so as to be capable of indicating at any moment the power transmitted, others being temporarily erected in position for the tests—as, for instance, the Siemens-Hefner-Alteneck belt transmission dynamometer. It is, however, manifestly more convenient to have a permanent arrangement, and the general principle of this type is to connect the driving pulley, which is loose on its shaft, by three or more helical springs, to a fixed collar or boss keyed to the shaft, and then to measure in some convenient way the "angular advance" of the shaft relatively to the pulley, due to the axial extension of the springs. This, as is well known, gives a measure of the power transmitted. Hitherto, however, the arrangements for observing this angular advance have not proved very satisfactory.

By the kind permission of the proprietors of *The Mechanical Engineer*, the author is enabled to give a reprint here of an article written by him in that journal of March 19, 1898, on a neat form of spring transmission dynamometer the recording arrangement of

which was devised by Professor W. Stroud some little time back, and is in use in the electrical engineering laboratories at the University, Leeds. It is accurate and extremely simple both in principle and working, and can be easily fitted to any shaft and kept permanently in position. It has the distinct advantage that the measurement of angular advance is solely an electrical one, and can consequently be obtained with considerable accuracy.

Fig. 288 shows an end elevation, and Fig. 289 a side elevation, of the arrangement, with the lower part of the driving pulley (y) cut away. It is on a counter-shaft, and rotates in the direction of the arrow (Fig. 288), driving a machine. It consists of a boss O, which is keyed to the shaft, and is provided with a flange Q, having an extension at one part of its circumference in the form of a short projection or arm. One end of the steel-driving spring M, which is of square cross-section, is bent so as to partly embrace this arm and be driven by it; the other end is similarly bent, only in the opposite sense, to partly embrace one arm of y, which is loose on the shaft, and drive it. Only these two bent ends of M are shown in Fig. 289, two turns being cut away, as shown, in which is a part sectional elevation about a vertical diameter through shaft centre. The belt can be thrown on to a loose pulley (u) by a fork not shown. To the short projection on the flange Q (Fig. 289) is bolted a light but strong bent arm A into a saw-cut, in the end of which are sweated the ends of two rigid strips of brass side by side. The right-hand one ends in a hinge, which carries a similar strip J, into the side of which is fixed a rigid pin, which passes freely through a slot cut in the end of the left-hand fixed strip. This latter is merely for the purpose of preventing J being pulled too far towards the pulley (y) by the spiral spring V. The hinged strip J carries at its end a light brass block W, to which is attached, but electrically insulated from it by means of ebonite or vulcanized fibre (D), a rounded brass contact block T. This makes electrical contact with a curved resistance frame RS, consisting of a curved piece of wood of somewhat smaller radius than the pulley rim, and of section something similar to that shown at R (Fig. 289). It embraces an angle of about 120 deg., and is provided with shallow saw-cuts on the inner and outer peripheries. Into these are

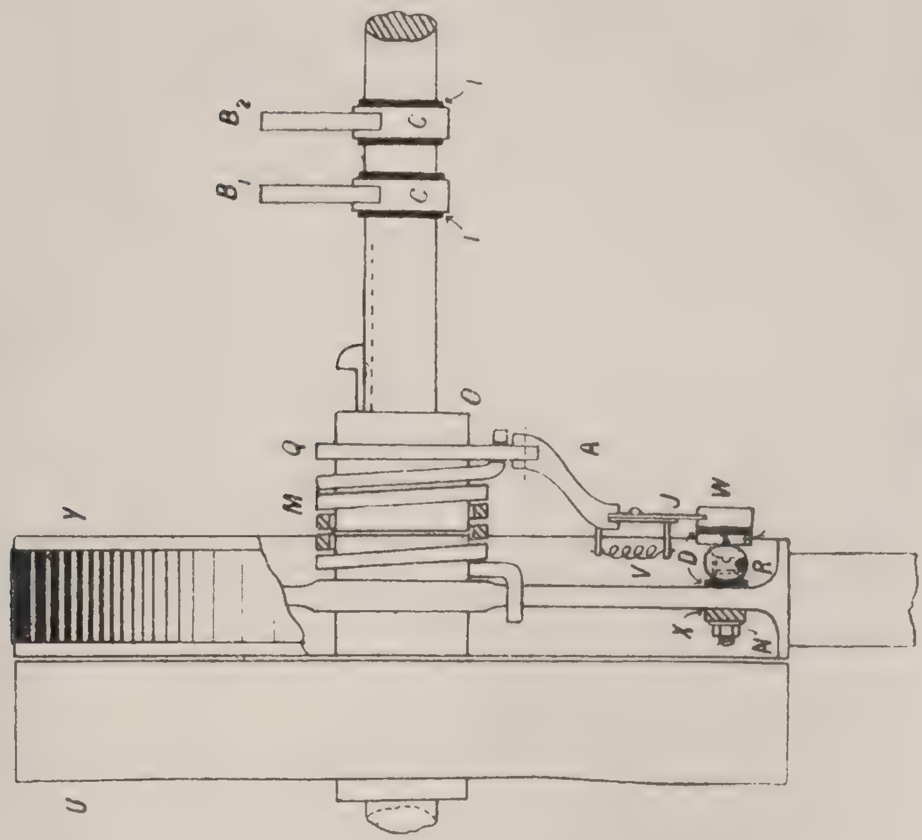

FIG. 289.

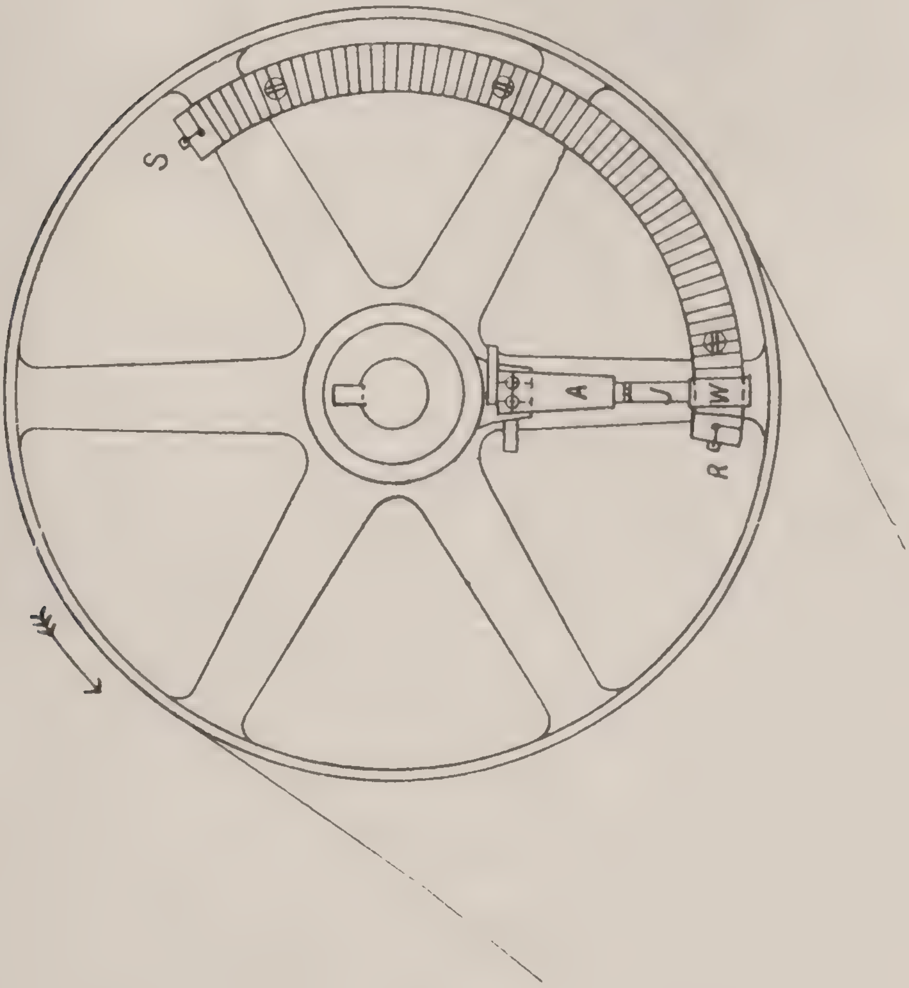

FIG. 288.

pressed the turns of wire with which it is wound, consisting in the present instance of between 280 and 300 turns of No. 18 or No. 20 B. W. G. platinoid wire, double silk covered, the ends being led out to brass terminal blocks at the extremities R and S. The arrangement is securely clasped to the arms of the pulley by the counter-sunk bolts N, which pass through a similarly curved piece of wood of section shown at X (Fig. 289), placed on the other side of the arms. A thin piece of soft insulating material (D) is interposed between these latter and R to prevent some of the turns of wire being short-circuited, and thereby rendered useless, or damaged by pressure against the arms. The turns of wire are bared of their silk insulation, along the line on which T makes contact with them between R and S. Two thin strips of

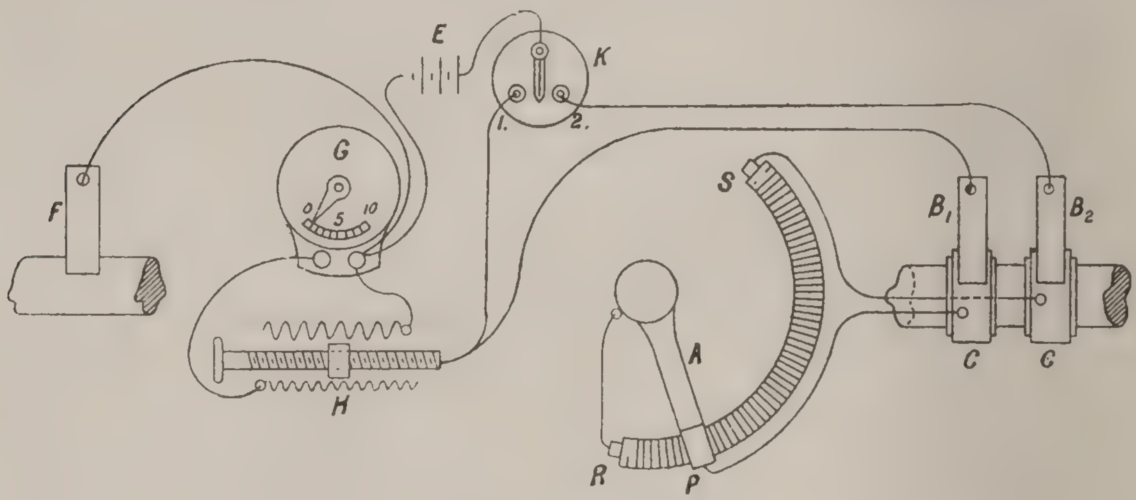

FIG. 290.

vulcanite I I are bent completely round the shaft, and over them are stretched two thin strips of brass CC, the ends of each being soldered together so as to form continuous rings, against which, as they rotate, press two small copper gauze brushes B_1B_2. It now remains to describe the method of indicating the angular advance, and so the measure of power transmitted. This is accomplished wholly electrically, a symbolical diagram of connections being shown in Figs. 290 and 291, corresponding parts being lettered alike. The brass rings CC are permanently and electrically connected by insulated copper wires to S and T, as shown, the shaft and arm A being depicted as taking up a position P on RS in advance of the pulley (Y). R is in electric connection with Y, and therefore with the shaft. F is a fixed brush, which

simply rubs on and makes contact with the shaft. K is a two-way key; E a battery of about three cells, preferably secondaries; G is a sensitive galvanometer or potential difference indicator, as dead beat as possible, and preferably of the moving coil or D'Arsonval type, in order that, firstly, its deflections may be directly proportional to the potential difference at its terminals; and, secondly, that by winding the moving coil on a light but broad aluminium frame, it can be made very dead beat. G is provided with a "constant total current shunt," shown symbolically at H (Fig. 290), which is for the purpose of adjusting the sensibility of G, to which it is shunted without altering the gross resistance of the combination, and therefore the P.D. between the two points to which it is applied.

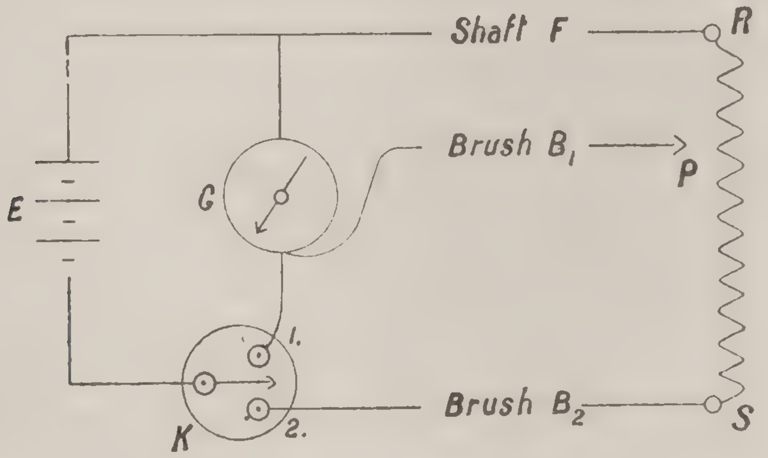

FIG. 291.

The principle of action of H is merely that the screw, fixed as regards end play, when turned, actuates the contact block, rubbing between the top and bottom resistances, which are suitably proportioned to one another and to the galvanometer as well. The resistance between the terminals of the combination should be at least 20 times that of RS.

In starting the calibration, switch the lever of key (K) on to stop 1 (Figs. 290 and 291). G is then directly across E, and the number of cells together with the sensitiveness of G (by using H) should be adjusted to give a full scale deflection. This must always be re-obtained before starting subsequent tests. Now, switch to 2 on K, so completing the main circuit by way of F, shaft, R, S, B_2, K, E, and back to F. A certain current will flow,

depending, of course, on the total resistance and pressure at E, and for this current a definite P.D. will exist between R and S, and also between R and P, causing a deflection on G, as shown. The calibration is now finally effected by hanging known dead weights from the face of pulley (Y), thereby twisting up M (Fig. 289), and noting the corresponding deflections on G. The mean nett pull with any particular dead weight will best be obtained by taking the mean of two readings on G, corresponding to the two extreme positions of equilibrium of the pulley, this latter being helped to take up these positions.

Fig. 292 shows two calibration curves A and B, for two totally distinct transmission dynamometers of this type, every detail in each being precisely the same with the exception of the springs M, being different as regards number of spirals and area of cross-section of these only. A few details about these may be worthy of note. The spring of the dynamometer giving curve A (Fig. 292) consists of three complete turns of tempered steel of normal internal diameter $= 6\frac{1}{16}$ in., and square cross-section $= \frac{11}{32}$ in. $\times \frac{11}{32}$ in. A pull = weight of 50 lbs. at the pulley face wound it up tight on the surface of the bosses $5\frac{1}{2}$ in. diameter, giving a deflection on G of 9·1 (10 being full). This result is clearly shown by the bending of the curve at the top part. The spring of the dynamometer giving curve B consists of five complete turns of same diameter, but square cross sections $= \frac{1}{2}$ in. $\times \frac{1}{2}$ in. Its internal diameter was approximately $\frac{1}{4}$ in. in excess of that of its bosses ($5\frac{1}{2}$ in.), for the largest weight used (180 lbs.). Hence, not being tight up, there is no bending of the curve B. The turns of both springs nearly touch.

It may now be useful to note, with regard to the design of such a helical spring, the relation that subsists between the pull of the weight at the pulley face, the decrease of the diameter of the spiral, and the angular deflection of one end of the spring in a plane perpendicular to its axis, represented by the position of the point of contact P (Figs. 290 and 291) on RS. Let $W =$ dead weight applied, *i.e.* the tangential force at the circumference of the pulley of radius R.

Then the moment (M) of the twisting couple in a plane perpendicular to the axis of the spring is $M = WR$. If the coils are quite flat, and their planes at right angles to this axis, there will

be no torsion in the spring itself, and it will be wholly subjected to a bending action due to M.

If r_0 = mean radius of the helix before W is applied,

r = mean radius of the helix after W is applied,

and θ = angle of twist, *i. e.* the angle through which it is wound up. Then, since the moment of the twisting couple must just balance that due to the elastic force of the spring when the arm A (Fig 290) has come to some steady position P on RS, we have

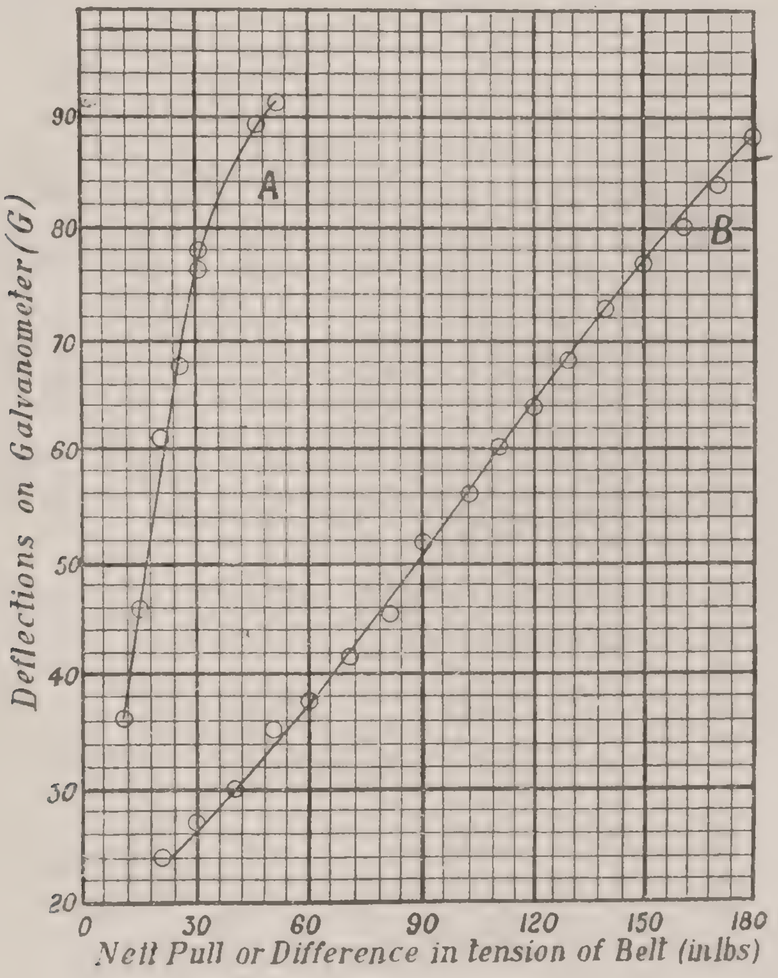

FIG. 292.

$M = EI\left(\frac{1}{r} - \frac{1}{r_0}\right) = EI\frac{\theta}{l} = WR$, where l = length of the spring and EI represents the flexural rigidity of the material of which the spring is made.

E being the modulus of elasticity for the material, *i. e.* tempered steel.

I being the moment of inertia of the section of the material, *i. e.* for a square.

In the present case, therefore, for square section $I = \frac{b^4}{12}$ where

(b) = length of a side of that square section. Substituting in the above equation we have

$$WR = \frac{Eb^4}{12}\left(\frac{1}{r} - \frac{1}{r_0}\right) = \frac{Eb^4}{12} \cdot \frac{\theta}{l}$$

from which it can be seen that for a given pulley, material of the spring, and angle of twist, $W \propto \frac{(\text{sectional area of spring})^2}{\text{length.}}$

We have also that the deflection or distance through which W acts $= R\theta$.

Writing the first equation in another form, we get an expression for the amount of coiling of the spring for a particular weight W.

Thus $$r = \frac{EIr_0}{r_0\, WR + E\,I} = \frac{Er_0 b^4}{12r_0\, WR + Eb^4}$$

W is evidently the nett pull on the circumference of pulley, and if T = tension (in lbs.) of the tight or driving side of the belt, and t = tension (in lbs.) of the slack side; then $W = (T - t)$ = nett pull of belt in pounds, which is one factor of the horse-power transmitted. Also, if the pulley of the machine driven by (Y) is of radius (f) ft. and makes (n) revolutions per minute, then, for no slipping of the belt, velocity of belt $(v) = (2\pi fn)$ ft. per minute, and horse-power

$$\text{transmitted} = \frac{V(T-t)}{33{,}000} = \frac{2\pi fn\,(T-t)}{33{,}000} = H_2.$$

If the driven machine be a dynamo developing A amperes of current at a pressure of V volts on the external circuit at that speed, then the useful horse-power developed $H_1 = \frac{A\,V}{746}$

Its commercial or nett efficiency is therefore $= \frac{H_1}{H_2}\,100$ per cent.

In conclusion, it may be mentioned that the dynamometer forms a simple and accurate means of measuring the horse-power transmitted to any machine, and works very satisfactorily provided there is no skidding of the belts, and also that the spring (V) (Fig. 289) exerts sufficient tension on J to give a good reliable contact between the contact piece (T), to which the brass ring under B_1 is electrically attached, and the curved resistance RS.

Absorption Dynamometer.

Fig. 293 illustrates an absorption dynamometer which is a modification of a Prony brake for making brake tests of the horse-power developed by electromotors. One great trouble in-

FIG. 293.

herent in all such tests is the high speed (relatively to other prime movers) at which these machines run, especially small ones. This tends to cause a jerkiness of the brake and trouble conse-

quently in reading the indications of the various parts. In Fig. 293, three plies of cord make a half-lap over the top side of the brake pulley, at one side supporting a scale pan, and at the other (the nearest to the observer looking at the figure) being attached to a single cord passing round a nearly frictionless pulley and hanging from the lower end of the spring-balance seen at the top of the illustration. This double bend of the cord is merely for convenience in having the balance in a position where it can be read easily. The three plies of cord over the pulley are kept in position by light brake blocks as shown. Thus the weights in the pan will represent the tension on the tight side, and the reading of the balance that on the slack side, so that their difference gives the nett load on the brake. The brake pulley shown is a special form of box pulley devised by the author, and water-cooled by an inlet or outlet pipe passing through a central opening.

Another form of Prony brake, giving excellent results with small motors from about ½ B.H P. upwards at speeds up to

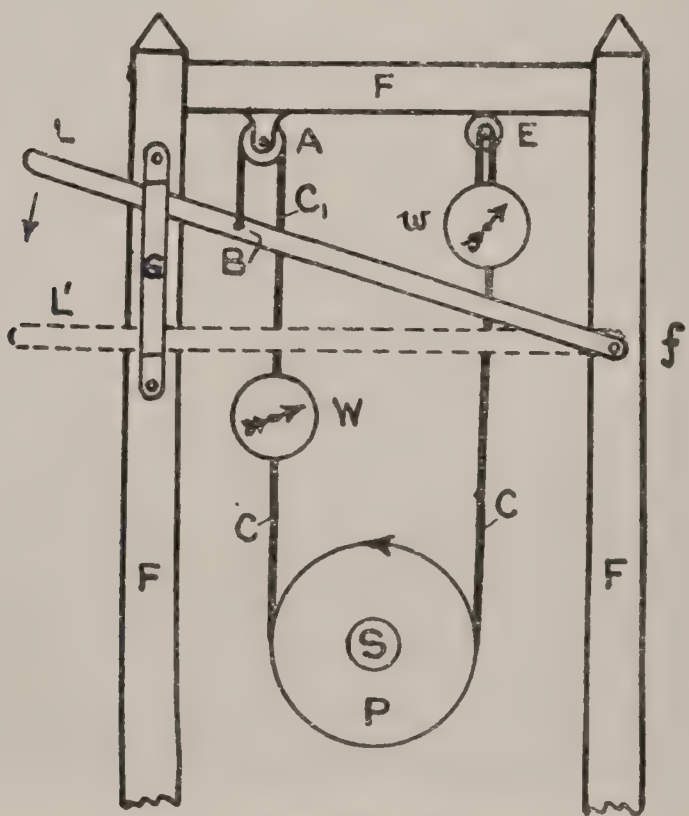

Fig. 294.

2,000 revs. per min., is shown diagrammatically in Fig. 294, and consists of a light cast-iron flanged pulley P keyed to the shaft S of the motor to be tested and rotating truly on it. A light

framework FFF carries an eyelet E, from which hangs a spring balance w. From the lower end of w a pliable cord passes $1\frac{1}{2}$ times round P and is attached to another spring balance W. From the upper end of W a cord C_1 passes round a small grooved pulley A (supported from F) to a fixed point B on an iron lever Lf. This lever is pivoted at f and is capable of moving in a guide G between the limits L and L^1. If P rotates counter-clockwise as shown, (W) will read the tension on the *tight* side, and (w) that on the *slack* side, of the brake rope C, so that $(W - w) =$ nett pull in lbs., and if $(r) =$ radius of (pulley face $+ \frac{1}{2}$ diam. of C) in feet, then the torque exerted $= (W - w)\ r$ lb.-ft., and the B.H.P. $= \dfrac{2\pi nr(W - w)}{33{,}000}$, where $n =$ speed of P in revs. per min. The ranges of W and w may be as 3 : 1, and if the face of P rotates quite *truly*, the deflections of w and W are quite steady and easy to read. The hand pressure on L can be both easily and very gradually applied, and when released, the weight of W raises L and releases P from the pressure of CC automatically—a feature of some value in preserving the rope, and a convenience in motors which do not race on removal of load.

Eddy Current Brake.

One of the most convenient methods of measuring the brake horse-power of electro-motors, petrol, and other motors, especially when the speed is high, is by means of an electro-magnetic or eddy-current brake. While the principle has been used in commercial work to an enormous extent in light apparatus, *e.g.* in electricity meters and instruments, so far as the author is aware, Messrs. Morris & Lister of Charlton Works, Coventry, were the first to patent and put on the market a form suitable for absorbing and measuring the B.H.P. of electro-motors, etc. One of these, used by the author for many years, gave excellent results. Unfortunately the firm have ceased to make them, and it is therefore of little use to describe this particular form. The general principle underlying the construction and action of all such brakes will readily be understood from Fig. 295.

A cylindrical ring C of copper is fixed securely to the outer surface of an iron drum I keyed to the shaft (S) of the motor, and rotates in a multipolar electro-magnetic field system F.

Fixed to F, and in a line passing through the centre of S, is a light graduated lever L, from two to five feet from tip to shaft centre, and made of thin aluminium or steel tube. This moves opposite a fixed index scale B between stops A, and carries a light slider D from which known weights W can be hung. A small sliding weight w, which can be clamped by a set screw on a light tail rod (l), serves to counterbalance the weight of L and D. A direct current, flowing through the field coils of F, produces a powerful multipolar magnetic field through the gap between I and the poles in which C rotates. E.M.F.'s are therefore induced in C, causing eddy or Foucault currents

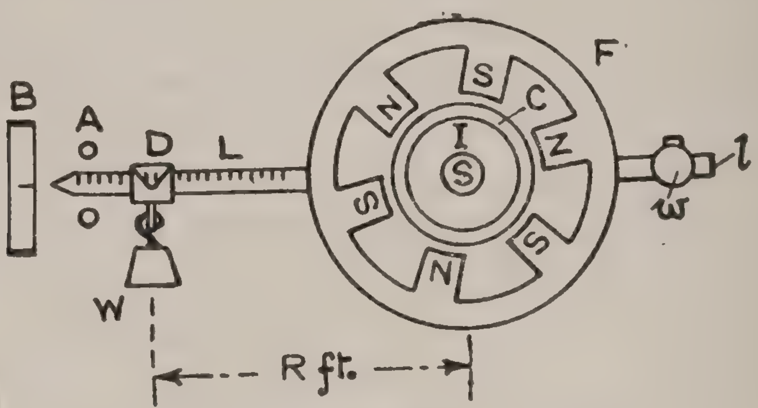

Fig. 295.

to flow in C, which oppose the driving source, and the output of the motor is thus expended in heating C. The strength of these currents, and hence the B.H.P. of the motor absorbed, depends on the strength of field, which is readily controlled by means of a rheostat in the exciting circuit.

The force or torque resulting from the interaction between field and currents in C tends to drag the floating-field system F round, and this is opposed and balanced by a gravitational force due to the weighted lever L.

When a position of D (at a radius R feet from the shaft centre) in conjunction with the adjustment of excitation, is found, such that the above opposing forces balance, the lever L will float between the stops A. The torque or turning moment (in lb.-ft.) exerted by the motor then $= WR$ and its B.H.P. $=$

$\frac{2\pi RnW}{33,000}$ at a speed of (n) revs. per min.

The field frame F can either be carried by a separate support, giving freedom of oscillation, or on a sleeve by the motor spindle itself; but even in this method the weight due to the whole brake is considerably less than the pull due to a bell drive. For example, a 3-H.P. (continuous rating) size, made by Messrs. Morris & Lister, and running at 1,500 revs. per min., weighed 45 lbs., while a 35-H.P. (continuous rating) brake at 250 revs. per min. weighed 250 lbs. complete. With a suitable provision in the design for ventilation, such brakes could be used for temperature load tests, but these are more economically effected by one or other of the well-known regenerative methods. A brake suitable for a certain H.P. and speed (intermittent rating) will, of course, absorb smaller powers for longer periods, or one for continuous rating larger powers, up to 100%, for short periods. Higher speeds require smaller brakes. Anyone who has used this class of brake will have realized the many practical advantages it possesses over other kinds of absorption brakes, particularly in the matter of sensitiveness, control, smoothness of operation, and accuracy in repeating readings, to say nothing about the absence of wear, burnt blocks, bands, and rope and water-cooling arrangements.

Soames Motor Testing Brake.

This appliance belongs to the class of mechanical power measurers known as absorption dynamometers, and is a modified form of Prony brake made by Messrs. Nalder Bros. and Co. It is of extremely simple construction, and gives perfectly definite weighing of the torque on the pulley under test against ordinary dead weights. It consists of a steel lever working on knife edges, which can be raised or lowered by the hand-wheel at the top of the brake. Holes are drilled in the lever at equal distances from the centre, corresponding to the ordinary sizes of pulley in use.

The centre of the brake is placed over the centre of the pulley, and from the two holes corresponding to the diameter of the pulley under test is suspended a piece of webbing, which passes round the pulley as in the diagram, Fig. 296.

When the pulley is running, a weight, say from $\frac{1}{4}$ to 30 lbs., is suspended on the end of the arm as shown.

The whole is then raised by turning the hand-wheel, tightening the belt on the pulley until sufficient friction is put on the belt to raise the arm to a horizontal position and keep it floating there; the speed of the pulley being taken at the same time.

The weight is hung at either of the two holes at the end of the bar marked respectively H.P. $K = \frac{1}{4000}$ and Watts $K = \frac{1}{6}$, the one

Fig. 296.

hole giving H.P. direct by multiplying weight in lbs. by revolutions per minute, and dividing by 4000, the other hole similarly giving Watts direct by dividing by 6.

The constant for H.P. is $2\pi L \div 33{,}000$, L being the distance of the weight from the centre.

The size of the pulley does not enter into the equation, so long as the distance between the holes in the arm is equal to its diameter.

The band *must*, in all cases, be hung from two holes equidistant from the centre.

The pulley should be perfectly smooth and flat, and it is convenient but not necessary to have it flanged.

No lubricant is generally required, but a little black-lead may be applied if necessary ; *no oil.*

This brake is extremely accurate, and every reading can be repeated with certainty, each one not taking more than ten seconds at the outside.

TABLES OF

CONSTANTS, LOGARITHMS, ETC.

Standards for Copper Conductors.

(*Adopted by the Electrical Standards Committee representing the General Post Office, Institution of Electrical Engineers, and all the leading cable manufacturers of Great Britain.*) *Revised March* 1910.

A wire 1 metre long, weighing 1 gramme, at 60° F. (15·6° C.) has Matthiessen's value of resistance—

0·1539 standard ohms for hard-drawn, high conductivity commercial copper.

0·150822 standard ohms for annealed, high conductivity commercial copper.

These figures at 60° F. being calculated from 0·1469 per metre-gramme for hard-drawn, and 0·1440 for annealed copper at 32° F. by Matthiessen's formula—

$$R_{t^\circ} = \frac{R_{32^\circ}}{1 - 0{\cdot}00215006\,(t - 32) + 0{\cdot}00000278\,(t - 32)^2}$$

Hard-drawn copper is defined as that which will not elongate more than 1% without fracture.

Copper is taken as weighing 555 lbs. per cubic foot at 60° F., and its corresponding specific gravity = 8·912.

The average temperature coefficient is = 0·00238 per degree F.

A *lay* of 20 times the pitch diameter is taken as the standard for calculating all tables.

The resistance and weight of conductors is calculated from the actual length of wire, or 1·01226 times the length of the cable for all except the centre wire.

Maximum variation of resistance or weight of any wire allowing for losses in manufacture is 2%.

An allowance of 1% increased resistance as calculated from the diameter is permissible on all tinned copper between Nos. 22 and 12 gauges inclusive.

The following tables of figures are deduced from the above constants—

TABLE XV.

COPPER WEIGHING 555 LBS. PER CUBIC FOOT AT 60° F.

Solid Wires.	Resistance in Standard Ohms of high conductivity Commercial Copper.	
	Annealed.	Hard-Drawn.
Resistance per cubic inch	0·00000066788	0·000000681327
,, ,, ,, cm.	0·00000169639	0·00000173054
,, of 100 ins. weighing 100 grains.	0·150158	0·153181
Resistance per mile	0·042317 ÷ area in □″	0·0431689 ÷ area in □″
,, per yard	0·000024044 ÷ ,,	0·0000245277 ÷ ,,
,, per mil. foot	10·2044	10·4099

Weight per mile . . . . . 2·0350 × area in □″
,, ,, yard . . . . . 11·5625 × ,,

The following table refers to copper cables with a lay = 20 times the pitch diameter.

TABLE XVI.

Cable.	Resistance in Standard Ohms.	Weight.
3-Strand	0·33742 $\times r$	3·03678 $\times w$
4- ,,	0·253065 $\times r$	4·04904 $\times w$
7- ,,	0·1443557 $\times r$	7·07356 $\times w$
12- ,,	0·084355 $\times r$	12·1471 $\times w$
19- ,,	0·0532424 $\times r$	19·2207 $\times w$
37- ,,	0·0273493 $\times r$	37·4414 $\times w$
61- ,,	0·0165911 $\times r$	61·7356 $\times w$
91- ,,	0·0111222 $\times r$	92·1034 $\times w$

where r = the resistance of each wire,
and w = ,, weight ,, ,, ,,
The resistance of a cable being equal to the resistance in parallel of the wires.

International Standards of Resistance for Copper.

Extract from the British Engineering Standards Association Report, No. 7, *July* 1919.

1. The following standards, fixed by the International Electro-Technical Commission, have been taken as normal values for standard annealed copper :—

(i) At a temperature of 20° C. the resistance of a wire of standard annealed copper, one metre in length and of a uniform section of one square millimetre, is 1/58 ohm (0·017241 . . . ohm).

(ii) At a temperature of 20° C. the density of standard annealed copper is 8·89 grammes per cubic centimetre.

(iii) At a temperature of 20° C. the "constant-mass" temperature coefficient of resistance of standard annealed copper, measured between two potential points rigidly fixed to the wire, is—

$$0{\cdot}00393 = 1/254{\cdot}45 \text{ . . . per degree Cent.}$$

(iv) As a consequence it follows from (i) and (ii) that at a temperature of 20° C. the resistance of a wire of standard annealed copper of uniform section, one metre in length and weighing one gramme, is—

$$(1/58) \times 8{\cdot}89 = 0{\cdot}15328 \text{ ohm.}$$

Coefficient of Linear Expansion of Standard Annealed Copper.

2. The coefficient of linear expansion of standard annealed copper between 60° F. (15·6° C.) and 68° F. (20° C.) has been taken as—

0·00000944 per 1° F. (0·0000170 per 1° C.).

Density of Standard Annealed Copper at 60° F.

3. The density of standard annealed copper at a temperature of 60° F. has been taken as 8·892015, and the weight per one cubic foot of copper as 555·1108 lbs.

Resistance of a Solid Conductor at 60° F.

4. For the purpose of calculating the tables, the resistance of a solid conductor of standard annealed copper at 60° F., 1000 yards in length and of uniform cross-sectional area of one square inch, has been taken as 0·0240079 ohm.

TABLE XVII.

RELATION BETWEEN E.M.F. AND TEMPERATURE OF THE CLARK STANDARD CELL—MADE ACCORDING TO REGULATIONS, OF THE CARHART-CLARK, AND OF THE WESTON CADMIUM STANDARD CELLS (see pp. 13 and 17).

Temperature °C.	E.M.F. in Legal Volts.			Temperature °C.	E.M.F. in Legal Volts.		
	Clark.	Carhart-Clark.	Weston Cadmium.		Clark.	Carhart-Clark.	Weston Cadmium.
4	1·4461	1·4395	1·01895	16	1·4329	1·4335	1·01846
5	1·4450	1·4390	91	17	1·4318	1·4330	42
6	1·4439	1·4385	87	18	1·4307	1·4325	38
7	1·4428	1·4380	83	19	1·4296	1·4320	34
8	1·4417	1·4375	79	20	1·4285	1·4315	30
9	1·4406	1·4370	75	21	1·4274	1·4310	26
10	1·4395	1·4365	71	22	1·4263	1·4305	22
11	1·4384	1·4360	66	23	1·4252	1·4300	18
12	1·4373	1·4355	62	24	1·4241	1·4295	14
13	1·4362	1·4350	58	25	1·4230	1·4290	10
14	1·4351	1·4345	54	26	1·4219	1·4285	06
15	1·4340	1·4340	1·01850	27	1·4208	1·4280	1·01802

TABLE XVIII.

RELATION BETWEEN PRACTICAL AND C.G.S. (ABSOLUTE) UNITS.

	Practical units.	Absolute C.G.S. units.		Dimensions.	
		Electromagnetic.	Electrostatic.	Electrostatic.	Electromagnetic.
Quantity	1 coulomb	10^{-1}	$v\times10^{-1}=3\times10^{9}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}$
Current	1 ampere	10^{-1}	$v\times10^{-1}=3\times10^{9}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$
Potential	1 volt	10^{8}	$10^{8}\div v=\frac{1}{3}\times10^{-2}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$
Resistance	1 ohm	10^{9}	$\frac{1}{v^2}\times10^{9}=\frac{1}{9}\times10^{-11}$	$L^{-1}T$	LT^{-1}
Capacity	1 farad	10^{-9}	$v^2\times10^{-9}=9\times10^{11}$	L	$L^{-1}T^{2}$
Self-induction	1 secohm	10^{9}	—	—	L
Mutual induction	1 secohm	10^{9}	—	—	L
Power	1 watt	10^{7}	—	—	$ML^{2}T^{-3}$
Work	1 joule	10^{7}	—	—	$ML^{2}T^{-2}$
Induction	1 weber	10^{8}	$v=3\times10^{10}$	—	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$

v = velocity of light $=3\times10^{10}$ cms. per second.

Table XIX.

Electro-chemical Equivalents, Specific Gravities, etc.

Metal.	Atomic weight.	Chemical equivalent $= \frac{\text{at. wt.}}{\text{valency}}$	Electro-chemical equivalent, grams per coulomb.	Weight in grams per hour deposited by 1 amp.	Specific gravity in grams per cub. cm.
Aluminium	27·0	9·0	0·00009317	0·3354	2·67
Copper (monad)	63·1	63·1	0·00065735	2·3665	8·912
,, (dyad)	63·1	31·6	0·00032867	1·1832	8·912
Gold	196·7	65·6	0·00067806	2·4410	19·3
Iron (dyad)	56	28	0·00028986	1·0435	7·85
Lead	206·4	103·2	0·0010714	3·8571	11·4
Nickel	58·6	29·3	0·00030538	1·0994	8·5
Silver	108	108	0·0011180	4·0249	10·57
Tin (dyad)	117·8	58·9	0·00061077	2·1988	7·3
Zinc	65	32·5	0·00033644	1·2112	7·15
Hydrogen	1	1	0·000010352	0·03738	0·0000896

Table XX.

Temperature Coefficients and Specific Resistances of Pure Metals and Alloys, determined by Professors J. A. Fleming and J. Dewar.

Metals, pure, soft, and annealed.	Specific resistance, ρ, in microhms, per c.c. at 0° C.	Mean temperature coefficient, a, between 0° and 100°C.	Alloys, usual proportions.	Specific resistance, ρ, in microhms, per c.c. at 0° C.	Temperature coefficient, a, at 15° C.
Platinum	10·917	0·003669	Platinum-silver	31·582	0·000243
Gold	2·197	0·003770	,, -iridium	30·896	0·000822
Palladium	10·219	0·003540	,, -rhodium	21·142	0·00143
Silver	1·468	0·004000	Gold-silver	6·280	0·00124
Copper	1·561	0·004280	Manganese steel	67·148	0·00127
Aluminium	2·665	0·004350	Nickel steel	29·452	0·00201
Iron [1]	9·065	0·006250	German silver	29·982	0·000273
Nickel	12·323	0·006220	Platinoid	41·731	0·000310
Tin	13·048	0·004400	Manganin	46·678	−0·00000
Magnesium	4·355	0·003810	Silverene	2·064	0·00285
Zinc	5·751	0·004060	Aluminium-silver	4·641	0·00238
Cadmium	10·023	0·004190	,, -copper	2·904	0·00381
Lead	20·380	0·004110	,, -bronze [2]	12·300	0·0010
Thallium	17·633	0·003980	Reostene [2]	76·468	0·00110
Mercury	94·070	0·000720	Brass [2]	7·2	0·0010
Bismuth [2]	119·160	0·00420	Nickelin [2]	38·50	—
Cobalt [2]	9·71	0·00330	Carbon [2] retort	67000	—
Tantalum [2]	14·6	0·00330	,, arc light Carré	7000	−0·0005
Tungsten [2]	5·0	0·0051	,, glow lamp Ediswan	4000	−0·00054
Osmium [2]	9·5	—	Phosphor bronze [2] (commercial)	8·479	0·00064
			Eureka [2]	47·48	0·000022
			Constantan [2]	—	−0·00004 to + 0·00001
			Nickel Chrome	100	·00042

[1] Approximately pure. [2] Not determined by Fleming and Dewar.

Table XXI.

Approximate Specific Resistance of the Commoner Liquids, and which Diminishes with Increase of Temperature about 1·5% per 1° C.

Solution.	Specific Resistance (Legal ohms per c.c.).	At a temperature (Degrees Cent.).
Sulphuric acid, 5% acid, sp. gr. 1·033	4·82	18
,, ,, 10% ,, ,, 1·070	2·84	,,
,, ,, 20% ,, ,, 1·1414	1·54	,,
,, ,, 25% ,, ,, 1·170	0·99	,,
,, ,, 30% ,, ,, 1·220	1·36	,,
,, ,, 40% ,, ,, 1·310	1·48	,,
Common salt (ordinary)—saturated solution	5·10	,,
Zinc sulphate ,, ,, ,,	20·3	,,
Copper ,, ,, ,, ,,	30·0	,,
Sal-ammoniac solution, sp. gr. 1·07	5·50	,,
Water	$0·3 \times 10^6$	11
,,	120×10^6	76

Table XXII.

Comparative Data for H.C. Copper and Aluminium.

	Copper.	Aluminium.
Conductivity (Electrical)	100	61·5
Tensile Strength (unannealed)	25–30 tons	12–15 tons
Melting-Point	1200° C.	700° C.
Specific Gravity	8·9	2·7
Sectional Area for Equal Conductivity . .	1	1·66
Diameter ,, ,, ,, . . .	1	1·28
Weight ,, ,, ,, . . .	1	0·495
Temperature Co-efficient per 1° C. . . .	0·00428	0·0032

Table XXIII.

Eureka Resistance Material

A Cupro-Nickel Alloy (supplied by the London Electric Wire Co. and Smith's Ltd.), prepared with great care to secure a Non-corrodible and Stable Alloy.

Temperature Co-efficient . .	0·000022 per deg. C.
Specific Resistance	48 Microhms per cm. cube.
,, Gravity	8·9.
Melting-Point	1250° C.
Comparative Resistance . .	28 times copper.
Tensile Strength	40 tons per sq. inch.
Weight per cubic inch . . .	0·32 lb.

Resistance and carrying capacity for "open spirals" of Eureka wire in air, well ventilated with free radiation.

Gauge S.W.G.	Diameter.		Approx. Amps giving temperatures of			Resistance in Standard Ohms per 1000 yds. at 60° F. (15·5° C.)
	Inches.	m/m.	100° C.	200° C.	300° C.	
6	·192	4·877				23·3
7	·176	4·470				27·7
8	·160	4·06	33·0	52	58·5	33·5
9	·144	3·65	26·0	43	50	41·3
10	·128	3·25	22 8	36	41·5	52·3
11	·116	2·94	19·0	30	35·5	63·7
12	·104	2·64	16·8	24	29·5	79·3
13	·092	2·33	12·7	20	24·2	101·3
14	·080	2·03	9·5	15	19·5	133·9
15	·072	1·82	7·4	12·6	16·8	165·3
16	·064	1·62	6·0	10·4	14·3	209·4
17	·056	1·42	5·3	8·8	11·3	273·3
18	·048	1·21	4·3	7·0	9·1	871·8
19	·040	1·01	3·7	5·5	6·8	535·6
20	·036	·914	3·0	4·7	5·9	661·3
21	·032	·812	2·8	4·0	5·0	837·2
22	·028	·711	2·2	3·2	4·1	1093
23	·024	·609	1·3	2·6	3·3	1457
24	·022	·558	1·5	2·3	2·8	1770
25	·020	·508	1·25	2 0	2·5	2142
26	·018	·457	1·0	1·68	2·1	2645
27	·0164	·416	·9	1·47	1·9	3186
28	·0148	·375	·76	1·37	1·58	3914
29	·0136	·345	·68	1·15	1·47	4634
30	·0124	·314	·59	1·0	1·25	5575
31	·0116	·294	·52	·9	1·05	6370
32	·0108	·274	·47	·81	·95	7350
33	·010	·253	·42	·74	·85	8571
34	·0092	·233	·37	·64	·75	10128
35	·0084	·213	·33	·56	·65	12149
36	·0076	·193	·28	·48	·57	14840
37	·0068	·172	·26	·43	·51	18536
38	·006	·152	·19	·31	·40	23808
39	·0052	·132	·16	·26	·31	31696
40	·0048	·121	·15	·24	·28	37184
41	·0044	·111	·14	·21	·26	44268
42	·004	·101	·13	·18	·23	53564
43	·0086	·091	·11	·17	·20	66136
44	·0032	·081	·10	·14	·17	83664
45	·0028	·071	·08	·13	·15	108648
46	·0024	·061	·07	·10	·12	148764
47	·002	·050	·05	·08	·10	214284

TABLE XXIV.

Nickel Chrome Resistance Material

A HIGH RESISTANCE ALLOY (SUPPLIED BY THE LONDON ELECTRIC WIRE CO. AND SMITH'S LTD.).

Temperature Co-efficient . .	·00042 per deg. C.
Specific Resistance	100 Microhms per cm. cube.
,, Gravity	8·15.
Melting-Point	1550° C.
Comparative Resistance . .	58 times copper.
Tensile Strength	47·12 tons per sq. inch.
Weight per cubic inch . . .	·29 lb.

Resistance and carrying capacity of Nickel Chrome Wire.

Size S.W.G.	Approximate Resistance in Standard Ohms per 1000 yds. at temperatures of			Approximate Amperes giving temperatures of		
	200° C.	400° C.	600° C.	200° C.	400° C.	600° C.
16	452	494	538	7·1	12	18
17	591	646	703	6·0	9·6	14
18	802	879	957	4·3	7·7	11
19	1154	1266	1378	3·7	5·7	8·4
20	1426	1590	1700	3·3	4·7	6·8
21	1809	1978	2151	2·7	4·2	6·2
22	2360	2583	2820	2·2	3·5	5·1
23	3237	3535	3860	1·8	2·8	4·1
24	3828	4187	4555	1·6	2·4	3·3
25	4732	5061	5505	1·4	2·1	3·1
26	5720	6250	6870	1·1	1·9	2·6
27	6890	7535	8400	1·0	1·6	2·4
28	8460	9250	10070	·93	1·4	2·0
29	10000	10950	11920	·78	1·3	1·8
30	12040	13170	14320	·68	1·1	1·6
31	13760	15040	16370	·61	·88	1·3
32	15880	17360	18900	·55	·80	1·2
33	18530	20250	22050	·50	·72	1·1
34	21880	23920	26100	·43	·63	·93
35	26250	28700	31500	·37	·56	·83
36	32200	35070	38380	·32	·49	·72
37	40100	43800	49000	·29	·43	·63
38	51400	56300	61270	·21	·34	·49
39	68500	74900	81500	·17	·26	·39
40	80200	87900	95700	·10	·24	·35

Table XXV.

The following Table shows the Current that will give certain rises in Temperature in various gauges of Reostene Resistance Material, and also the Resistances and Watts absorbed per yard in each case (W. T. Glover and Co.).

S. W. G.	Ohms per yard at 15·5° C.	50° C. rise in temperature.			100° C. rise in temperature.			150° C. rise in temperature.			200° C. rise in temperature.		
		Ohms per yard at 65·5° C.	Amperes giving a rise of 50° C.	Watts consumed per yard.	Ohms per yard at 115·5° C.	Amperes giving a rise of 100° C.	Watts consumed per yard.	Ohms per yard at 165·5° C.	Amperes giving a rise of 150° C.	Watts consumed per yard.	Ohms per yard at 215·5° C.	Amperes giving a rise of 200° C.	Watts consumed per·yard.
8	·0541	·0571	20·1	23·1	·0601	33·0	65·5	·0631	39	96	·0661	53	186
9	·0669	·0706	17·8	22·4	·0743	28·1	58·6	·0780	34	90	·0817	43	151
10	·0845	·0890	15·1	20·3	·0938	22·8	48·75	·0984	27·8	76·00	·103	35·6	131
11	·1024	·1080	13·2	18·8	·1137	19·6	43·70	·119	23·7	66 7	·125	30·5	116
12	·1290	·136	11·3	17·4	·143	16·6	39·35	·150	20·2	61·2	·157	26·0	106
13	·1646	·174	9·4	15·4	·183	13·7	34·35	·192	16·6	52·8	·201	21·7	95
14	·2180	·230	7·6	13·3	·242	11·6	32·55	·254	13·9	49·0	·266	18·0	86
15	·2670	·282	6·7	12·7	·296	10·0	29·60	·311	12·1	45·5	·326	16·0	83
16	·3382	·357	5·7	11·6	·377	8·60	27·90	·394	10·5	43·4	·413	13·4	74
17	·4406	·465	4·7	10·3	·489	7·20	25·4	·513	8·8	39·7	·547	11·3	70
18	·6008	·634	4·0	10·15	·667	5·93	23·5	·700	7·35	37·8	·733	9·27	63
19	·8677	·915	3·2	9·38	·963	4·80	22·2	1·01	5·91	35·3	1·06	7·45	59
20	1·068	1·13	2·84	9·12	1·19	4·25	21·5	1·24	5·25	34·2	1·30	6·66	58
21	1·3528	1·43	2·5	8·94	1·50	3·75	21·1	1·59	4·61	33·8	1·65	5·80	56
22	1·7666	1·86	2·15	8·60	1·96	3·30	21·4	2·06	4·03	33·5	2·16	5·07	55·5
23	2·4030	2·54	1·83	8·50	2·67	2·80	20·9	2·80	3·44	33·2	2·93	4·33	55·0
24	2·8658	3·02	1·62	7·93	3·18	2·53	20·4	3·34	3·12	32·5	3·50	3·96	54·8
25	3·4665	3·66	1·49	8·13	3·85	2·29	20·2	4·04	2·82	32·2	4·23	3·57	54·0
26	4·2764	4·51	1·32	7·86	4·75	2·05	20·0	4·98	2·52	31·6	5·22	3·19	53·2
27	5·1531	5·44	1·2	7·83	5·72	1·84	19·4	6·00	2·29	31·4	6·29	2·90	52·8
28	6·3279	6·68	1·07	7·65	7·02	1·65	19·1	7·37	2·05	31·0	7·73	2·59	52·3
29	7·4938	7·91	·98	7·59	8·32	1·51	19·0	8·73	1·87	30·4	9·14	2·36	50·8
30	9·01125	9·51	·895	7·61	10·00	1·36	18·5	10·50	1·69	29·9	11·00	2·15	50·8

Table XXVI.

Relation between gauges of covered Manganin wire and currents giving a rise of Temperature of 100° C. above the Outside Air (W. T. Glover and Co.).

No. B.W.G.	Amperes.	No. B.W.G.	Amperes.	No. B.W.G.	Amperes.	No. B.W.G.	Amperes.
2	50·6	12	12·0	22	1·57	32	0·386
3	49·0	13	9·8	23	1·32	33	0·335
4	39·0	14	7·97	24	1·093	34	0·31
5	34·6	15	6·47	25	0·952	35	0·272
6	30·6	16	5·55	26	0·807	36	0·235
7	25·5	17	4·68	27	0·677	37	0·209
8	22·5	18	3·63	28	0·555	38	0·188
9	19·0	19	2·90	29	0·497	39	0·167
10	16·4	20	2·2	30	0·440	40	0·148
11	13·9	21	1·9	31	0·410		

Note.—For bare wires in air Mr. L. B. Atkinson allows 1 sq. in. of total external surface per Watt to be dissipated for a temperature of 150° C.

Table XXVII.

Showing the Currents that will Produce 212° F. (100° C.) rise in Temperature above the Surrounding Air for bare Manganin wire stretched horizontally and freely exposed to the Air (W. T. Glover and Co.).

S. W. G . . .	No.	8	9	10	11	12	13	14
Amperes. . .		60	50	40	35	30	25	20
S.W. G. . . .	No.	15	16	17	18	19	20	21
Amperes . . .		15	12	10	9	8	6	5

Note.—If placed vertically or coiled, an allowance must be made.

Table XXVIII.

Densities of Dry Air in lbs. per cubic foot at different Temperatures and Pressures calculated by the Relation (lbs. per cubic foot) $= \left\{ \frac{0 \cdot 001293}{1 + 0 \cdot 00367 \times T} \times \frac{H}{760} \right\} \times 62 \cdot 43$, where T = Temp. in ° C. and H = Pressure in mm. of Mercury at 0° C., Lat. 45°, $g = 980 \cdot 62$. $0 \cdot 001293 =$ Density in grams per c.c. at 0° C. and 760 mm. Pressure of Mercury. 1 gram per c.c. = 62·43 lbs. per cubic foot.

Temp. T° C.	Barometric Pressure in Millimetres of Mercury = H.							
	710 mm. 27·95 in.	720 28·34	730 28·74	740 29·13	750 29·52	760 29·92	770 30·31	780 30·70
0	0·07541	0·07647	0·07754	0·07859	0·07968	0·08072	0·08179	0·08285
5	7406	7509	7617	7716	7821	7929	8030	8134
10	7274	7381	7480	7578	7685	7784	7893	7991
15	7148	7249	7348	7447	7554	7647	7754	7847
20	7023	7124	7223	7323	7425	7521	7617	7716
25	6904	7008	7098	7197	7300	7392	7492	7584
30	6794	6885	6984	7079	7173	7274	7367	7461

Table XXIX.

Comparison of Wire Gauges in Common Use.

No.	S.W.G. inch.	B.W.G. inch.	B. & S. inch.	No.	S.W.G. inch.	B W.G. inch.	B. & S. inch.	No.	S.W.G. inch.	B.W.G. inch.	B. & S. inch.
4/0	·400	·454	·4600	15	·072	·072	·0571	33	·0100	·008	·0071
3/0	·372	·425	·4096	16	·064	·065	·0508	34	·0092	·007	·0063
2/0	·348	·380	·3648	17	·056	·058	·0453	35	·0084	·005	·0056
0	·324	·340	·3249	18	·048	·049	·0403	36	·0076	·004	·0050
1	·300	·300	·2893	19	·040	·042	·0359	37	·0068		·0045
2	·276	·284	·2576	20	·036	·035	·0320	38	·0060		·0040
3	·252	·259	·2294	21	·032	·032	·0285	39	·0052		·0035
4	·232	·230	·2043	22	·028	·028	·0253	40	·0048		·0031
5	·212	·220	·1819	23	·024	·025	·0226	41	·0044		
6	·192	·203	·1620	24	·022	·022	·0201	42	·0040		
7	·176	·180	·1443	25	·020	·020	·0179	43	·0036		
8	·160	·165	·1285	26	·018	·018	·0159	44	·0032		
9	·144	·148	·1144	27	·0164	·016	·0142	45	·0028		
10	·128	·134	·1019	28	·0148	·014	·0126	46	·0024		
11	·116	·120	·0907	29	·0136	·013	·0113	47	·0020		
12	·104	·109	·0808	30	·0124	·012	·0100	48	·0016		
13	·092	·095	·0720	31	·0116	·010	·0089	49	·0012		
14	·080	·083	·0641	32	·0108	·009	·0079	50	·0010		

Photometer Bench.

Table XXX.

Ratios of Squares of Distances from Screen to Sources of Light for different Distances (D) between the latter (for facilitating Calculations of Candle Powers).

C.P. to be determined = C.P. of standard $\times \frac{(D-d)^2}{d^2}$ where (d) = distance between screen and standard.

Note.—Intermediate values may very approximately be found by proportion, but should be obtained from a curve between d and $\frac{(D-d)^2}{d^2}$ when required more accurately.

$\frac{(D-d)^2}{d^2}$	$D=$ 200	300	400	500	600	$\frac{(D-d^2)}{d^2}$	$D=$ 200	300	400	500	600
	d	d	d	d	d		d	d	d	d	d
81·00	20·0	30·0	40	50·0	60·0	17·04	39·0	58·5	78	97·5	117·0
76·67	0·5	0·75	1	1·25	1·5	16·51	9·5	9·25	9	8·75	8·5
72·68	1·0	1·5	2	2·50	3·0	16·00	40·0	60·0	80	100·0	120·0
68·93	1·5	2·25	3	3·75	4·5	15·51	0·5	0·75	1	1·25	1·5
65·46	2·0	3·0	4	5·0	6·0	15·04	1·0	1·5	2	2·5	3·0
62·23	2·5	3·75	5	6·25	7·5	14·59	1·5	2·25	3	3·75	4·5
59·24	3·0	4·5	6	7·5	9·0	14·15	2·0	3·0	4	5·0	6·0
56·41	3·5	5·25	7	8·75	70·5	13·74	2·5	3·75	5	6·25	7·5
53·77	4·0	6·0	8	60·0	2·0	13·32	3·0	4·5	6	7·5	9·0
51·31	4·5	6·75	9	1·25	3·5	12·94	3·5	5·25	7	8·75	130·5
49·00	5·0	7·5	50	2·5	5·0	12·57	4·0	6·0	8	110·0	2·0
46·81	5·5	8·25	1	3·75	6·5	12·21	4·5	6·75	9	1·25	3·5
44·78	6·0	9·0	2	5·0	8·0	11·87	5·0	7·5	90	2·5	5·0
42·85	6·5	9·75	3	6·25	9·5	11·53	5·5	8·25	1	3·75	6·5
41·04	7·0	40·5	4	7·5	81·0	11·21	6·0	9·0	2	5·0	8·0
39·33	7·5	1·25	5	8·75	2·5	10·89	6·5	9·75	3	6·25	9·5
37·73	8·0	2·0	6	70·0	4·0	10·59	7·0	70·5	4	7·5	141·0
36·21	8·5	2·75	7	1·25	5·5	10·30	7·5	1·25	5	8·75	2·5
34·77	9·0	3·5	8	2·5	7·0	10·02	8·0	2·0	6	120·0	4·0
33·40	9·5	4·25	9	3·75	8·5	9·754	8·5	2·75	7	1·25	5·5
32·11	30·0	5·0	60	5·0	90·0	9·497	9·0	3·5	8	2·5	7·0
30·89	0·5	5·75	1	6·25	1·5	9·247	9·5	4·25	9	3·75	8·5
29·72	1·0	6·5	2	7·5	3·0	9·00	50·0	5·0	100	5·0	150·0
28·62	1·5	7·25	3	8·75	4·5	8·535	1	6·5	2	7·5	3
27·56	2·0	8·0	4	80·0	6·0	8·102	2	8·0	4	130·0	6
26·56	2·5	8·75	5	1·25	7·5	7·691	3	9·5	6	2·5	9
25·61	3·0	9·5	6	2·5	9·0	7·300	4	81·0	8	5·0	162
24·69	3·5	50·25	7	3·75	100·5	6·950	5	2·5	110	7·5	5
23·83	4·0	1·0	8	5·0	2·0	6·613	6	4·0	2	140·0	8
23·01	4·5	1·75	9	6·25	3·5	6·295	7	5·5	4	2·5	171
22·22	5·0	2·5	70	7·5	5·0	5·985	8	7·0	6	5·0	4
21·47	5·5	3·25	1	8·75	6·5	5·710	9	8·5	8	7·5	7
20·75	6·0	4·0	2	90·0	8·0	5·44	60	90·0	120	150·0	180
20·06	6·5	4·75	3	1·25	9·5	5·193	1	1·5	2	2·5	3
19·41	7·0	5·5	4	2·5	111·0	4·909	2	3·0	4	5·0	6
18·78	7·5	6·25	5	3·75	2·5	4·730	3	4·5	6	7·5	9
18·17	8·0	7·0	6	5·0	4·0	4·51	4	6·0	8	160·0	192
17·60	8·5	7·75	7	6·25	5·5	4·313	5	7·5	130	2·5	5

$\frac{(D-d)^2}{d^2}$	D = 200	D = 300	D = 400	D = 500	D = 600	$\frac{(D-d)^2}{d^2}$	D = 200	D = 300	D = 400	D = 500	D = 600
	d	d	d	d	d		d	d	d	d	d
4·123	66·0	99·0	132	165·0	198·0	0·0918	153·5	230·25	307	383·75	460·5
3·941	7	100·5	4	7·5	201	0·0892	4·0	1·0	8	5·0	2·0
3·769	8	2·0	6	170·0	4	·0867	4·5	1·75	9	6·25	3·5
3·606	9	3·5	8	2·5	7	·0842	5·0	2·5	310	7·5	5·0
3·45	70·0	5·0	140	5·0	210	·0819	5·5	3·25	1	8·75	6·5
3·091	2·5	8·75	145	181·25	7·5	0·0795	6·0	4·0	2	390·0	8·0
2·777	5·0	112·5	150	7·5	225·0	·0773	6·5	4·75	3	1·25	9·5
2·500	7·5	6·25	155	193·75	232·5	·0751	7·0	5·5	4	2·5	471·0
2·25	80·0	120·0	160	200·0	240·0	·0728	7·5	6·25	5	3·75	2·5
2·029	2·5	3·75	165	206·25	7·5	·0707	8·0	7·0	6	5·0	4·0
1·831	5·0	7·5	170	212·5	255·0	0·0685	8·5	7·75	7	6·25	5·5
1·654	7·5	131·25	175	218·75	262·5	·0665	9·0	8·5	8	7·5	7·0
1·49	90·0	5·0	180	225·0	270·0	·0645	9·5	9·25	9	8·75	8·5
1·350	2·5	8·75	185	231·25	7·5	·0625	160·0	240·0	320	400·0	480·0
1·222	5·0	142·5	190	237·5	285·0	·0606	0·5	0·75	1	1·25	1·5
1·106	7·5	6·25	195	243·75	292·5	0·0587	1·0	1·5	2	2·5	3·0
1	100·0	150·0	200	250·0	300·0	·0568	1·5	2·25	3	3·75	4·5
0·9045	2·5	3·75	205	256·25	7·5	·0551	2·0	3·0	4	5·0	6·0
0·8185	5·0	7·5	210	262·5	315·0	·0533	2·5	3·75	5	6·25	7·5
·7407	7·5	161·25	215	268·75	322·5	·0515	3·0	4·5	6	7·5	9·0
0·6711	110·0	5·0	220	275·0	330·0	0·0499	3·5	5·25	7	8·75	490·5
·6046	2·5	8·75	225	281·25	7·5	·0482	4·0	6·0	8	410·0	2·0
0·5461	5·0	172·5	230	287·5	345·0	·0466	4·5	6·75	9	1·25	3·5
·4929	7·5	6·25	235	293·75	352·5	·0450	5·0	7·5	330	2·5	5·0
0·44	120·0	180·0	240	300·0	360·0	·0435	5·5	8·25	1	3·75	6·5
·400	2·5	3·75	245	306·25	7·5	·0420	6·0	9·0	2	5·0	8·0
0·3601	5·0	7·5	250	312·5	375·0	·0405	6·5	9·75	3	6·25	9·5
·3236	7·5	191·25	255	8·75	382·5	0·0391	7·0	250·5	4	7·5	501·0
0·29	130·0	5·0	260	325·0	390·0	·0377	7·5	1·25	5	8·75	2·5
·2773	1	6·5	2	7·5	3	·0363	8·0	2·0	6	420·0	4·0
·2653	2	8·0	4	330·0	6	·0349	8·5	2·75	7	1·25	5·5
·2537	3	9·5	6	2·5	9	·0336	9·0	3·5	8	2·5	7·0
·2425	4	201·0	8	5·0	402	·0324	9·5	4·25	9	3·75	8·5
·2318	5	2·5	270	7·5	5	·03114	170·0	255·0	340	5·0	510·0
·2217	6	4·0	2	340·0	8	0·0299	0·5	5·75	1	6·25	1·5
·2114	7	5·5	4	2·5	411	·0288	1·0	6·5	2	7·5	3·0
·2037	8	7·0	6	5·0	4	·0276	1·5	7·25	3	8·75	4·5
·1926	9	8·5	8	7·5	7	·0265	2·0	8·0	4	430·0	6·0
0·1838	140·0	210·0	280	350·0	420	·0254	2·5	8·75	5	1·25	7·5
·1751	1	1·5	2	2·5	3	·0244	3·0	9·5	6	2·5	9·0
·1670	2	3·0	4	5·0	6	·0233	3·5	260·25	7	3·75	520·5
·159	3	4·5	6	7·5	9	·0223	4·0	1·0	8	5·0	2·0
·1512	4	6·0	8	360·0	432	·0214	4·5	1·75	9	6·25	3·5
·1439	5	7·5	290	2·5	5	·0204	5·0	2·5	350	7·5	5·0
·1370	6	9·0	2	5·0	8	0·0195	5·5	3·25	1	8·75	6·5
·1300	7	220·5	4	7·5	441	·0186	6·0	4·0	2	440·0	8·0
·1235	8	2·0	6	370·0	4	·0177	6·5	4·75	3	1·25	9·5
·1172	9	3·5	8	2·5	7	·0169	7·0	5·5	4	2·5	531·0
0·1111	150·0	5·0	300	5·0	450	·0161	7·5	6·25	5	3·75	2·5
·1081	0·5	5·75	1	6·25	1·5	·0153	8·0	7·0	6	5·0	4·0
·1053	1·0	6·5	2	7·5	3·0	·0145	8·5	7·75	7	6·25	5·5
·1025	1·5	7·25	3	8·75	4·5	·0138	9·0	8·5	8	7·5	7·0
·0982	2·0	8·0	4	380·0	6·0	·0130	9·5	9·25	9	8·75	8·5
·0971	2·5	8·75	5	1·25	7·5	0·0124	180·0	270·0	360	450·0	540·0
·0944	3·0	9·5	6	2·5	9·0						

TABLE XXXI.

TABLE OF SIZES, WEIGHTS AND RESISTANCES FOR SOME IMPORTANT HIGH RESISTANCE MATERIALS AT 60° F.

Size.			Weight per 1000 Yds.	Yds. to the lb.			Copper.		German Silver.		Platinoid.		Eureka.		Manganin.		Reostene.	Iron.
				Bare.	Covered (approx.)		Resistance bare (approx).		Resistance bare (approx.)		Resistance bare (approx.)		Resistance.		Resistance.		Resistance.	Resistance bare.
L.S.G.	Inch.	m/m.			Silk covd.	Cottn covd.	per. lb.	pr 1000 yds.	per lb.	per 1000 yds.	per lb.	per 1000 yds.	per lb.	per 1000 yds.	per lb.	per 1000 yds.	per 1000 yds.	per lb.
			lbs.				ohms.	ohms.	ohms.	ohms.	ohms.	ohms.	ohms.	ohms.	ohms.	ohms.	ohms.	ohms.
8	·160	4·064	232·47	4·3	4·25	4·25	·0051	1·178	·068	15·929	·1241	28·852	·152	35·4	...	...	54·1	...
9	·144	3·658	188·30	5·3	5·18	5·02	·0077	1·455	·104	19·665	...	...	...	...	...	...	66·9	...
10	·128	3·251	148·78	6·7	6·56	6·36	·0124	1·842	·167	24·890	·3031	45·084	·372	55·4	...	...	84·5	...
11	·116	2·946	122·19	8·2	8·06	7·85	·0183	2·242	·248	30·305	...	...	...	...	...	...	102·4	...
12	·104	2·642	98·22	10·2	10·02	9·82	·0284	2·788	·384	37·702	·6957	68·292	·855	66·6	...	...	129·0	...
13	·092	2·336	76·86	13·0	12·70	12·50	·0464	3·564	·627	48·179	...	...	...	...	...	...	164·6	...
14	·080	2·032	58·12	17·2	16·90	16·70	·0811	4·714	1·090	63·719	1·9869	115·416	2·442	141·8	2·027	117·85	218·0	0·457
15	·072	1·829	47·08	21·2	20·67	19·20	·1237	5·819	1·670	78·662	...	...	...	...	...	...	267·0	·806
16	·064	1·626	37·19	26·9	26·30	24·80	·1981	7·367	2·670	99·561	4·8520	180·338	5·963	221·6	4·952	184·17	338·2	1·215
17	·056	1·422	28·48	35·1	34·20	32·40	·3379	9·620	4·560	130·033	...	...	...	...	...	...	440·6	1·916
18	·048	1·219	20·92	47·8	47·00	45·00	·6261	13·097	8·460	176·99	15·3312	320·601	18·840	394·1	15·652	327·42	600·8	3·762
19	·040	1·016	14·53	68·8	71·00	69·00	1·298	18·857	17·550	254·87	31·7952	461·664	39·070	567·3	...	...	867·7	6·970
20	·036	·914	11·77	85·0	83·00	80·00	1·979	23·280	26·750	314·66	48·4602	569·952	59·550	700·4	49·475	582·0	1068·0	14·455
21	·032	·813	9·29	107·6	105·20	100·30	3·169	21·490	42·850	398·25	77·6480	721·368	95·430	886·5	...	...	1352·8	20·684
22	·028	·711	7·12	140·4	138·40	129·40	5·407	38·480	73·110	520·16	132·4272	942·192	162·700	1157	135·175	962·0	1766·6	35·288
23	·024	·610	5·23	191·2	188·00	179·00	10·017	52·390	135·4	707·98	245·3280	1282·392	301·500	1576	...	...	2403·0	55·525
24	·022	·559	4·39	227·8	224·30	215·30	14·188	62·410	191·8	843·58	347·4720	1526·184	427·000	1875	354·700	1560·25	2865·8	92·588
25	·020	·508	3·63	275·5	272·20	262·70	20·772	75·480	280·8	1019·50	508·7280	1846·656	625·200	2269	...	...	3466·5	135·558
26	·018	·457	2·94	340·1	336·70	316·70	31·659	93·200	428·1	1258·60	775·3680	2279·808	952·900	2802	791·475	2330·0	4276·5	206·613
27	·0164	·417	2·44	409·8	398·80	376·80	45·943	112·300	621·2	1516·20	1125·2160	2746·440	1382	3375	...	...	5153·1	330·950
28	·0148	·376	1·99	502·8	490·80	467·20	69·270	137·7	937	1851·7	1696·4880	3372·264	2085	4144	1731·750	3442·5	6327·9	554·592
30	·0124	·315	1·40	716·3	694·10	670·10	140·570	196·3	1900	2652·1	3442·8000	4803·984	4231	5904	3514·25	4907·5	9011·25	1045·978
32	·0108	·274	1·06	944·3	914·20	870·20	244·290	258·7	3302	3496·2	5982·7200	6332·904	7352	7783	6107·25	6467·5	...	1490·08
34	·0092	·2337	·77	1301	1239	1163	463·910	356·5	6272	4818·1	11362	8727·120	13964	10725	11597·75	8912·5	...	2678·47
36	·0076	·1930	·52	1906	1791	1613	996·180	522·4	13469	7060·3	24398	12789·640	29985	15717	24904·50	13060·0	...	5601·09
38	·0060	·1524	·33	3059	2806	...	2564	838·2	34673	11327	62805	20518·560	77187	25217	64100·0	20955·0	...	10202·07
40	·0048	·1219	·21	4800	4365	...	6261	1315	84652	17699	153333	32060·160	188440	39403	156525·0	32875·0	...	19277·8
42	·0040	·1016	·145	6882	5743	...	12982	1886	175509	25487	317904	46166	390700	56740	324550·0	47150·0	...	85166·9
44	·0032	·0813	·093	10755	8297	...	31695	2947	428571	39825	784280	72136	963880	88654	792375·0	73675·0	...	...
46	·0024	·0610	·052	19120	13712	...	100171	5239	1354415	70798	2453280	128239	3015100	157600	...	...	...	...
47	0020	·0508	·036	27548	20700	...	207720	7548	2808602	101950	5087280	184665	6252300	226950	...	...	...	...

Resistance of Copper calculated on a basis of 100% of pure Copper at 60° F. German Silver calculated on F. Jenkins' Table.

Table XXXII.

The Table given below shows the sizes of Various Wires of Different Materials which will fuse at the Currents given in the first column (Sir W. H. Preece).

Current in Amperes.	Tin Wire.		Lead Wire.		Copper Wire.		Iron Wire.	
	Diameter Inches.	Approx. S.W.G.	Diameter Inches.	Approx. S.W.G.	Diameter Inches.	Approx. S.W.G.	Diameter Inches.	Approx. S.W.G.
1	0·0072	36	0·0081	35	0·0021	47	0·0047	40
2	0·0113	31	0·0128	30	0·0034	43	0·0074	36
3	0·0149	28	0·0168	27	0·0044	41	0·0097	33
4	0·0181	26	0·0203	25	0·0053	39	0·0117	31
5	0·0210	25	0·0236	23	0·0062	38	0·0136	29
10	0·0334	21	0·0375	20	0·0098	33	0·0216	24
15	0·0437	19	0·0491	18	0·0129	30	0·0283	22
20	0·0529	17	0·0595	17	0·0156	28	0·0343	20·5
25	0·0614	16	0·0690	15	0·0181	26	0·0398	19
30	0·0694	15	0·0779	14	0·0205	25	0·0450	18·5
35	0·0769	14·5	0·0864	13·5	0·0227	24	0·0498	18
40	0·0840	13·5	0·0944	13	0·0248	23	0·0545	17
45	0·0909	13	0·1021	12	0·0268	22	0·0589	16·5
50	0·0975	12·5	0·1095	11·5	0·0288	22	0·0632	16
60	0·1101	11	0·1237	10	0·0325	21	0·0714	15
70	0·1220	10	0·1371	9·5	0·0360	20	0·0791	14
80	0·1334	9·5	0·1499	8·5	0·0394	19	0·0864	13·5
90	0·1443	9	0·1621	8	0·0426	18·5	0·0935	13
100	0·1548	8·5	0·1739	7	0·0457	18	0·1003	12
120	0·1748	7	0·1964	6	0·0516	17·5	0·1133	11
140	0·1937	6	0·2176	5	0·0572	17	0·1255	10
160	0·2118	5	0·2379	4	0·0625	16	0·1372	9·5
180	0·2291	4	0·2573	3	0·0676	16	0·1484	9
200	0·2457	3·5	0·2760	2	0·0725	15	0·1592	8
250	0·2851	1·5	0·3203	0	0·0841	13·5	0·1848	6·5

Note.—The above numbers can only be taken as approximate, as the actual current required to fuse any gauge will depend on the length of fuse and cooling effects of the fuse block in which it is placed.

In Allo-tin diameters 3·0 % greater than those of lead fuse at the *same* currents.

Useful Numbers.

Metres	×	3·2809	=	feet.
Feet	×	0·3048	=	metres.
Centimetres	×	0·3937	=	inches.
Inches	×	2·54	=	cms.
Mils.	×	0·0254	=	millimetres.
Sq. cms.	×	0·155	=	sq. inches.
Sq. inches	×	0·00155	=	sq. mm.
Sq. inches	×	6·451	=	sq. cms.
Cubic ,,	×	16·387	=	cub. ,,
,, cms.	×	0·061027	=	,, inches.

Kilogrammes	×	2·2046	=	pounds.
Miles	×	1·609	=	kilometres.
Kilometres	×	1094	=	yards.
Pounds (avoir.)	×	0·4536	=	kilogrammes.
Gallons		0·1604	=	cubic ft.
Cubic ft.		6·2355	=	gallons.
Gallons (water)	×	10·0	=	pounds.
Cub. ft. („)	×	62·27	=	„
Metres	×	39·3704	=	inches.
Foot	×	30·4797	=	cms.
Pounds (avoir.)	×	453·593	=	grammes.
Revs. per sec.	×	6·2832	=	radians per sec.
Feet „ min.	×	0·005	=	metres per „
Metres „ sec.	×	197	=	ft. per min.

Joules Equivalent = 1390 lb. cent. units
= $4{\cdot}156 \times 10^7$ ergs per gram. °C.

Acceleration due to Gravity (g) at Greenwich
= 32·1908 ft. sec. units
= 981·17 cm. „ „

Density of mercury = 13·596 grammes per c.c.

Watts	×	0·7373	=	foot lbs. per sec.
Joules (*i. e.* Watt secs.)	×	10^7	=	ergs.
„ „	×	0·7373	=	ft. lbs.
„ „	×	0·239	=	calorics.
Calorics	×	4·158	=	Joules.
Ft. lbs.	×	1·35	=	„
Kilogrammeters	×	7·233	=	ft. lbs.
Ft. lbs.	×	0·138	=	kilogrammeters.
Horse-power	×	746	=	Watts.
„ „	×	33000	=	ft. lbs. per min.
„ „	×	550	=	„ „ sec.
„ „	×	76	=	kilogrammeters per sec.
Watts	×	44·25	=	ft. lbs. per min.
„	×	0 1	=	kilogrammeters per sec.
H.P. hours	×	1980000	=	ft. lbs.
„ „	×	2685600	=	Joules.
Kilowatt hours	×	1·34	=	H.P. hours.
„ „ „	×	2656400	=	ft. lbs.

Length of circumference of circle radius (r) = $2\pi r = \pi d$.

Area of circumference of circle radius $(r) = \pi r^2 = \dfrac{\pi d^2}{4} = \cdot 7854 d^2$.

Ratio of circumference of circle to its diam.

$$(\pi) = 3\cdot 14159 = \tfrac{22}{7} \text{ approx.}$$

$$\log.\ \pi = 0\cdot 4971499.$$

Base of hyperbolic or Napierian logarithms $\epsilon = 2\cdot 71828 \ldots$

To convert *Napierian into Common logarithms* multiply by 0·43429.

To convert *Common into Napierian logarithms* multiply by 2·30258.

Lbs. per yard of pure copper wire = area in sq. ins. × 11·5625.

Ohms per yard of pure copper wire at 60° F. (15·5°C.)

= 0·0000244657 ÷ area in sq. ins.

Pounds per 1000 yards + 10 % = pounds per kilometre.

„ „ 1000 „ ÷ 2 = kilograms per „

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0820	0804	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1401	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7859	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8170	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Anti-logarithms.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTI-LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

SQUARES OF NUMBERS FROM 1 TO 10,000, CORRECT TO FOUR SIGNIFICANT FIGURES.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1000	1020	1040	1061	1082	1102	1124	1145	1166	1188	3	5	7	9	11	13	15	17	19
11	1210	1232	1254	1277	1300	1323	1346	1369	1392	1416	3	5	7	10	12	14	17	19	21
12	1440	1464	1488	1513	1538	1563	1588	1613	1638	1664	3	5	8	10	13	15	18	20	23
13	1690	1716	1742	1769	1796	1823	1850	1877	1904	1932	3	6	9	11	14	17	19	22	25
14	1960	1988	2016	2045	2074	2103	2132	2161	2190	2220	3	6	9	12	15	18	21	24	27
15	2250	2280	2310	2341	2372	2403	2434	2465	2496	2528	4	7	10	13	16	19	22	25	28
16	2560	2592	2624	2657	2690	2723	2756	2789	2822	2856	4	7	10	14	17	20	24	27	30
17	2890	2924	2958	2993	3028	3063	3098	3133	3168	3204	4	7	11	14	18	21	25	28	32
18	3240	3276	3312	3349	3386	3423	3460	3497	3534	3572	4	8	12	15	19	23	26	30	34
19	3610	3648	3686	3725	3764	3803	3842	3881	3920	3960	4	8	12	16	20	24	28	32	36
20	4000	4040	4080	4121	4162	4203	4244	4285	4326	4368	5	9	13	17	21	25	29	33	37
21	4410	4452	4494	4537	4580	4623	4666	4709	4752	4796	5	9	13	18	22	26	31	34	39
22	4840	4884	4928	4973	5018	5063	5108	5153	5198	5244	5	9	14	18	23	27	32	36	41
23	5290	5336	5382	5429	5476	5523	5570	5617	5664	5712	5	10	15	19	24	27	33	38	43
24	5760	5808	5856	5905	5954	6003	6052	6101	6150	6200	5	10	15	20	25	30	35	40	45
25	6250	6300	6350	6401	6452	6503	6554	6605	6656	6708	6	11	16	21	26	31	36	41	46
26	6760	6812	6864	6917	6970	7023	7076	7129	7182	7236	6	11	16	22	27	32	38	43	48
27	7290	7344	7398	7453	7508	7563	7618	7673	7728	7784	6	11	17	22	28	33	39	44	50
28	7840	7896	7952	8009	8066	8123	8180	8237	8294	8352	6	12	18	23	29	35	40	46	52
29	8410	8468	8526	8585	8644	8703	8762	8821	8880	8940	6	12	18	24	30	36	42	48	54
30	9000	9060	9120	9181	9242	9303	9364	9425	9486	9548	7	13	19	25	31	37	43	49	55
31	9610	9672	9734	9797	9860	9923	9986	1005^1	1011^1	1018^1	7	13	19	26	32	38	45	51	57
32	1024	1030	1037	1043	1050	1056	1063	1069	1076	1082	1	1	2	3	3	4	5	5	6
33	1089	1096	1102	1109	1116	1122	1129	1136	1142	1149	1	1	2	3	4	4	5	6	6
34	1156	1163	1170	1176	1183	1190	1197	1204	1211	1218	1	2	2	3	4	4	5	6	6
35	1225	1232	1239	1246	1253	1260	1267	1273	1282	1289	1	2	2	3	4	4	5	6	7
36	1296	1303	1310	1318	1325	1332	1340	1347	1354	1362	1	2	2	3	4	5	5	6	7
37	1369	1376	1384	1391	1399	1406	1414	1421	1429	1436	1	2	2	3	4	5	5	6	7
38	1444	1452	1459	1467	1475	1482	1490	1498	1505	1513	1	2	2	3	4	5	6	6	7
39	1521	1529	1537	1544	1552	1560	1568	1576	1584	1592	1	2	3	3	4	5	6	6	7
40	1600	1608	1616	1624	1632	1640	1648	1656	1665	1673	1	2	3	3	4	5	6	7	7
41	1681	1689	1697	1706	1714	1722	1731	1739	1747	1756	1	2	3	3	4	5	6	7	8
42	1764	1772	1781	1789	1798	1806	1815	1823	1832	1840	1	2	3	4	4	5	6	7	8
43	1849	1858	1866	1875	1884	1892	1901	1910	1918	1927	1	2	3	4	5	5	6	7	8
44	1936	1945	1954	1962	1971	1980	1989	1998	2007	2016	1	2	3	4	5	5	6	7	8
45	2025	2034	2043	2052	2061	2070	2079	2088	2098	2107	1	2	3	4	5	6	7	7	8
46	2116	2125	2134	2144	2153	2162	2172	2181	2190	2200	1	2	3	4	5	6	7	8	9
47	2209	2218	2228	2237	2247	2256	2266	2275	2285	2294	1	2	3	4	5	6	7	8	9
48	2304	2314	2323	2333	2343	2352	2362	2372	2381	2391	1	2	3	4	5	6	7	8	9
49	2401	2411	2421	2430	2440	2450	2460	2470	2480	2490	1	2	3	4	5	6	7	8	9
50	2500	2510	2520	2530	2540	2550	2560	2570	2581	2591	1	2	3	4	5	6	7	8	9
51	2601	2611	2621	2632	2642	2652	2663	2673	2683	2694	1	2	3	4	5	6	7	8	9
52	2704	2714	2725	2735	2746	2756	2767	2777	2788	2798	1	2	3	4	5	6	8	9	10
53	2809	2820	2830	2841	2852	2862	2873	2884	2894	2905	1	2	3	4	6	7	8	9	10
54	2916	2927	2938	2948	2959	2970	2981	2992	3003	3014	1	2	3	5	6	7	8	9	10

Squares from 1 to 3 contain 1 figure.
" " 4 to 9 " 2 figures.
" " 10 to 31 " 3 "
" 32 to 99 " 4 "

Squares from 100 to 316 contain 5 figures.
" " 317 to 999 " 6 "
" " 1000 to 3162 " 7 "
" " 3163 to 10000 " 8 "

The differences for squares from 3171 to 3199 are 1, 1, 2, 3, 3, 4, 5, 5, 6.

Squares of Numbers from 1 to 10,000, Correct to Four Significant Figures.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	3025	3036	3047	3058	3069	3080	3091	3102	3114	3125	1	2	3	5	6	7	8	9	10
56	3136	3147	3158	3170	3181	3192	3204	3215	3226	3238	1	2	4	5	6	7	8	9	10
57	3249	3260	3272	3283	3295	3306	3318	3329	3341	3352	1	2	4	5	6	7	8	9	11
58	3364	3376	3387	3399	3411	3422	3434	3446	3457	3469	1	2	4	5	6	7	8	10	11
59	3481	3493	3505	3516	3528	3540	3552	3564	3576	3588	1	3	4	5	6	7	8	10	11
60	3600	3612	3624	3636	3648	3660	3672	3684	3697	3709	1	3	4	5	6	7	9	10	11
61	3721	3733	3745	3758	3770	3782	3795	3807	3819	3832	1	3	4	5	6	8	9	10	11
62	3844	3856	3869	3881	3894	3906	3919	3931	3944	3956	1	3	4	5	6	8	9	10	11
63	3969	3982	3994	4007	4020	4032	4045	4058	4070	4083	1	3	4	5	7	8	9	10	12
64	4096	4109	4122	4134	4147	4160	4173	4186	4199	4212	1	3	4	5	7	8	9	10	12
65	4225	4238	4251	4264	4277	4290	4303	4316	4330	4343	1	3	4	5	7	8	9	11	12
66	4356	4369	4382	4396	4409	4422	4436	4449	4462	4476	1	3	4	5	7	8	9	11	12
67	4489	4502	4516	4529	4543	4556	4570	4583	4597	4610	2	3	4	6	7	8	10	11	12
68	4624	4638	4651	4665	4679	4692	4706	4720	4733	4747	2	3	4	6	7	8	10	11	12
69	4761	4775	4789	4602	4816	4830	4844	4858	4872	4886	2	3	4	6	7	8	10	11	13
70	4900	4914	4928	4942	4956	4970	4984	4998	5013	5027	2	3	4	6	7	9	10	11	13
71	5041	5055	5069	5084	5098	5112	5127	5141	5155	5170	2	3	4	6	7	9	10	12	13
72	5184	5198	5213	5227	5242	5256	5271	5285	5300	5314	2	3	5	6	7	9	10	12	13
73	5329	5344	5358	5373	5388	5402	5417	5432	5446	5461	2	3	5	6	8	9	10	12	13
74	5476	5491	5506	5520	5535	5550	5565	5580	5595	5610	2	3	5	6	8	9	11	12	14
75	5625	5640	5655	5670	5685	5700	5715	5730	5746	5761	2	3	5	6	8	9	11	12	14
76	5776	5791	5806	5822	5837	5852	5868	5883	5898	5914	2	3	5	6	8	9	11	12	14
77	5929	5944	5960	5975	5991	6006	6022	6037	6053	6068	2	3	5	6	8	9	11	13	14
78	6084	6100	6115	6131	6147	6162	6178	6194	6209	6225	2	3	5	6	8	10	11	13	14
79	6241	6257	6273	6288	6304	6320	6336	6352	6368	6384	2	3	5	7	8	10	11	13	14
80	6400	6416	6432	6448	6464	6480	6496	6512	6529	6545	2	3	5	7	8	10	11	13	15
81	6561	6577	6593	6610	6626	6642	6659	6675	6691	6708	2	3	5	7	8	10	12	13	15
82	6724	6740	6757	6773	6790	6806	6823	6839	6856	6872	2	3	5	7	8	10	12	13	15
83	6889	6906	6922	6939	6956	6972	6989	7006	7022	7039	2	3	5	7	9	10	12	14	15
84	7056	7073	7090	7106	7123	7140	7157	7174	7191	7208	2	4	5	7	9	10	12	14	15
85	7225	7242	7259	7276	7293	7310	7327	7344	7362	7379	2	4	5	7	9	10	12	14	16
86	7396	7413	7430	7448	7465	7482	7500	7517	7534	7552	2	4	5	7	9	11	12	14	16
87	7569	7586	7604	7621	7639	7656	7674	7691	7709	7726	2	4	5	7	9	11	12	14	16
88	7744	7762	7779	7797	7815	7832	7850	7868	7885	7903	2	4	5	7	9	11	13	14	16
89	7921	7939	7957	7974	7992	8010	8028	8046	8064	8082	2	4	6	7	9	11	13	14	16
90	8100	8118	8136	8154	8172	8190	8208	8226	8245	8263	2	4	6	7	9	11	13	15	16
91	8281	8299	8317	8336	8354	8372	8391	8409	8427	8446	2	4	6	7	9	11	13	15	17
92	8464	8482	8501	8519	8538	8556	8575	8593	8612	8630	2	4	6	8	9	11	13	15	17
93	8649	8668	8686	8705	8724	8742	8761	8780	8798	8817	2	4	6	8	10	11	13	15	17
94	8836	8855	8874	8892	8911	8930	8949	8968	8987	9006	2	4	6	8	10	11	13	15	17
95	9025	9044	9063	9082	9101	9120	9139	9158	9178	9197	2	4	6	8	10	12	14	15	17
96	9216	9235	9254	9274	9293	9312	9332	9351	9370	9390	2	4	6	8	10	12	14	16	18
97	9409	9428	9448	9467	9487	9505	9526	9545	9565	9584	2	4	6	8	10	12	14	16	18
98	9604	9624	9643	9663	9683	9702	9722	9742	9761	9781	2	4	6	8	10	12	14	16	18
99	9801	9821	9841	9860	9880	9900	9920	9940	9960	9980	2	4	6	8	10	12	14	16	18

Squares from 1 to 3 contain 1 figure.
,, ,, 4 to 9 ,, 2 figures.
,, ,, 10 to 31 ,, 3 ,,
,, ,, 32 to 99 ,, 4 ,,

Squares from 100 to 316 contain 5 figures.
,, ,, 317 to 999 ,, 6 ,,
,, ,, 1000 to 3162 ,, 7 ,,
,, ,, 3163 to 10000 ,, 8 ,,

Reciprocals of Numbers from 1 to 9999.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	$\bar{9}901$	$\bar{9}804$	$\bar{9}709$	$\bar{9}615$	$\bar{9}524$	$\bar{9}434$	$\bar{9}346$	$\bar{9}259$	$\bar{9}174$	9	18	27	36	45	54	63	72	81
11	9091	9009	8929	8850	8772	8696	8621	8547	8475	8403	8	16	23	31	38	46	53	61	68
12	8333	8264	8197	8130	8065	8000	7937	7874	7813	7752	6	13	19	26	32	38	45	51	57
13	7692	7634	7576	7519	7463	7407	7353	7299	7246	7194	5	11	16	22	27	32	38	43	49
14	7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5	9	14	19	23	28	33	37	42
15	6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	5	9	13	17	21	25	29	34	38
16	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	8	11	15	19	22	26	30	33
17	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	6	10	13	16	19	23	26	29
18	5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3	6	8	11	14	17	20	23	26
19	5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3	5	8	10	13	16	18	21	23
20	5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19	21
21	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	6	9	11	13	15	17	19
22	4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	11	13	15	17
23	4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2	3	5	7	9	11	12	14	16
24	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	4	5	7	9	10	12	14	15
25	4000	3984	3968	3953	3937	3921	3906	3891	3876	3861	1	3	4	6	7	9	10	12	13
26	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1	2	4	5	7	8	9	11	12
27	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	2	4	5	6	8	9	10	12
28	3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	2	3	4	5	6	8	9	10	11
29	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	3	4	5	6	7	8	9	11
30	3333	3322	3311	3300	3289	3279	3268	3257	3247	3237	1	3	4	5	6	7	8	9	10
31	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	2	3	4	5	6	7	8	9	10
32	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8	9
33	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	4	5	6	7	8
34	2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	0	1	2	3	4	5	5	6	7
35	2857	2849	2841	2833	2825	2817	2809	2801	2793	2785	1	2	3	3	4	5	6	7	7
36	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	3	3	4	5	5	6	7
37	2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	2	3	3	4	5	5	6	7
38	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	0	1	2	2	3	4	4	5	6
39	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	2	2	3	4	4	5	6	6
40	2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	5
41	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	2	2	3	3	4	5	5	6
42	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	5	5
43	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	0	1	1	2	2	3	3	4	4
45	2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	1	1	2	2	3	3	4	4	5
46	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	0	1	1	2	2	3	3	4
47	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	2	3	3	4
48	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	1	1	2	2	2	3	3	4	4
49	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	1	2	2	3	3	3
51	1961	1957	1953	1949	1946	1942	1938	1934	1931	1927	1	1	1	2	2	3	3	3	4
52	1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	1	1	1	2	2	3	3	3	4
53	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
54	1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	1	1	1	2	2	2	3	3	3

Reciprocals from 2 to 10 = 0·
„ „ to 100 = 0·0

Reciprocals from 101 to 1000 = 0·00
„ „ 1001 to 9999 = 0·000

NOTE.—Numbers in difference columns to be subtracted, not added.

Reciprocals of Numbers from 1 to 9999.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	1	1	1	2	2	2	3	3	3
56	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	1	1	1	1	2	2	2	3	3
57	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	1	2	2	2	3
58	1724	1721	1718	1715	1712	1709	1706	1704	1701	1698	0	0	1	1	1	1	2	2	2
59	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	1	1	1	2	2	2	2	3	3
60	1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	2	2	2	2	3
61	1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
62	1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	1	1	1	1	2	2	2	2	3
64	1563	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	0	0	1	1	1	1	2	2
65	1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	1	1	1	1	2	2	2	2	2
66	1515	1513	1511	1508	1506	1504	1502	1499	1497	1495	1	1	1	1	1	2	2	2	2
67	1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	0	0	1	1	1	1	2
68	1471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	1	1	1	1	2	2	2	2
69	1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	1	1	1	1	1	2	2	2
70	1428	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	0	0	0	1	1	1	1	1
71	1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	1	1	1	1	1	2	2	2	2
72	1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	0	0	1	1	1	1	1	1
73	1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	0	0	0	0	1	1	1	1
74	1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	0	0	1	1	1	1	1	1
75	1333	1332	1330	1328	1326	1325	1323	1321	1319	1317	0	0	0	0	0	1	1	1	1
76	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	0	1	1	1	1	1	1
77	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	0	1	1	1	1	1
78	1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	1	1	1	1	1	1	2	2
79	1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	1	1	1	1	1	1	2	2
80	1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	0	1	1	1	1	1
81	1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	0	1	1	1	1	1	1	1
82	1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
83	1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	1	1	1	1	1	1	1	2	2
84	1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	0	0	1	1	1	1
85	1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	0	0	0	0	0	0	1	1
86	1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
87	1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	1	1	1	1	1	1	1	1
88	1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	0	1	1	1	1	1	1	1
89	1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	0	0	0	0	1	1	1	1
90	1111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0	0	0	1	1	1	1	1	1
91	1099	1098	1096	1095	1094	1093	1092	1091	1089	1088	0	0	1	1	1	1	1	1	1
92	1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	0	0	0	1	1	1	1	1
93	1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	0	0	0	0	0	0	0	1
94	1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	0	1	1	1	1
95	1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
96	1042	1041	1040	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	0	0	1	1	1
97	1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	1	1	1	1	1	1	1	1	1
98	1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	0	0	1	1	1
99	1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	1	1	1	1	1

Reciprocals from 2 to 10 =0·
,, ,, 11 to 100=0·0

Reciprocals from 101 to 1000=0·00
,, ,, 1001 to 9999=0·000

Note.—Numbers in difference columns to be subtracted, not added.

TABLE OF DOUBLED SQUARE ROOTS FOR

	0	100	200	300	400	500	600	700	800	900	
0	0·000	20·00	28·28	34·64	40·00	44·72	48·99	52·92	56·57	60·00	0
1	2·000	20·10	28·35	34·70	40·05	44·77	49·03	52·95	56·60	60·03	1
2	2·828	20·20	28·43	34·76	40·10	44·81	49·07	52·99	56·64	60·07	2
3	3·464	20·30	28·50	34·81	40·15	44·86	49·11	53·03	56·67	60·10	3
4	4·000	20·40	28·57	34·87	40·20	44·90	49·15	53·07	56·71	60·13	4
5	4·472	20·49	28·64	34·93	40·25	44·94	49·19	53·10	56·75	60·17	5
6	4·899	20·59	28·71	34·99	40·30	44·99	49·23	53·14	56·78	60·20	6
7	5·292	20·69	28·77	35·04	40·35	45·03	49·27	53·18	56·82	60·23	7
8	5·657	20·78	28·84	35·10	40·40	45·08	49·32	53·22	56·85	60·27	8
9	6·000	20·88	28·91	35·16	40·45	45·12	49·36	53·25	56·89	60·30	9
10	6·325	20·98	28·98	35·21	40·50	45·17	49·40	53·29	56·92	60·33	10
11	6·633	21·07	29·05	35·27	40·55	45·21	49·44	53·33	56·96	60·37	11
12	6·928	21·17	29·12	35·33	40·60	45·25	49·48	53·37	56·99	60·40	12
13	7·211	21·26	29·19	35·38	40·64	45·30	49·52	53·40	57·03	60·43	13
14	7·483	21·35	29·26	35·44	40·69	45·34	49·56	53·44	57·06	60·46	14
15	7·746	21·45	29·33	35·50	40·74	45·39	49·60	53·48	57·10	60·50	15
16	8·000	21·54	29·39	35·55	40·79	45·43	49·64	53·52	57·13	60·53	16
17	8·246	21·63	29·46	35·61	40·84	45·48	49·68	53·55	57·17	60 56	17
18	8·485	21·73	29·53	35·67	40·89	45·52	49·72	53·59	57·20	60·60	18
19	8·718	21·82	29·60	35·72	40·94	45·56	49·76	53·63	57·24	60·63	19
20	8·944	21·91	29·66	35·78	40·99	45·61	49·80	53·67	57·27	60·66	20
21	9·165	22·00	29·73	35·83	41·04	45·65	49·84	53·70	57·31	60·70	21
22	9·381	22·09	29·80	35·89	41·09	45·69	49·88	53·74	57·34	60·73	22
23	9·592	22·18	29·87	35·94	41·13	45·74	49·92	53·78	57·38	60·76	23
24	9·798	22·27	29·93	36·00	41·18	45·78	49·96	53·81	57·41	60·79	24
25	10·000	22·36	30·00	36·06	41·23	45·83	50·00	53·85	57·45	60·83	25
26	10·198	22·45	30·07	36·11	41·28	45·87	50·04	53·89	57·48	60·86	26
27	10·392	22·54	30·13	36·17	41·33	45·91	50·08	53·93	57·52	60·89	27
28	10·583	22·63	30·20	36·22	41·38	45·96	50·12	53·96	57·55	60·93	28
29	10·770	22·72	30·27	36·28	41·42	46·00	50·16	54·00	57·58	60·96	29
30	10·954	22·80	30·33	36·33	41·47	46·04	50·20	54·04	57·62	60·99	30
31	11·136	22·89	30·40	36·39	41·52	46·09	50·24	54·07	57·65	61·02	31
32	11·314	22·98	30·46	36·44	41·57	46·13	50·28	54·11	57·69	61·06	32
33	11·489	23·07	30·53	36·50	41·62	46·17	50·32	54·15	57·72	61·09	33
34	11·662	23·15	30·59	36·55	41·67	46·22	50·36	54·18	57·76	61·12	34
35	11·832	23·24	30·66	36·61	41·71	46·26	50·40	54·22	57·79	61·16	35
36	12·000	23·32	30·72	36·66	41·76	46·30	50·44	54·26	57·83	61·19	36
37	12·166	23·41	30·79	36·72	41·81	46·35	50·48	54·30	57·86	61·22	37
38	12·329	23·49	30·85	36·77	41·86	46·39	50·52	54·33	57·90	61·25	38
39	12·490	23·58	30·92	36·82	41·90	46·43	50·56	54·37	57·93	61·29	39
40	12·649	23·66	30·98	36·88	41·95	46·48	50·60	54·41	57·97	61·32	40
41	12·806	23·75	31·05	36·93	42·00	46·52	50·64	54·44	58·00	61·35	41
42	12·961	23·83	31·11	36·99	42·05	46·56	50·68	54·48	58·03	61·38	42
43	13·115	23·92	31·18	37·04	42·10	46·60	50·71	54·52	58·07	61·42	43
44	13·266	24·00	31·24	37·09	42·14	46·65	50·75	54·55	58·10	61·45	44
45	13·416	24·08	31·30	37·15	42·19	46·69	50·79	54·59	58·14	61·48	45
46	13·565	24·17	31·37	37·20	42·24	46·73	50·83	54·63	58·17	61·51	46
47	13·711	24·25	31·43	37·26	42·28	46·78	50·87	54·66	58·21	61·55	47
48	13·856	24·33	31·50	37·31	42·33	46·82	50·91	54·70	58·24	61·58	48
49	14·000	24·41	31·56	37·36	42·38	46·86	50·95	54·74	58·28	61·61	49
50	14·142	24·49	31·62	37·42	42·43	46·90	50·99	54·77	58·31	61·64	50

LORD KELVIN'S STANDARD ELECTRIC BALANCES.

	0	100	200	300	400	500	600	700	800	900	
51	14·283	24·58	31·69	37·47	42·47	46·95	51·03	54·81	58·34	61·68	51
52	14·422	24·66	31·75	37·52	42·52	46·99	51·07	54·85	58·38	61·71	52
53	14·560	24·74	31·81	37·58	42·57	47·03	51·11	54·88	58·41	61·74	53
54	14·697	24·82	31·87	37·63	42·61	47·07	51·15	54·92	58·45	61·77	54
55	14·832	24·90	31·94	37·68	42·66	47·12	51·19	54·95	58·48	61·81	55
56	14·967	24·98	32·00	37·74	42·71	47·16	51·22	54·99	58·51	61·84	56
57	15·100	25·06	32·06	37·79	42·76	47·20	51·26	55·03	58·55	61·87	57
58	15·232	25·14	32·12	37·84	42·80	47·24	51·30	55·06	58·58	61·90	58
59	15·362	25·22	32·19	37·89	42·85	47·29	51·34	55·10	58·62	61·94	59
60	15·492	25·30	32·25	37·95	42·90	47·33	51·38	55·14	58·65	61·97	60
61	15·620	25·38	32·31	38·00	42·94	47·37	51·42	55·17	58·69	62·00	61
62	15·748	25·46	32·37	38·05	42·99	47·41	51·46	55·21	58·72	62·03	62
63	15·875	25·53	32·43	38·11	43·03	47·46	51·50	55·24	58·75	62·06	63
64	16·000	25·61	32·50	38·16	43·08	47·50	51·54	55·28	58·79	62·10	64
65	16·125	25·69	32·56	38·21	43·13	47·54	51·58	55·32	58·82	62·13	65
66	16·248	25·77	32·62	38·26	43·17	47·58	51·61	55·35	58·86	62·16	66
67	16·371	25·85	32·68	38·31	43·22	47·62	51·65	55·39	58·89	62·19	67
68	16·492	25·92	32·74	38·37	43·27	47·67	51·69	55·43	58·92	62·23	68
69	16·613	26·00	32·80	38·42	43·31	47·71	51·73	55·46	58·96	62·26	69
70	16·733	26·08	32·86	38·47	43·36	47·75	51·77	55·50	58·99	62·29	70
71	16·852	26·15	32·92	38·52	43·41	47·79	51·81	55·53	59·03	62·32	71
72	16·971	26·23	32·98	38·57	43·45	47·83	51·85	55·57	59·06	62·35	72
73	17·088	26·31	33·05	38·63	43·50	47·87	51·88	55·61	59·09	62·39	73
74	17·205	26·38	33·11	38·68	43·54	47·92	51·92	55·64	59·13	62·42	74
75	17·321	26·46	33·17	38·73	43·59	47·96	51·96	55·68	59·16	62·45	75
76	17·436	26·53	33·23	38·78	43·63	48·00	52·00	55·71	59·19	62·48	76
77	17·550	26·61	33·29	38·83	43·68	48·04	52·04	55·75	59·23	62·51	77
78	17·664	26·68	33·35	38·88	43·73	48·08	52·08	55·79	59·26	62·55	78
79	17·776	26·76	33·41	38·94	43·77	48·12	52·12	55·82	59·30	62·58	79
80	17·889	26·83	33·47	38·99	43·82	48·17	52·15	55·86	59·33	62·61	80
81	18·000	26·91	33·53	39·04	43·86	48·21	52·19	55·89	59·36	62·64	81
82	18·111	26·98	33·59	39·09	43·91	48·25	52·23	55·93	59·40	62·67	82
83	18·221	27·06	33·65	39·14	43·95	48·29	52·27	55·96	59·43	62·71	83
84	18·330	27·13	33·70	39·19	44·00	48·33	52·31	56·00	59·46	62·74	84
85	18·439	27·20	33·76	39·24	44·05	48·37	52·35	56·04	59·50	62·77	85
86	18·547	27·28	33·82	39·29	44·09	48·41	52·38	56·07	59·53	62·80	86
87	18·655	27·35	33·88	39·34	44·14	48·46	52·42	56·11	59·57	62·83	87
88	18·762	27·42	33·94	39·40	44·18	48·50	52·46	56·14	59·60	62·86	88
89	18·868	27·50	34·00	39·45	44·23	48·54	52·50	56·18	59·63	62·90	89
90	18·974	27·57	34·06	39·50	44·27	48·58	52·54	56·21	59·67	62·93	90
91	19·079	27·64	34·12	39·55	44·32	48·62	52·57	56·25	59·70	62·96	91
92	19·183	27·71	34·18	39·60	44·36	48·66	52·61	56·28	59·73	62·99	92
93	19·287	27·78	34·23	39·65	44·41	48·70	52·65	56·32	59·77	63·02	93
94	19·391	27·86	34·29	39·70	44·45	48·74	52·69	56·36	59·80	63·06	94
95	19·494	27·93	34·35	39·75	44·50	48·79	52·73	56·39	59·83	63·09	95
96	19·596	28·00	34·41	39·80	44·54	48·83	52·76	56·43	59·87	63·12	96
97	19·698	28·07	34·47	39·85	44·59	48·87	52·80	56·46	59·90	63·15	97
98	19·799	28·14	34·53	39·90	44·68	48·91	52·84	56·50	59·93	63·18	98
99	19·900	28·21	34·58	89·95	44·63	48·95	52·88	56·53	59·97	63·21	99
100	20·000	28·28	34·64	40·00	44·72	48·99	52·92	56·57	60·00	63·25	100

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